MAXIMUM LIKELIHOOD ESTIMATION OF BLUR FROM MULTIPLE OBSERVATIONS

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ABSTRACT

A limitation of the existing maximum likelihood (ML) based methods for blur identification is that the estimate of blur is poor when the blurring is severe. In this paper, we propose an ML-based method for blur identification from multiple observations of a scene. When the relations among the blurring functions of these observations are known, we show that the estimate of blur obtained by using the proposed method is very good. The improvement is particularly significant under severe blurring conditions. With an increase in the number of images, direct computation of the likelihood function, however, becomes difficult as it involves calculating the determinant and the inverse of the cross-correlation matrix. To tackle this problem, we propose an algorithm that computes the likelihood function recursively as more observations are added.

1. INTRODUCTION

In actual practice, the blur has to be estimated from the degraded image itself. The earliest work on blur identification concentrated on point spread functions (PSFs), the Fourier transforms of which have zeros on the unit bi-circle [1]. In more recent work, the original image is first modeled as a 2-D autoregressive (AR) process and the identification problem is formulated as a maximum likelihood (ML) problem. Tekalp *et al.* [2] showed that the ML estimation problem could be interpreted as an autoregressive moving average (ARMA) model identification problem. Lagendijk *et al.* formulated blur identification and image restoration as constrained ML estimation problems [3]. An iterative approach called the expectation-maximization (EM) algorithm has been used in [4, 5] to find the ML estimates of the image and blur parameters. In [6], a hierarchial blur identification method based on the EM algorithm is proposed to identify a severe blur. Pavlovic *et al.* [7] propose parametric modeling of the blur in the continuous spatial co-ordinates to identify the extent of the PSF. An overview of the development in image and blur identification under the ML framework is given in [8].

Recently, the recovery of an image from its *multiple*, distorted observations has been receiving much attention. Multiple, blurred views (same or different distortions) of a common object provides information that can be used to advantage in image restoration. In [9], Katsaggelos *et al.* suggest an algorithm that incorporates a number of distorted versions of a signal and results in a restoration error approaching zero with fewer iterations. In [10], Ghighlia develops a scheme for image restoration from multiple, blurred images based on the constrained least squares approach in the frequency domain. Ward [11] considers restoration from differently blurred versions of an image in the presence of noise. In the above schemes, the PSF is assumed to be either fully or partially known. In [12], Subbarao *et al.* recover the original focused image from two blurred images using the spatial domain convolution/deconvolution transform and the Wiener filter.

Existing ML-based methods yield a poor estimate of blur when the blurring is severe [5, 6]. In this paper, we propose a method that uses multiple, blurred views of the original image to obtain an improved estimate of the blur when the relations among the blurring functions are known. Some of the practical cases of interest where the relation between the PSFs is known are depth from defocused images [13], electron microscopy [14] etc. Since, computation of the likelihood function becomes cumbersome as the number of blurred images increases, we propose a method to compute it recursively.

2. ML-BASED BLUR IDENTIFICATION FROM MULTIPLE IMAGES

Let the discrete original image $f(i,j)$ be modeled by a 2-D AR process having coefficients $a(i,j)$, with a causal support and driven by a zero mean homogeneous Gaussian white noise process $v(i,j)$. Let the observed blurred image $g(i,j)$ be modeled as the output of a 2-D linear space-invariant system with point spread function (PSF) $h(i,j)$ and the observation noise $w(i,j)$ be an additive, zero-mean white Gaussian process independent of $v(i,j)$. Under the assumption of circular convolution, we get in the frequency domain, $\overline{F} = (I - \Lambda_A)^{-1}\overline{V}$ and $\overline{G} = \Lambda_H\overline{F} + \overline{W}$ where $\overline{F}$, $\overline{G}$, $\overline{V}$ and $\overline{W}$ are the DFTs of the sequences $f(i,j)$, $g(i,j)$, $v(i,j)$ and $w(i,j)$, respectively. Matrices Λ_A and Λ_H are diagonal with entries that correspond to the DFTs $A(k,l)$ and $H(k,l)$ of the sequences $a(i,j)$ and $h(i,j)$, respectively.

