MAGNETIC RESONANCE IMAGE RECONSTRUCTION FROM NON-EQUIDISTANTLY SAMPLED DATA

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ABSTRACT

In this paper, we consider the problem of magnetic resonance (MR) image reconstruction from non-uniformly sampled data acquired by echo-planar imaging (EPI) and spiral scan imaging (SSI) techniques. In EPI, mismatches in the timings of the odd and even echoes collected with readout gradients of alternating polarity will result in "$N/2$" ghosting in the reconstructed images. We propose a new method using a calibration data set to correct this mismatch and hence to calculate accurate k-space trajectories. In order to reconstruct images at real-time speeds, we utilize a high speed optoelectronic device to perform two-dimensional discrete Fourier transform (DFT). Images reconstructed from EPI data are presented to demonstrate that our method can successfully suppress the "$N/2$" ghosting and provide good contrast at real-time speeds. Image reconstructed from SSI data is also presented to show that our method can provide better sharp features and more details in the reconstructed images.

1. INTRODUCTION

The relatively long scan times with the traditional MR systems restrict the range of magnetic resonance imaging (MRI) applications and can lead to image artifacts from patient motion during the scan. Echo-planar imaging, proposed by Mansfield in 1977 [2], is a method by which the MRI data are collected extremely fast in comparison to conventional MRI. This is accomplished by using rapidly oscillating gradients that permit the spatial encoding of the body and acquisition of the data in a single excitation. With EPI, a single image can be acquired in scan times as short as $20-100$ msec [3] and hence whole-brain functional imaging becomes possible.

The k-space position at any time is determined by the time integral of the applied gradient. In conventional MRI, the applied gradients are constant, hence the analog-to-digital converter (ADC) captures the signal uniformly through the k-space. This makes it straightforward to apply the inverse fast Fourier transform (FFT) and to reconstruct the image. With EPI and SSI, this simplicity is lost. They sample data along time-varying k-space trajectories with the same ADC rate, resulting in sampled points being non-equidistantly spaced. In order to take advantage of the great computational speed offered by the FFT, the data must be resampled onto a Cartesian grid. The method currently in use for reconstruction from non-equidistantly sampled data is using gridding with an appropriate windowing function, typically, a Kaiser-Bessel window function [4], [5]. Gridding process is time consuming and can potentially degrade the reconstructed image quality. The most direct and optimal way for image reconstruction from time-varying k-space data is to use direct DFT [6] which is too slow for real-time reconstruction. In this work, to achieve real-time image reconstruction, we use a high-speed optic device – **ImSyn**^{TM} [1] to perform the 2D DFT. Another issue, with EPI, is that since the odd and even echoes are collected with readout gradients of alternating polarity, there are mismatches of echo centers. Without correction, this leads to "$N/2$" ghosting propagating along the phase-encoding direction by half of the field of view in the reconstructed image. We propose a new method to correct this mismatch and hence to calibrate the k-space trajectory by using a calibration data. The results show that although gridding and FFT can suppress "$N/2$" ghosting, without calibration, a geometric distortion will exist in the reconstructed images. We show that by using the calibration data set and a high speed optoelectronic device, we can successfully reconstruct EPI and SSI images at real-time speeds. In the next section, we present an overview of ultra-fast EPI and SSI imaging techniques, introduce our methodology in section 3, and present results showing the successful application of the method in section 4.

2. ULTRA-FAST MR IMAGING TECHNIQUES

MR imaging has become one of the most important diagnostic tools in modern medicine over the last decade. The rapid success and general acceptance of MRI stem from the fact that it provides flexibility in imaging and permits design of techniques that emphasize soft tissue contrast between normal and abnormal tissues. While MRI can produce images with excellent spatial resolution and contrast, the relatively long scan times can lead to image artifacts from patient motion during the scan. The challenge to obtain high-quality MR images with short acquisition time has motivated the development of ultra-fast imaging techniques. These ultra-fast imaging techniques shorten the scan time to milliseconds, and therefore reduce motion artifacts. Furthermore, with short scan times, ultra-fast-imaging can do precise motion detection and measurement, functional brain imaging, and certain types of cardiac imaging.

2.1. Echo-planar Imaging

EPI is an ultra-fast imaging technique by which the MR data are acquired in a single excitation rather than N separate excitations. Comparing Figs. 1 (a) and 2 (a), we can see that the first half of the echo planar pulse sequence is essentially identical to a standard spin echo pulse sequence. The second half of the pulse sequence is where it significantly differs from a standard spin echo acquisition. The readout (frequency) gradient oscillates rapidly from positive to negative to form a train of gradient echoes. Typical readout gradient waveforms are trapezoidal (as shown in Fig. 2 (a)) or sinusoidal. Each echo in this "echo train" is phase encoded differently by phase encode "blips" that occur on the phase axes. This type of echo planar acquisition is referred to as *blipped EPI* [7].

