ON DESCRIPTION OF IMPULSIVE NOISE REMOVAL USING PWL FILTER MODEL

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ABSTRACT

The classical method for impulsive noise removal in image signals consists of two steps, i.e., a step for impulse detection and a step for estimation of the pixels corrupted by impulsive noise. In this paper, we show that the step of impulse detection can be described very suitably by piecewise-linear (PWL) functions which explains intuitively the process of detection as a partition of the signal domain space. A PWL filter model is then proposed. In such a filter model, we can use either a linear or a nonlinear subfilter to estimate the corrupted pixels. For the latter, using a median subfilter yields a multi-level PWL filter. The filter model is simulated and its excellent result is compared to that of a median filter.

I. INTRODUCTION

Impulsive noise arises commonly during generation and transmission of signals. It appears as discrete isolated pixel variations that are not spatially correlated [1], and can degrade the quality of the signal strongly. An impulsive noise-corrupted image can be described by

$$x_{ij} = \begin{cases} s_{ij} & , \quad 1-p, \\ s_{ij} + i_{ij} & , \quad p, \end{cases} \tag{1}$$

where $\{s_{ij}\}$ is the useful image signal, $\{i_{ij}\}$ is the disturbing impulse and p is the probability of impulse occurrence. The corrupted signal $\{x_{ij}\}$ is a piecewise-linear (PWL) function of two random variable sequences $\{s_{ij}\}$ and $\{i_{ij}\}$ and is a function with strong nonlinearity. Thus, for the removal of such noises, the use of nonlinear techniques is suggested.

There are existing two major kinds of nonlinear techniques for impulsive noise removal. The first kind is the median filter (or median based filters) that, due to its statistical and robustness properties [2], is very effective in removing impulsive noise. It suffers, however, from the known fact that it not only removes impulses but also smoothes simultaneously signal details. Such an effect becomes more serious with the increase of the filter window size which, in case of larger noise probability, must be used. To improve the property of the median filter, a structure such as the L-filter was proposed and provides a compromise between median and moving average filters [2]. A further structure is based on the use of hybrid-median filters [3].

The other kind, which is called the classical method, is based on the concept that only the corrupted pixels should be estimated; those pixels that are not corrupted are taken directly as the filter output [1]. Such a concept, compared to that of the median filter, is expected to provide a more satisfactory result because only the corrupted pixels are processed and the pixels that are not corrupted are preserved. In doing such a processing, two steps, i.e., impulse detection and pixel estimate, are need to be carried out.

Recently, a new class of nonlinear filters, called canonical piecewise-linear (CPWL) filters, were also found to be very efficient in removing impulsive noise [4]. In [4] a general CPWL function was used as the input/output function of a mapping network and it was trained for removing such noises. In studying the internal mechanism of such filters in impulsive noise removal, we find that the result is not accidental. In fact, an impulsive noise-corrupted signal has a piecewise property, as already implied in (1). This implication suggests that a piecewise function and hence also a PWL function are very suitable for this task.

Based on this observation, the objective of this paper is to give an explicit PWL filter model for impulsive noise removal. It is a model performing both impulse detection and pixel estimation and it is the PWL implementation of the classical method. In principle, it can use either a linear or a nonlinear subfilter. For the latter, it represents a more general case. By applying median filter as subfilter, the input/output function of the filter corresponds to a multi-level PWL function (for multi-level PWL functions, refer to [6]). Both simulation and implementation of the filter model are discussed.

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II. FILTER MODEL

A continuous CPWL function $f : D \subset \mathbb{R}^n \to \mathbb{R}$ is expressed as

$$f(\mathbf{x}) = a + \langle \mathbf{b}, \mathbf{x} \rangle + \sum_{q=1}^{Q} c_q \mid \langle \boldsymbol{\alpha}_q, \mathbf{x} \rangle + \beta_q \mid \qquad (2)$$

where $\mathbf{x} \in D, \mathbf{b} \in \mathbb{R}^n$, $a, c_q \in \mathbb{R}$ and $\boldsymbol{\alpha}_q \in \mathbb{R}^n, \beta_q \in \mathbb{R}$; $\langle \cdot \rangle$ denotes the inner product and $\langle \boldsymbol{\alpha}_q, \mathbf{x} \rangle + \beta_q = 0, q = 1, 2, \cdots, Q$ are called boundary equations. The domain space D is partitioned by the boundaries, i.e., by Q $(n-1)$-dimensional hyperplanes into a finite number of regions, in each of which f takes the form of a linear function. By viewing f as a filter function, $\mathbf{x}$ then corresponds to the filter support and n is the support length.