Henceforth, we shall derive mathematical relationships for 1-D signals for notational simplicity. Given M differently blurred versions of the image $f(j)$, we have,

$$\overline{G^i} = \Lambda_{H_i}\overline{F} + \overline{W^i}, \; i = 1, 2 \ldots, M, \qquad (1)$$

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where Λ_{H_i} is a diagonal matrix whose entries are the DFT $H_i(k)$ of the i^{th} PSF $h_i(j)$ and $\overline{W^i}$ is a white Gaussian noise process such that $\overline{W^i}$ and $\overline{W^j}$ are statistically independent for $i \neq j$.

We first examine the joint probability density function (pdf) $p(\overline{G^1}, \overline{G^2}, \ldots, \overline{G^M})$. Notationally, the bar in $\overline{G^i}$ represents the process while G^i is a realization of the process. Let $\overline{G_M} = \left[\overline{G^{1^T}}\ \overline{G^{2^T}} \ldots \overline{G^{M^T}}\right]^T$ and $G_M = \left[G^{1^T}\ G^{2^T} \ldots G^{M^T}\right]^T$. Here, '$T$' represents transpose.

It is straightforward to show that $\overline{G_M}$ is jointly Gaussian. Therefore,

$$p(\overline{G_M}) = \frac{1}{(2\pi)^{\frac{MN}{2}} (\det P_M)^{\frac{1}{2}}} \exp\left(-\frac{1}{2} G_M^H P_M^{-1} G_M\right) ,$$

where E is the expectation operator, the block matrix $P_M = E\left[\overline{G_M}\, \overline{G_M}^H\right] = \left[P^{i,j}\right]$, $i,j = 1,2,\ldots,M$, and $P^{i,j} = E\left[\overline{G^i}\, \overline{G^j}^H\right]$. Here, '$H$' represents the Hermitian operator and N corresponds to the length of the sequence. It can be shown that

$$P^{i,j} = \sigma_v^2 \Lambda_{H_i} (I - \Lambda_A)^{-1} (I - \Lambda_A)^{-H} \Lambda_{H_j}^H + \sigma_w^2\, \delta_{i,j}\, I ,$$

where σ_v^2 and σ_w^2 are the AR model noise and observation noise variances, respectively. It may be noted that $P^{i,j}$ is a diagonal matrix whose k^{th} diagonal element is given by

$$P^{i,j}(k,k) = \frac{H_i(k) H_j^*(k)}{|1 - A(k)|^2} \sigma_v^2 + \sigma_w^2\, \delta_{i,j},\ k = 0,1,\ldots,N-1 . \quad (2)$$

Let the unknown parameters be denoted by the vector $\theta = \{a(j), h_1(j), \ldots, h_M(j), \sigma_w^2, \sigma_v^2\}$. The blur identification problem focuses on estimating the unknown parameters $h_i(m)$, $i = 1,\ldots,M$ from the M noisy and blurred observations. The maximum likelihood estimator of θ is then given by

$$\min_\theta F_M(\theta) \quad \text{where} \quad F_M(\theta) = \log\left(\det P_M\right) + G_M^H P_M^{-1} G_M . \quad (3)$$

Unfortunately, (3) specifies a complicated non-linear optimization problem in several variables (the PSFs, the parameters of the image model, and the noise variance), mainly because of the non-quadratic behavior of $\log\det P_M$. Since an analytic solution for $\hat{\theta}$ cannot be found in general, one has to consider numerical solution strategies like the gradient descent algorithm to minimize $F_M(\theta)$. Low order parametric image and blur models when incorporated into the identification scheme make the identification algorithm applicable to more realistic blurs and improve the identification results. We propose to reduce the number of unknown parameters in θ by assuming that we have some knowledge about the structure of the PSF in different channels. For example, in the scheme on recovery of depth from defocused images [15, 16], the camera blur is usually modeled as a 2-D Gaussian function and for different lens settings, the blur parameter σ_i corresponding to the i^{th} blurred image is related by $\sigma_1 = \alpha_i \sigma_i + \beta_i$, $i = 2,\ldots,M$, where α_i and β_i are known constants.