Fig. 2 (b) illustrates how the k-space data are collected. At the beginning of the echo train, the acquisition com-

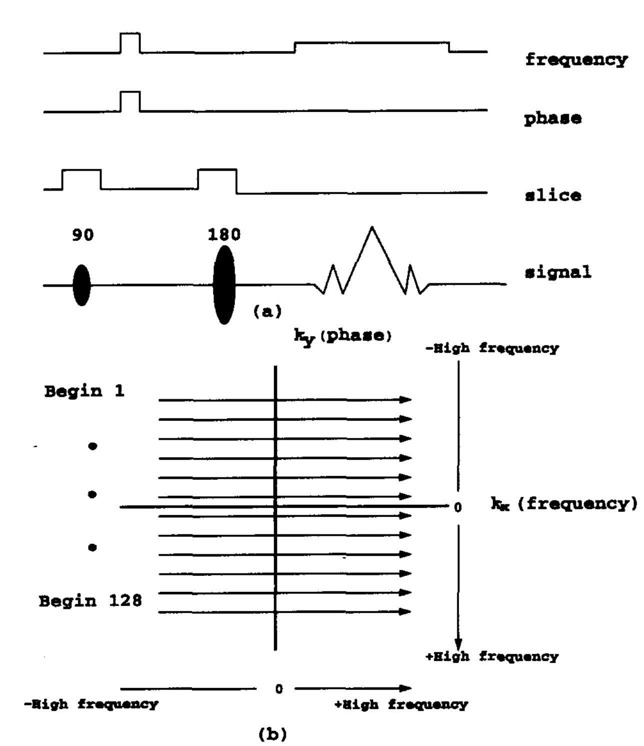

Figure 1. (a) Spin echo pulse sequence for single slice and (b) its associated k-space diagram

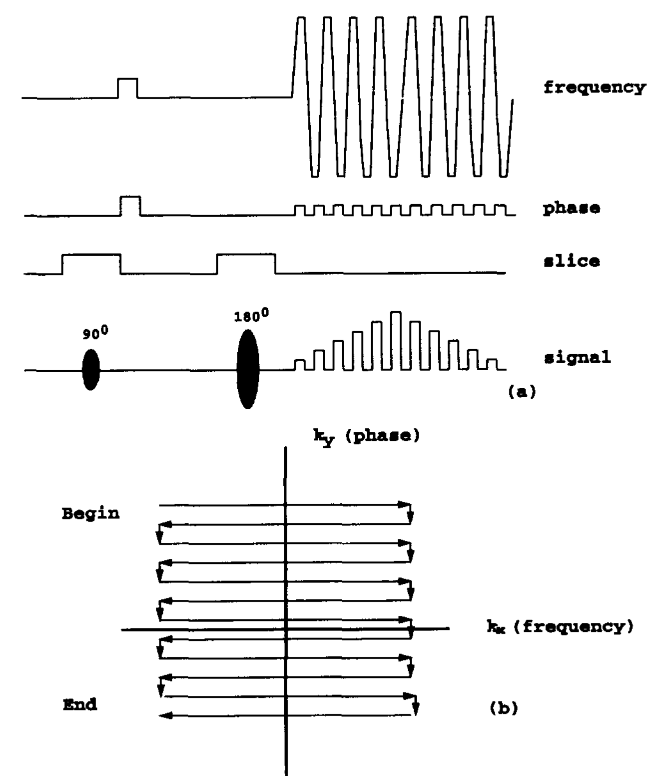

Figure 2. (a) Echo-planar pulse sequence and (b) its associated k-space diagram

mences from the top left of k-space and proceeds to sweep over and down, as indicated by the arrows. Each oscillation of the readout gradient corresponds to one line in k-space and each phase encode blip corresponds to transition from one line to the next.

2.2. Spiral Scan Imaging

Although single-shot echo planar imaging is capable of achieving high temporal resolution, it requires special purpose hardware that is generally not available on most conventional MR systems. An alternative for overcoming these hardware limitations is to collect the data for each image in a relatively small number of interleaves, such as in spiral scan imaging [5]. Also, N interleaved spiral can improve the signal to noise ratio by $\sqrt{N}$ [5]. Data points are sampled along a spiral trajectory and generally do not fall on a Cartesian grid.

