HYBRID IMAGE COMPRESSION WITH IMPLICIT FRACTAL TERMS

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ABSTRACT

The performance of any block based image coder can be improved by applying fractal terms to selected blocks. Two novel methods are used to achieve this. Firstly the coder determines whether a local fractal term will improve each image block by examining its rate/distortion contribution, so that only beneficial fractal terms are used. Secondly, the decoder deduces the offset parameters for the local fractal transform from the basis functions alone, by inferring the dominant edge position, so that no offset information is required. To illustrate the method, we use a quadtree decomposed image with a truncated DCT basis. Using a standard test image, the proportion of the picture area enhanced by fractals decreases from 16.1% at 0.6 bpp to 8.1% at a high compression ratio of 80:1 (0.1bpp). The fractal terms contribute less than 5% of the compressed code in all cases. The PSNR is improved slightly, and edge detail is visually enhanced.

1. BACKGROUND

To compress an image, define an Iterated Function System (IFS)[1-5] of order N to be $W = \{w_k;\ k = 1, \ldots, N\}$, where the w_k are contraction mappings, each defined on a subset A_k of the image support. The attractor of W is a non-overlapping tiling of the image, as in Figure 1. A fractal function $f(x, y)$, is then defined which approximates the brightness $g(x, y)$. An image block taken from the location A_k is referred to as the parent and an image block taken from $w_k(A_k)$ is referred to as the child. For each tile the function is specified by a recursive mapping v_k such that

$$f(w_k(x, y)) = v_k(x, y, f(x, y)) \quad \text{for } (x, y) \text{ in } A_k. \qquad (1$$

In this work we use mappings of the form

$$v_k(x, y, f) = p_k(x_0 + \delta x,\ y_0 + \delta y) + e_k \tilde{f}_k(\delta x, \delta y) \qquad (2$$

where (x_0, y_0) is the bottom left corner of A_k and $p_k(x_0 + \delta x,\ y_0 + \delta y) = \sum_{i=1}^{n} c_i\, b_i(\delta x, \delta y)$, is an approximation by basis functions $\{b_i\}$, e_k is the single fractal coefficient

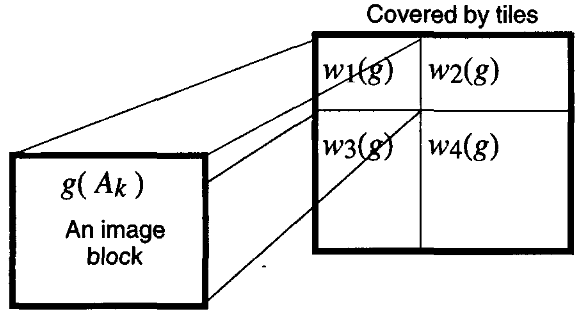

Figure 1. Fractal transforms apply contraction mappings of parent blocks onto child blocks.

and $\tilde{f}_k$ is the parent block $f_k(\delta x, \delta y) = f(x_0 + \delta x, y_0 + \delta y)$ orthogonalized with respect to the basis using

$$\tilde{f}_k = f_k - \sum_{i=1}^{n} \langle b_i, f_k \rangle\, b_i \qquad (3$$

$$\text{where} \quad \langle b_i, f_k \rangle = \iint_{A_k} f_k(x, y)\, b_i(x, y)\, dx\, dy \qquad (4$$

and the basis functions are normalized by $\langle b_i, b_i \rangle = 1$

To solve, the known image $g(x, y)$ is used in place of the unknown fractal function $f(x, y)$ and the approximation is known to be valid by the Collage Theorem [1].

The process is fractal because of the self-similarity inherent in v_k. The mappings form an ensemble of functions which, when iterated or otherwise rendered [5], form an approximation to the image. Usually the tiling of the image is by square or rectangular child blocks, and it is often assumed that p_k is a simple brightness level. Much work has concentrated on reducing the complexity of searching for the best parent to map onto each child [6, 7].

An alternative approach uses more complex basis functions [7, 8] and restricts or even eliminates searching. Such an approach is the Bath Fractal Transform (BFT) [9, 10] with which a pre-determined tiling without searching gives the greatest accuracy at a given compression ratio, when used with a quadratic basis. In combination with the Accurate Fractal Rendering Algorithm (AFRA) [5], the BFT has been used for real time fractal video [11].