3. RECURSIVE COMPUTATION OF THE LIKELIHOOD FUNCTION

To compute $F_M(\theta)$, direct evaluation of $\det P_M$ and P_M^{-1} in equation (3) would be very cumbersome for increasing M. In this section, we propose a method that recursively computes $F_M(\theta)$.

The block matrix P_M can be written in partitioned form as

$$P_M = \begin{bmatrix} P_{M-1} & D_M \\ D_M^H & P^{M,M} \end{bmatrix} ,$$

where $D_M^H = \left[P^{M,1} \ldots P^{M,M-1}\right]$. It should be noted that $P^{M,M}$ is always diagonal.

From the partitioned matrix inversion lemma [17], we obtain

$$P_M^{-1} = \begin{bmatrix} A_M & B_M \\ B_M^H & C_M \end{bmatrix}$$

where $C_M = (P^{M,M} - D_M^H P_{M-1}^{-1} D_M)^{-1}$, $B_M = -P_{M-1}^{-1} D_M C_M$ and $A_M = P_{M-1}^{-1} + P_{M-1}^{-1} D_M C_M D_M^H P_{M-1}^{-1}$. It may be noted that C_M is always diagonal. Hence, it is trivially determined. However, block matrices A_M and B_M are not diagonal, in general, and are given by $A_M = \left[A_M^{i,j}\right]$, $i,j = 1,\ldots,M-1$ and $B_M = \left[B_M^i\right]^T$, $i = 1,\ldots M-1$, where $A_M^{i,j}$ and B_M^i are diagonal matrices for all i,j.

Using the determinant lemma for partitioned matrix [17], we get $\det P_M = (\det P_{M-1})\ \det(P^{M,M} - D_M^H P_{M-1}^{-1} D_M) = (\det P_{M-1})\ \det(C_M^{-1})$. Therefore, from equation (3), we obtain

$$F_M(\theta) = \log(\det P_{M-1}) + \log(\det C_M^{-1}) + G_M^H P_M^{-1} G_M . \quad (4)$$

Now, it can be shown that $G_M^H P_M^{-1} G_M = G_{M-1}^H A_M G_{M-1} + G_{M-1}^H B_M G^M + G^{M^H} B_M^H G_{M-1} + G^{M^H} C_M G^M$. By substituting for A_M and using equations (3) and (4) we get the important recursive relation

$$\begin{aligned} F_M(\theta) = F_{M-1}(\theta) \ &+\ \log\det C_M^{-1} + G_{M-1}^H P_{M-1}^{-1} D_M C_M \\ &D_M^H P_{M-1}^{-1} G_{M-1} + G_{M-1}^H B_M G^M + G^{M^H} B_M^H G_{M-1} + \\ &G^{M^H} C_M G^M . \quad (5) \end{aligned}$$

We now proceed to compute the likelihood function for different values of M.

3.1. Computation of $F_1(\theta)$

This case is quite straightforward. From (3),

$$F_1(\theta) = \log\det P_1 + G_1^H P_1^{-1} G_1 .$$

Now, $P_1 = P^{1,1}$. But $P^{1,1}$ is a diagonal matrix. Hence, its inversion is trivial. Therefore,

$$F_1(\theta) = \sum_{k=0}^{N-1} \log\left(P^{1,1}(k,k)\right) + \frac{\left|G^1(k)\right|^2}{P^{1,1}(k,k)} . \quad (6)$$

This expression is exactly the same as the one derived in [8] for the case of a single image. Thus, our formulation provides a general framework in which $M = 1$ is a special case.