For describing the process of impulse detection using CPWL functions, we first consider the domain partition of an impulsive noise-corrupted signal. According to the piecewise property of such a signal, the signal amplitude can be divided into three levels, i.e., if a positive impulse exists, the amplitude is greater than the local mean; if a negative impulse exists, the amplitude is smaller than the local mean; and if no impulse exists, the amplitude equals approximately the local mean. Therefore, the signal falls randomly into one of the three regions, in each of which it shows various behaviours. In doing a domain partition for such signals, we need only two nonlinear terms in (2) since in general two hyperplanes partition the domain space into four regions. If the two hyperplanes are furthermore parallel, the domain is then partitioned into three regions which satisfies our requirement. Therefore, the first function model is stated as

$$\begin{aligned} y_{ij} &= \sum_{k_1,k_2 \setminus i,j} \alpha_{k_1k_2} x_{i-k_1,j-k_2} \\ &- \frac{1}{2} \mid \sum_{k_1,k_2 \setminus i,j} \alpha_{k_1k_2} x_{i-k_1,j-k_2} + \beta - x_{ij} \mid \\ &+ \frac{1}{2} \mid \sum_{k_1,k_2 \setminus i,j} \alpha_{k_1k_2} x_{i-k_1,j-k_2} - \beta - x_{ij} \mid \qquad (3) \end{aligned}$$

where $\alpha_{k_1k_2}, \beta \in \mathbb{R}$ and $\beta > 0$. Here, we use y_{ij} instead of f to emphasize that it is the output of a filter. It is easy to see that the two hyperplanes are parallel with offsets $-\beta$ and β, respectively. The linear term $m_{ij} := \sum_{k_1,k_2 \setminus i,j} \alpha_{k_1k_2} x_{i-k_1,j-k_2}$ corresponds to the estimate of the pixel at (i,j). The three partitioned regions for this filter model are

$$\begin{aligned} R_1 &: \quad m_{ij} + \beta < x_{ij}, \\ R_2 &: \quad m_{ij} - \beta < x_{ij} < m_{ij} + \beta, \\ R_3 &: \quad m_{ij} - \beta > x_{ij} \end{aligned}$$

In each of these regions, one can examine that (3) satisfies

$$y_{ij} = \begin{cases} m_{ij} + \beta, & \mathbf{x} \in R_1, \\ x_{ij} \quad , & \mathbf{x} \in R_2, \\ m_{ij} - \beta, & \mathbf{x} \in R_3, \end{cases} \qquad (4)$$

which corresponds to the cases where a positive impulse is detected, no impulse is detected and a negative impulse is detected, respectively. β plays hereby the role of a threshold for detection. The model in (4) is very simple; however, it is locally biased since the threshold β contributes also to the output. This is due to that the decision point for detection is taken on the boundaries. In fact, for the case that the x_{ij} equals approximately to m_{ij}, a small transit region is hoped for the decision maker. Therefore, we derive a more suitable model

$$\begin{aligned} y_{ij} &= m_{ij} - \frac{\beta + \delta}{2\delta} \mid m_{ij} + \beta - x_{ij} \mid \\ &+ \frac{\beta + \delta}{2\delta} \mid m_{ij} - \beta - x_{ij} \mid \\ &- \frac{\beta}{2\delta} \mid m_{ij} - \beta - \delta - x_{ij} \mid \\ &+ \frac{\beta}{2\delta} \mid m_{ij} + \beta + \delta - x_{ij} \mid \qquad (5) \end{aligned}$$

with $\beta, \delta > 0$, which uses four parallel hyperplanes. In this model, the domain space is partitioned into five regions

$$\begin{aligned} R_1 &: \quad m_{ij} + \beta + \delta < x_{ij}, \\ R_2 &: \quad m_{ij} + \beta < x_{ij} < m_{ij} + \beta + \delta, \\ R_3 &: \quad m_{ij} - \beta < x_{ij} < m_{ij} + \beta, \\ R_4 &: \quad m_{ij} - \beta - \delta < x_{ij} < m_{ij} - \beta, \\ R_5 &: \quad m_{ij} - \beta - \delta > x_{ij} \end{aligned}$$

and the filter output becomes

$$y_{ij} = \begin{cases} m_{ij} & , \ \mathbf{x} \in R_1, \\ m_{ij} + \frac{\beta}{\delta}(m_{ij} + \beta + \delta - x_{ij}), & \mathbf{x} \in R_2, \\ x_{ij} & , \ \mathbf{x} \in R_3, \\ m_{ij} - \frac{\beta}{\delta}(m_{ij} + \beta + \delta - x_{ij}), & \mathbf{x} \in R_4, \\ m_{ij} & , \ \mathbf{x} \in R_5 \end{cases} \qquad (6)$$

Accordingly, by introducing two transit regions R_2, R_4, the local mean estimate becomes locally unbiased. Here, β is still the detection threshold, and δ is a parameter used to control the width of the transit regions and is usually chosen to be a small value.