The most commonly used spiral is Archimedean spiral. Based on the fact that the radius at any point on an Archimedean spiral is proportional to the cumulative angle traced out to that point [5], a spiral k-space trajectory can be written as:

$$\mathbf{k}(t) = k_x(t) + ik_y(t) = A\tau(t)e^{i\omega\tau(t)} \qquad (1)$$

where τ is a function of time, A and ω are constants. It is known that the k-space trajectory is the time integral of the applied gradients $\left(\mathbf{k}(t) = \int_0^t \mathbf{G}(\tau)d\tau\right)$, thus, spiral gradients can be generated by using $\mathbf{G}(t) = \frac{d\mathbf{k}(t)}{dt}$ and setting $\tau(t) = t$ leads to the constant-angular-velocity spiral [8], with the gradients $\mathbf{G}(t) = iA\omega te^{i\omega t}$. Setting $\tau(t) = \sqrt{t}$ leads to the constant-linear-velocity spiral [5] with the gradients $\mathbf{G}(t) = i\frac{1}{2}A\omega\sqrt{t}e^{i\omega\sqrt{t}}$.

3. IMAGE RECONSTRUCTION FROM NON-EQUIDISTANTLY SAMPLED DATA

At time t, the detected magnetic resonance signal in k-space is given by:

$$S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x,y)e^{-j2\pi\left(k_x(t)x+k_y(t)y\right)}dx\,dy \qquad (2)$$

where $\rho(x,y)$ is the unknown density function of excited spins and $k_x(t)$ and $k_y(t)$ denote the k-space trajectory in the spatial frequency domain. The k-space position $(k_x(t), k_y(t))$ in MRI at any time is determined by the time integral of the applied gradient. In conventional MRI, the applied gradients are constant, hence the signal is captured uniformly by the ADC through the k-space. Thus, FFT can be easily applied to reconstruct the image. With EPI and SSI, this simplicity is lost. Data are sampled along time-varying trajectories with the same ADC rate, resulting in non-equidistantly sampled data in k-space. If the data are sampled at time $t = t_0, t_1, \ldots$ along the time-varying trajectories $k_x(t)$ and $k_y(t)$, the spin density at point (x,y) is estimated by [6]:

$$\rho(x,y) = \sum_i S(t_i)e^{j2\pi\left(k_x(t_i)x+k_y(t_i)y\right)}w(t_i) \qquad (3)$$

where $w(t_i)$ is the weighting function to compensate for the varying density of the data points in k-space. The method currently in use for image reconstruction from non-equidistantly sampled data is gridding and then using inverse FFT. Since the data points sampled along the accurate k-space trajectory are in fractional coordinates, gridding can potentially degrade the image quality. Hence, the most direct and optimal way for image reconstruction from time-varying sampled data is to use DFT with an appropriate weighting function. Since the traditional implementation of DFT on a computer is too slow for real-time reconstruction, we use an optoelectronic device to perform 2D DFT after proper weighting and calibration of the data for reconstruction at real-time speeds.

3.1. EPI Reconstruction with Calibration Data

In EPI proposed by Mansfield [2] or the method proposed by Macovski [9], it was shown that the coordinates in EPI can be separated. In this study, we assume that $k_y(t)$ is equally spaced and consider $k_x(t)$ to be sinusoidally varying:

$$k_x(t) = k_{max}\cos(\Omega t) \qquad (4)$$

where k_{max} is proportional to the amplitude of the gradient and Ω denotes the modulation frequency. This is a reasonable assumption for the common types of EPI imaging. Since the sample density in Eq.(4) increases as $|k_x(t)|$ increases [10], the weighting function for EPI is chosen as:

$$w(t_i) = |\frac{dk_x(t)}{dt}|_{t=t_i} = |\Omega k_{max}\sin(\Omega t_i)|, \qquad (5)$$

to compensate for the variation of the sample points density along the k_x trajectory.

With EPI, odd and even echoes are collected with readout gradients of alternating polarity, which often results in mismatches of echo centers. Without correction, this leads to "$N/2$ ghosting" propagating along the phase-encoding direction by half of the field of view in the reconstructed image as shown in Fig. 4 (a). We propose a method to correct these mismatches and hence to calibrate the k-space trajectory by using the calibration data. A calibration data set is a special raw data set measured by switching off the phase encoding gradient. Ideally, the echo center in each view should follow a straight line. However, in reality, the positions of the individual echoes of the calibration data differ from each other as shown in Fig. 3 (a). The same echo shifting is presented in the corresponding phase encoded EPI data set. Using the calibration data set, we calculate the proper $k_x(t)$ and apply it to the phase encoded EPI data set to reconstruct images.