3.2. Computation of $F_2(\theta)$

From (5), we have the recursive relation

$$F_2(\theta) = F_1(\theta) \ + \log\det C_2^{-1} + G_1^H P_1^{-1} D_2 C_2 D_2^H P_1^{-1} G_1 \\ + G_1^H B_2 G^2 + G^{2^H} B_2^H G_1 + G^{2^H} C_2 G^2 \ .$$

Since, the matrices involved in the computation are all diagonal, it is easy to show that

$$F_2(\theta) = F_1(\theta) \ + \sum_{k=0}^{N-1} \log\left(\frac{1}{C_2(k,k)}\right) \\ + C_2(k,k)\left|G^1(k)\frac{P^{21}(k,k)}{P^{11}(k,k)} - G^2(k)\right|^2$$

$$\text{where} \quad C_2(k,k) = \frac{1}{\left(P^{2,2}(k,k) - \frac{|P^{2,1}(k,k)|^2}{P^{1,1}(k,k)}\right)}$$

$$\text{and} \quad B_2(k,k) = -\frac{P^{1,2}(k,k)C_2(k,k)}{P^{1,1}(k,k)} \ .$$

Thus, $F_2(\theta)$ is known. Note that the superscript 2 immediately on G corresponds to the FFT of the second observation, and not G raised to its 2^{nd} power.

3.3. Computation of $F_M(\theta)$

One may continue proceeding as above to obtain the general terms in the expression for $F_M(\theta)$. However, the expression becomes unwieldy beyond $M = 2$. Here we provide the steps involved in computing the likelihood function recursively.

Step 1: Initialize $F_1(\theta) = \log\det P_1 + G_1^H P_1^{-1} G_1$. Minimize $F_1(\theta)$ to obtain the estimate $\theta = \theta_1$ and set $M = 2$.

Step 2: Obtain $C_M = \left(P^{M,M} - D_M^H P_{M-1}^{-1} D_M\right)^{-1}$.

Step 3: Obtain $B_M = -P_{M-1}^{-1} D_M C_M$.

Step 4: Calculate the FFT of the M^{th} observation g^M.

Step 5: Compute $F_M(\theta) = F_{M-1}(\theta) + \log\det C_M^{-1} + G_{M-1}^H P_{M-1}^{-1} D_M C_M D_M^H P_{M-1}^{-1} G_{M-1} + G_{M-1}^H B_M G^M + G^{M^H} B_M^H G_{M-1} + G^{M^H} C_M G^M$.

Step 6: Minimize $F_M(\theta)$ using θ_{M-1} as the initial estimate and obtain θ_M.

Step 7: $M \leftarrow M + 1$ and goto step 2.