In (5), m_{ij} is the local mean with respect to x_{ij} and is estimated by a linear subfilter which is nested in the filter function model. The more accurate the estimate is, the better the filtering result will be. Theoretically, the estimate can be done not only by a linear subfilter but also by a nonlinear subfilter. In case of a linear subfilter, an adaptive method can provide a good result since the filter coefficients can trace the change in signal and noise distributions. In considering the situation where besides x_{ij} some of the pixels inside the filter window may also be corrupted by impulses, the estimated value m_{ij} requires further informations on signal pixels for determining the filter weighting coefficients, especially for high impulse probability. In this case, a nonlinear subfilter may also be prefered. Using e.g. a median filter (or an order statistic based filter) as subfilter, the influence of the 'bad' pixels inside the filter window can be reduced effectively and a satisfactory estimate value can be obtained.

By taking a median filter or an order statistic based filter as subfilter, the input/output function of the filter becomes a multi-level PWL function. To see how this is true, we review the result on the equivalent of order statistic based filters to PWL filters [5]. The function of a comparator (i.e., a 2nd order sorting network) which performs the comparison and ordering of two data, say x_1, x_2, can be described by

$$x_{(1)} = \frac{1}{2}(x_1 + x_2 + | x_1 - x_2 |), \quad (7)$$

$$x_{(2)} = \frac{1}{2}(x_1 + x_2 - | x_1 - x_2 |), \quad (8)$$

where $x_{(1)}, x_{(2)}$ are the ordered data. Obviously, this expression is a special case of (2) and thus a CPWL function. For a sorting network of $(n-1)$th order (without the pixel at (i,j)), its input/output function is just an $(n-1)$-level deep composite function of CPWL functions or an $(n-1)$-level PWL function. Therefore, by using a median filter as subfilter, the input/output function of the proposed filter model corresponds to an n-level PWL function.

III. SIMULATION RESULT AND MODEL IMPLEMENTATION

Using the derived filter model, we give now a simulation to it and compare the result with that of a median filter. As the input signal, we take a $256 \times 256 \times 8$bit image which is corrupted by impulsive noise with amplitude of 100 and 10% occurrence probability, and is shown in Fig. 1(b). The original image is given in Fig 1(a). The simulation is done by using a median filter and the proposed filter, respectively, yielding the results in Fig. 1(c) and (d), where both filters have a 3×3 window. One sees that both filters remove impulses and remain edge information effectively. The proposed filter model, furthermore, preserves signal details much better than the median filter. This excellent result can trace back to the concept that if no impulses are detected, the output of the filter takes the original signal and if impulses are detected, the output value is estimated. In contrast, the median filter replaces not only the corrupted pixels but also the non-corrupted pixels with their local median means.

Hereby the parameter β and δ control the detection threshold and the detection transit regions, respectively. Theoretically, they should be chosen in such a way that the least mean-square error or the mean-absolute error is minimized. Their optimal values are expected different from one image to another, depending on the statistical properties of the signal and noise. For the example given here, $\beta = 70$ and $\delta = 1$ are used, which provided a very satisfactory result. The mean-square and mean-absolute errors measured for the noisy image are MSE= 999, MAE= 9, for the median filter MSE= 109, MAE= 5 and for the proposed filter MSE= 46, MAE= 1.

A further advantage of the given filter model is its simplicity in implementation. In fact, since a median filter is mathematically equivalent to a PWL filter, we can in turn use the implementation technique of median filters for the implementation of the latter if a simple implementation for the given filter function is available. Here, it is easy to see that (5) can be arranged into

$$\begin{aligned} y_{ij} &= \frac{1}{2}(m_{ij} + \beta + x_{ij} - | m_{ij} + \beta - x_{ij} |) \\ &+ \frac{1}{2}(m_{ij} - \beta + x_{ij} + | m_{ij} - \beta - x_{ij} |) - x_{ij} \\ &\frac{\beta}{\delta} \quad \{\frac{1}{2}(m_{ij} + \beta + x_{ij} - | m_{ij} + \beta - x_{ij} |) \\ &+ \frac{1}{2}(m_{ij} - \beta + x_{ij} + | m_{ij} - \beta - x_{ij} |) \\ &+ \frac{1}{2}(m_{ij} - \beta - \delta + x_{ij} - | m_{ij} - \beta - \delta - x_{ij} |) \\ &+ \frac{1}{2}(m_{ij} + \beta + \delta + x_{ij} + | m_{ij} + \beta + \delta - x_{ij} |) \\ &- 2x_{ij} - 2m_{ij}\} \end{aligned} \quad (9)$$