We propose the following general expression for $k_x(t)$:

$$k_x(t) = -\frac{N}{\pi}\cos(\frac{\pi}{N}t + \Delta) + A + Bt + Ct^2 \qquad (6)$$

where N is the total number of points in each view, Δ is a phase shift, and $A + Bt + Ct^2$ is the quadratic term to represent the nonlinearity of the trajectory. To calculate the echo center of each view, we define the half power point of each view as the midpoint of the integral of the signal's power. Since the true half power point can be a fractional point, we interpolate N points of power into $10 \times N$ points at each view. Based on the fact that the echo center or the peak of the signal at every view should happen at $k_x(t) = 0$, the parameters $\Delta, A, B,$ and C are calculated by using nonlinear least squares fitting of the k-space data [11]. After data calibration, the echo center of the odd and even lines are perfectly aligned as shown in Fig. 3 (b).

The main steps of the algorithm can be summarized as follows:

1. Start with the first view $M = 0$.
2. Calculate the power for each point of the view M.
3. Find the true half power point of the signal at view M.
4. $M = M + 1$.
5. Repeat steps 2 to 4 until $M =$ last view.
6. Use curve fitting procedure to calculate the phase shift and the quadratic term in $k_x(t)$ given in Eq. (6).
7. Multiply each data point by the weighting function given in Eq. (5).
8. Perform 2D inverse DFT to obtain the MR image.

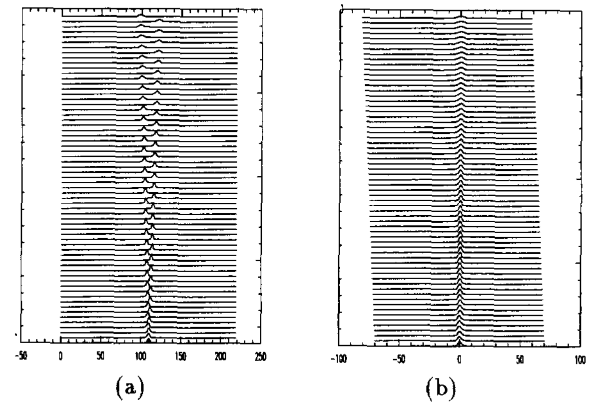

Figure 3. k_x trajectory for EPI calibration data: (a) original $k_x(t)$; and (b) corrected $k_x(t)$

3.2. Spiral Scan Imaging Reconstruction

As we discussed in the previous section, the gradients $\mathbf{G}(t) = G_x(t) + iG_y(t)$ are time-varying in both the x and y directions. It is shown that the behavior of the point spread function (PSF) is controlled by the density of the sample points $\mathbf{k}(t_j)$ in the two-dimensional spatial frequency domain [12]. Therefore, to achieve the sinc-function-like PSF, the weighting function $w(t)$ must be proportional to $|\mathbf{k}(t)|$. For spiral scan data, we choose the weighting function as [5]:

$$w(t) = |\mathbf{G}(t)| \cdot |\sin[\arg\{\mathbf{G}(t)\} - \arg\{\mathbf{k}(t)\}]| \qquad (7)$$

where $|\mathbf{G}(t)| = \sqrt{\frac{\partial^2 k_x(t)}{\partial t} + \frac{\partial^2 k_y(t)}{\partial t}}$, and arg denotes the angle of x and y coordinates at time t. The first term in Eq. (7) compensates for the varying k-space velocity and the second term compensates for the increased density of the spiral near the origin.

4. RESULTS AND DISCUSSION

A raw data set of 220×64 points collected on a Picker 1.5 Tesla scanner is reconstructed into a 256×256 image. To compare the image reconstructed by our method, we first implemented reconstruction by gridding and FFT. This is the method currently in use to resample non-equidistantly k-space onto a rectilinear grid and then to reconstruct the image by using inverse FFT. The data samples are convolved with a continuous Kaiser-Bessel window function [13] (we choose length $L = 3$, and the free parameter $\beta = \frac{\pi L}{2.0}$) and resampled onto a Cartesian grid. The image is obtained by dividing the transform of the window function by the inverse FFT of the resampled data [14]. This image is shown in Fig. 4 (b). The image reconstructed by using the calibration data as explained in the previous section and the conventional DFT is shown in Fig. 4 (c). As we can see from Figs. 4 (b) and 4 (c), both methods can successfully suppress "$N/2$ ghosting" with good tissue contrast. With the calibration data set, we can perfectly align the echo centers of the odd and even views of the raw data set and thus calculate the correct k-space trajectory. This results in the cerebral mid line of the brain on the reconstructed images as shown in Fig. 4 (c). On the other hand, the cerebral mid line tilts from the center line of the image (as shown in Fig. 4 (b)) by a few degrees resulting in geometric distortion when the gridding method is used without k-trajectory calibration.