4. SIMULATION RESULTS

In this section, we present simulation results on blur identification based on the proposed method, under severe blurring conditions. We illustrate the improvement in the estimate of the blur parameter with multiple blurred images. We compare the estimate of blur corresponding to $M = 1$, 2 and 3 distorted observations. Here, we consider the special problem of blur due to camera defocusing where the blurring can be parameterized in terms of the spread σ of a Gaussian function [15]. The relations between the standard deviations of the blurred images are assumed to be known. The order of the AR model was chosen to be 2. The proposed ML based identification algorithm was implemented on the blurred image in the presence of zero-mean, white Gaussian noise that is independent of the image, with $SNR = 40\ dB$ and $10\ dB$, respectively for two separate cases. The cameraman image of size 64×64 pixels was severely blurred by a 2-D Gaussian out-of-focus blur with standard deviation $\sigma_1 = 3.0$. The estimate of σ_1 obtained by using only this blurred image was 1.42 and 1.14 for SNRs of 40 dB and 10 dB, respectively. The estimate was poor, as expected, because of the absence of high frequency content in the image. Next, we generated a second blurred image by blurring the original image with Gaussian functions having standard deviations σ_2 ranging from 1.5 to 4.5. The errors in the estimates of σ_1 corresponding to these values of σ_2 are plotted in Fig. 1. As may be noted from the plot, the estimate of σ_1 improves (and so does its immunity to noise) when σ_2 is either small or large as compared to σ_1. The improvement is better when the second image is more focused than the first. We generated a third blurred image with σ_3 ranging widely from 0.1 to 9.0. We then estimated σ_1 for these values of σ_3 for the cases when $\sigma_2 = 1.5$ and $\sigma_2 = 4.5$. The corresponding estimates are plotted in Figures 2 and 3, respectively. From the plots, we again observe that the estimate of σ_1 improves significantly when the third image is relatively either more focused or more blurred than both the first and the second blurred images. The improvement in the estimate is better when σ_3 is smaller than both σ_1 and σ_2. Also, the estimate is better for $\sigma_2 = 1.5$ as compared to that of $\sigma_2 = 4.5$, as expected. As the third image becomes progressively more focused or blurred, the estimate of σ_1 exhibits a significant immunity to noise.

5. CONCLUSIONS

Existing maximum likelihood based methods for blur identification give poor estimates under severe blurring conditions. In this paper, an ML-based blur identification method that uses multiple blurred versions of the original image has been proposed for improving the estimate of blur. The degree of improvement in the estimate of the blur is dependent on the relative blurring among the images. The more the relative blurring, the better is the estimate. The estimate of blur exhibits better immunity to noise with multiple images. We have also proposed a scheme to find the maximum likelihood function recursively because direct computation of the likelihood function becomes difficult with increasing number of images.

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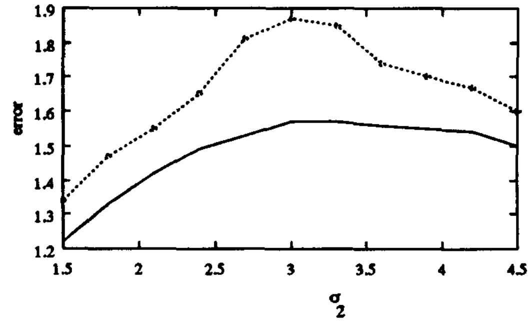

Figure 1. Magnitude of the error in the estimate of σ_1 for various values of σ_2 for the cameraman image. The continuous and the dotted lines correspond to two different $SNRs$ of 40 dB and 10 dB, respectively.

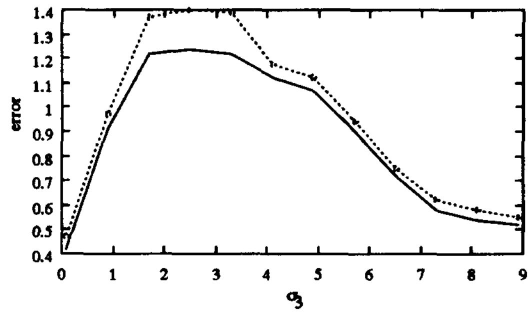

Figure 2. Magnitude of the error in the estimate of σ_1 for various values of σ_3 for the cameraman image. The value of σ_2 is 1.5.

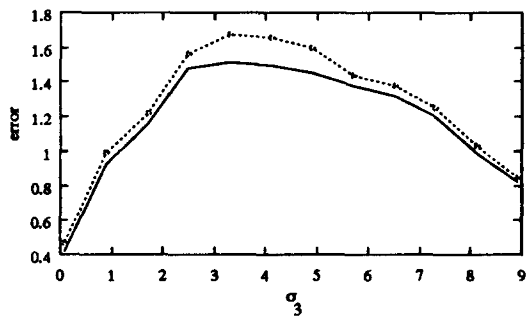

Figure 3. The same plot as in Fig. 2, but computed for $\sigma_2 = 4.5$.