By comparing the terms in bracket $(\cdot)$ with (7) and (8), it can be seen that they are just the operations of a comparator. Therefore, the whole filter model, including the median subfilter, can be implemented as shown in Fig. 2, in which one multiplier is used. As a comparison, a direct implementation of (5) requires two multipliers.

IV. CONCLUSION

A PWL filter model has been proposed which describes intuitively the detection process in impulsive noise removal as a partition of the domain space. The proposed model is a CPWL filter if a linear subfilter is used and

(a)

(b)

(c)

(d)

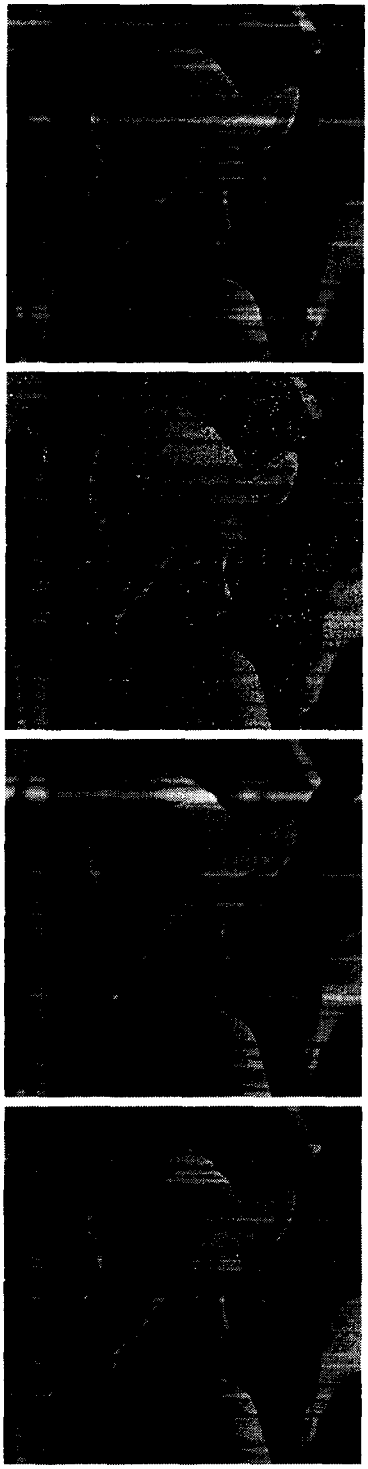

FIGURE 1. (a) Original image, (b) Impulsive noise-corrupted image with 10% impulse probability, (c) filtered image using a 3×3 median filter, (d) filtered image using the proposed filter model with a 3×3 window.

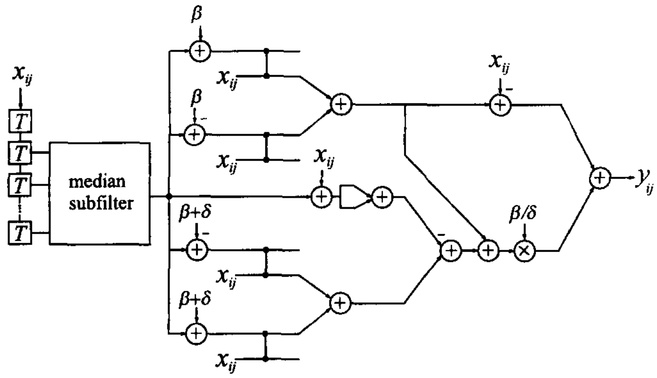

FIGURE 2. Implementation of the proposed filter model using comparators.

is a multi-level PWL filter if a median subfilter (or an order statistic based subfilter) is used. Using the filter model, a signal is filtered according to its behaviour in the partitioned domain by a corresponding local filter function. Simulation shows that such a filter model gives a very satisfactory result in removing impulsive noise while preserving signal details. Its implementation is simple by making profit from the implementation of a median filter.

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