To reconstruct images at real-time speeds, we perform the 2D DFT by utilizing a high speed optoelectronic device – **ImSyn**^{TM} [1] and obtain a 256×256 from the same data set in less than one second. Fig. 4 (d) shows that the

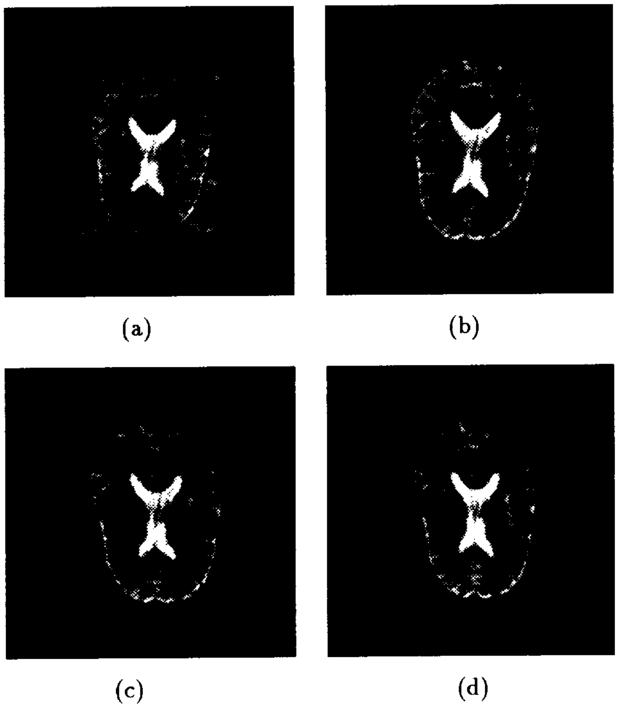

Figure 4. Images reconstructed from EPI data: (a) with "$N/2$ ghosting"; (b) by gridding + FFT; (c) by calibration + conventional DFT; and (d) by calibration + optical DFT

current **ImSyn**^{TM} prototype can reconstruct images with equivalent anatomical definition at real-time speeds.

The results shown in Fig 4 are implemented on a Sun Sparc Station 2. To reconstruct a 220×64 EPI raw data into a 256×256 image, gridding and FFT requires 21 seconds and gridding with conventional DFT 21 minutes 18 seconds. The optoelectronic device **ImSyn**^{TM}, on the other hand, transforms a 220×64 data into a 256×256 complex matrix in 0.3 seconds. Considering calibration and I/O of **ImSyn**^{TM}, approximately 15 seconds are required to reconstruct the same EPI image with 220×64 EPI raw data into a 256×256 image.

We also utilize a data set acquired by SSI technique to emphasize the importance of using exact k-space information for image reconstruction from non-uniformly sampled data. The data set used is provided by the National Institute of Health (NIH), Bethesda, MD. It contains 5400 sampled points and is to be reconstructed into a 128×128 image. Images reconstructed by using gridding and FFT and DFT are shown in Fig. 5. Since the raw data set contained only a single inter-leaf spiral, the overall spatial resolution is low. However, as shown in Fig. 5 (b) that the image reconstructed by using direct DFT provides sharper features and more details.

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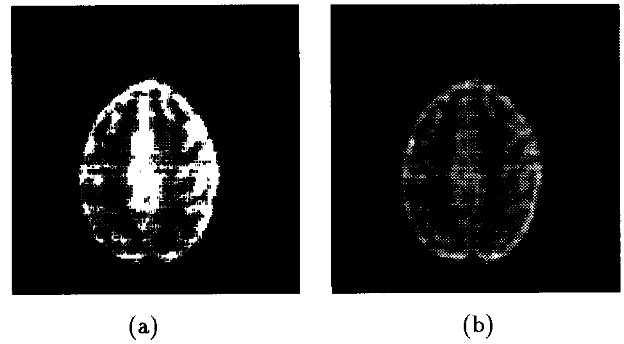

Figure 5. Images reconstructed from spiral scan imaging data: (a) by gridding + FFT; and (b) by conventional DFT

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