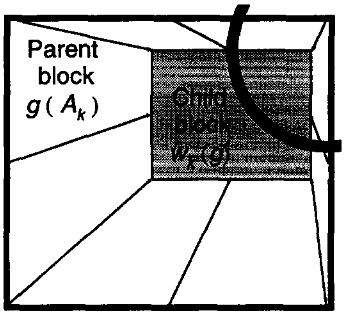

Figure 2. A local fractal transform, with the child block inside the parent block. For a strong edge, the parent/child relationship can be deduced from the basis function.

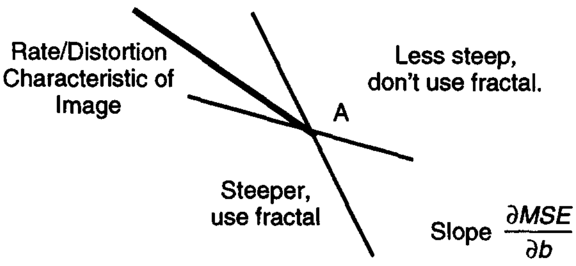

Figure 3. A block is selected for fractal enhancement by the terminal slope of the rate/distortion characteristic.

2. IMPLICIT FRACTALS

In this work we improve the performance of non-searching fractal compression by allowing any relative position of child block and parent block provided the child is local to the parent. We start from the observation that fractal terms enhance edges in images provided the alignment of the edges in the mapping is correct. To exploit this, we determine the dominant edge of the child block from the basis functions alone, using a mathematical model of the edge. We position the parent block so that the edge passes through it in the same relative position, as illustrated by Figure 2. The method applies to any choice of basis functions and to any image partition.

Once the coder has calculated the basis coefficients for a child block, the same process of edge determination and matching to find the parent can be carried out as will be used in the decoder. The coder can then decide whether using a fractal term will improve the rate/distortion characteristic of the image. The compressed code contains only the fractal coefficient e_k, because the offset of the child within the parent will be computed by the decoder from the same information that was used by the coder.

The coder froms a non-fractal approximation to all or part of the image by any coding method. The code for any image block adds Δb bits to the total image code, and increases its total MSE by ΔMSE. If the slope of the rate distortion curve is determined numerically by the coder as it compresses a partition of the image, then the terminal slope $\partial MSE/\partial b$ is know quite accurately, point A in Figure 3. The coder can then examine each image block to determine whether a fractal term will improve the compression. It is assumed that the terminal slope will not be altered appreciably by this process.

If a fractal is used, it will contribute further bits, Δb_{frac} and improve the image MSE by ΔMSE_{frac}. The fractal term is beneficial in rate/distortion terms if and only if

$$\frac{\Delta MSE_{frac}}{\Delta b_{frac}} < \frac{\partial MSE}{\partial b} \qquad (5$$

(i.e. is more negative at point A in Figure 3.)

as illustrated by Figure 3. The estimate of Δb_{frac} can be completely accurate, even if entropy coding is used. The collage theorem [1, 2] would normally be used to estimate ΔMSE_{frac}, which is not completely accurate although bounds on its accuracy can be computed. The gradient is also more difficult to compute if the basis functions and the fractal component are not orthogonal.

3. IMPLEMENTATION AND EVALUATION

To evaluate the method, for $p_k(x, y)$ we used a DCT basis limited to the 6 terms C_{00}, C_{01}, C_{02}, C_{10}, C_{11} and C_{20}. C_{00} was quantized to 7 bits, and the other coefficients to 6 bits, with a fixed Huffman table derived from a test set of images. An image is partitioned into 32 x 32 pixel blocks, and within each partition a quadtree structure was formed so that the basis approximation MSE was distributed as evenly as possible over the image. Because it seldom occurs that a fractal term is used with the initial partition, it is acceptable to carry out the fractal selection only as blocks are split.

Given the DCT coefficients, we can classify a block as being predominantly horizontal or predominantly vertical by comparing $|C_{10}|$ to $|C_{01}|$. To apply the implicit fractal, we search a 2D table using the edge model, which gives the edge location as a function of the ratios

$r_{xx} = \left|\frac{C_{20}}{C_{10}}\right|$ and $r_{yx} = \left|\frac{C_{01}}{C_{10}}\right|$ for a vertical edge and similarly $r_{yy} = \left|\frac{C_{02}}{C_{01}}\right|$ and $r_{xy} = \left|\frac{C_{10}}{C_{01}}\right|$ for horizontal.

It can be shown that these ratios are independent of the intensity on either side of the edge according to the model used, Figure 4. The location is reflected horizontally and/or vertically according to the signs of the ratios.

The fractal coefficient is then found by solving equation 2 for e_k. By the Collage Theorem, $f(x, y)$ is approximated by the original parent block $g(x, y)$, orthogonalized with respect to the basis [1, 7]. In our experiments, using a

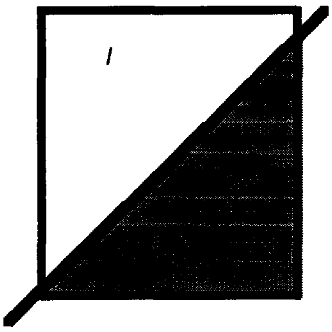

Figure 4. A simple model of an edge, in which l and r are the flat intensities on either side of the edge.

Huffman coded fractal coefficient e_k adds less than 2 bits to the code of each block in which it is used. Because the decoder can determine the fractal offset vectors from the basis approximation, the fractal terms are very efficiently coded.

Figure 5 shows the improvement in edge definition that can be obtained in a synthetic image. The implicit fractal method improves all orientations of the edge by computing the optimum parent location from the decoded basis function.

In Table 1 we list the results of coding the test image Gold Hill over a range of compression ratios. Figure 6 shows clearly the improvement in edge detail obtained in the Lenna image where the implicit fractal enhancement is applied.

4. DISCUSSION AND CONCLUSIONS

We have introduced a technique for applying fractal transforms in combination with other image coding methods. It could be used to improve any image coding system in which the original and approximated images are available to the coder, including wavelet compression. Once an image has been coded by the basis approximation, one can examine any partition of the image to decide where a fractal term will improve the rate/distortion characteristic.

We have incorporated a fractal term in the test examples only where the L2 measured rate/distortion performance is not degraded according to our prediction. While the PSNR is slightly improved over the whole image, in the blocks actually coded with a fractal the improvement will be several times larger, and the visual quality contributed by greater edge definition can be striking, as the examples show. This suggests that the condition for inclusion of a fractal term might be too severe, if the objective is optimum visual quality. It might be useful, for example, to select fractal enhancement for blocks with significant edges even if the PSNR is slightly degraded.

The implicit fractal method is particularly powerful because the offset information can be determined by the decoder from its reconstruction of the basis approximation. Over an entire picture, if only a proportion of blocks are fractally enhanced at an average compression penalty of 2 bits per block, the overall cost per pixel will be negligible.

bpp	PSNR Basis Approx	PSNR Centred Child	PSNR Implicit fractal	Fractal % of Picture Area	Fractal terms bpp
0.6	31.39	31.50	31.53	16.1	.028
0.5	30.74	30.88	30.90	16.1	.123
0.4	30.01	30.16	30.18	15.4	.018
0.3	29.15	29.33	29.35	14.8	.014
0.2	28.09	28.28	28.30	13.0	.009
0.1	26.60	26.70	26.71	8.1	.004

Table 1. Implicit fractal applied to the standard test image Gold Hill.

5. ACKNOWLEDGEMENTS

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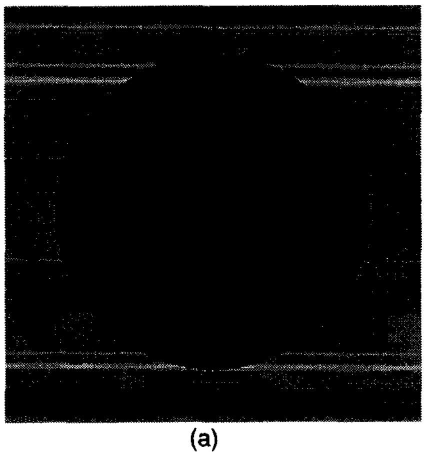
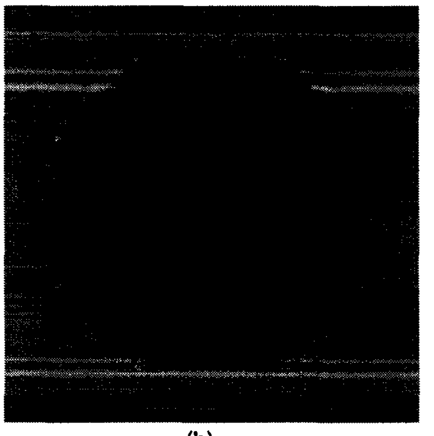
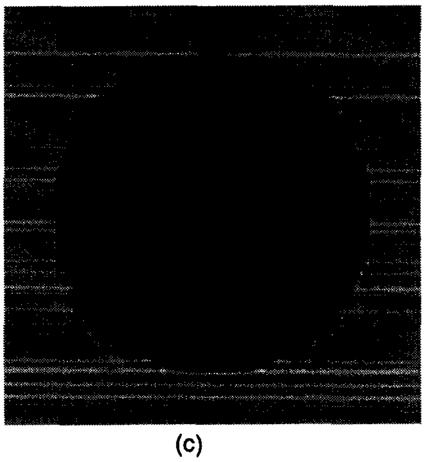


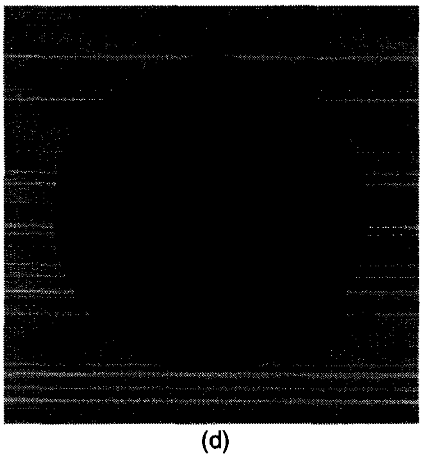
(a)



(b)

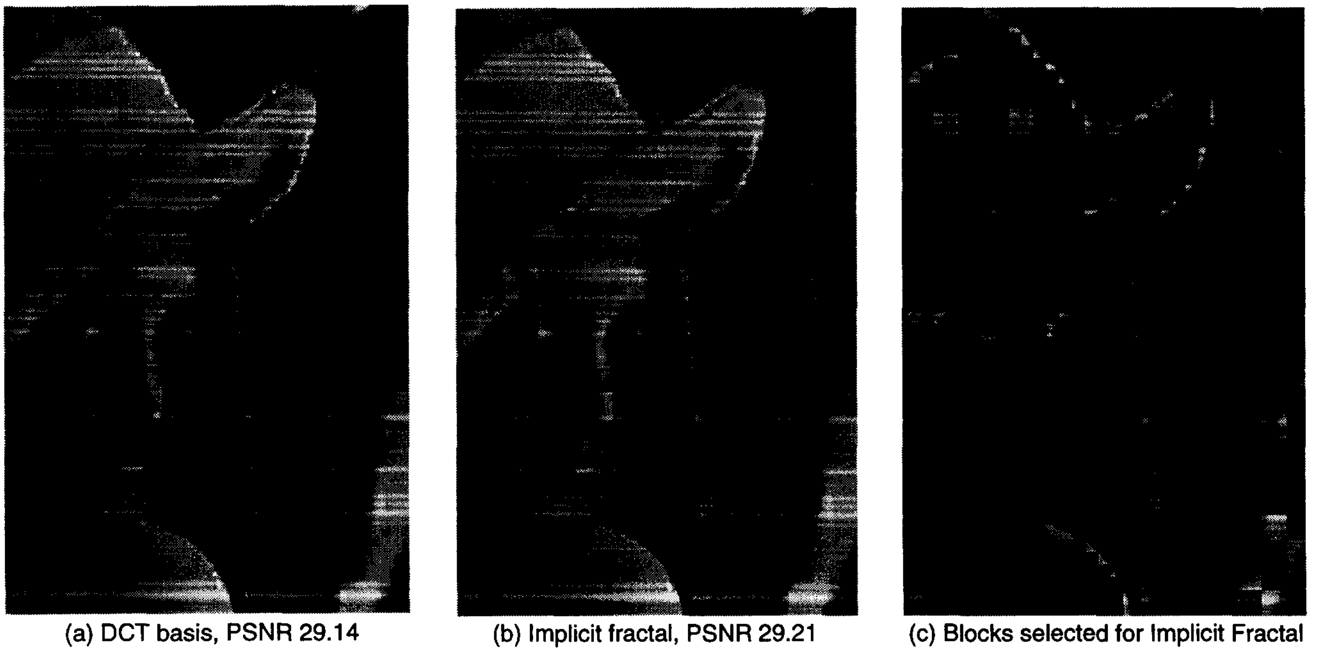


(c)



(d)

Figure 5. Edge enhancement with implicit fractals, fixed 16x16 blocks. (a) Original. (b) Basis approximation. (c) Local fractal with child centred on parent. (d) Local fractal with implicit alignment.



(a) DCT basis, PSNR 29.14 (b) Implicit fractal, PSNR 29.21 (c) Blocks selected for Implicit Fractal

Figure 6. Detail at 0.2 bpp. The PSNR difference is small, but the implicit fractal improves edges visually.