JOINTLY OPTIMAL CLASSIFICATION AND UNIFORM THRESHOLD QUANTIZATION IN ENTROPY CONSTRAINED SUBBAND IMAGE CODING

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ABSTRACT

A method for coding a source modeled by an infinite Gaussian mixture distribution is proposed. The source is first split into N classes. The samples of each class are then quantized by an infinite-level uniform threshold quantizer followed by an entropy coder designed for each class. The problem of joint optimization of this system's rate distortion performance is first solved theoretically, assuming an exponential mixing density. A comparison to a system optimal for high rates, using one common quantizer for all classes, showed that for a fixed distortion the rate was reduced by 11–12% at low rates for a fixed $N = 5$. A subband image coder, using the optimum theoretical parameter values was simulated. The resulting coder has high performance and low complexity.

1 INTRODUCTION

In recent years subband coding has become popular for image compression. The main idea behind subband coding is first to decompose the image by a filter bank, followed by quantization of the subband signals and possibly entropy coding of the resulting quantization indexes. It was shown in [1] that for a uniform 16 band decomposition scheme, there was negligible interband correlation and except for the lowest frequency subband, the intraband correlation was small. Thus, DPCM is commonly used to code the lowest frequency band [2, 3], while zero-memory scalar or vector quantization have usually been applied to code the higher subbands [1, 2, 4].

However, inspection of the subbands reveals that most of the energy is concentrated in areas which corresponds to edge activity in the original image. Some of the approaches which exist to exploit the local characteristics of the imagery are spatially adapting filter banks [5], spatially adaptive quantization [2, 6], and the zerotree algorithm [7]. Another approach is based on adaptive classification of the subbands followed by class-wise entropy constrained coding [8, 9, 10, 11].

This work belongs to the latter class of approaches. In contrast to [9, 10, 11], where the classification and coding processes are optimized separately, we propose a method for *joint* optimization of these tasks. In addition, the proposed method does not employ high rate approximations to find the optimal classification scheme like in [8, 9, 10, 11].

In this paper, we provide a mathematical formulation, solution, and analysis of the joint problem of optimal classification, uniform threshold quantization (UTQ), and entropy coding based on a Gaussian mixture distribution model of the subband signal statistics. UTQ has proved to perform very close to the optimal entropy constrained quantizers for a wide class of memoryless sources, and within 0.3 bits per sample from the rate-distortion lower bound at all rates [12].

This paper is organized as follows: An infinite mixture distribution source model is introduced in Section 2. Then in Section 3, a system for coding this source is given and the coder optimization problem is stated. In Section 4, the solution of this problem is found. A subband image coder employing the theoretical parameters is simulated in Section 5.

2 SOURCE MODEL

Consider designing a subband image coder. First, we need an appropriate model for the subband signal statistics. In [13] it was shown that a suitable statistical model for the subband signals is a memoryless infinite Gaussian mixture distribution. Furthermore, an exponential mixing density was shown to be a reasonable model for the variance statistics. Hence, the following two-dimensional probability density function (pdf) is introduced to characterize the subband signal statistics:

$$p_{X,\Sigma^2}(x,\sigma^2) = p_{\Sigma^2}(\sigma^2) \cdot p_{X|\Sigma^2}(x|\sigma^2) = \lambda e^{-\lambda\sigma^2} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}, \quad (1)$$

where X and Σ^2 are the stochastic variables for the amplitude and the variance of the subband signals, respectively, and x and σ^2 are the corresponding parameters. The mixing density, $p_{\Sigma^2}(\sigma^2)$, is exponential with parameter λ, and the component density, $p_{X|\Sigma^2}(x|\sigma^2)$, is Gaussian $\sim \mathcal{N}(0,\sigma^2)$. It can be shown that $E[X^2] = 1/\lambda$.

3 PROBLEM FORMULATION

The source which was introduced in Section 2 is composed of an infinite number of Gaussian distributions. To minimize the coder mismatch when coding these densities, sources with approximately the same statistics are coded with the same coder.

The multiple entropy coder system shown in Figure 1 is used to code the subband signals. The subband signal, X, is first classified into N classes according to its variance[1], such that the samples with variance in the interval: $[\sigma_i^2, \sigma_{i+1}^2)$, $i \in \{0, 1, \ldots, N-1\}$, belong to class i. The samples belonging to class $i \in \{0, 1, \ldots, N-1\}$, are then quantized by a mid-tread UTQ, $Q_i(\cdot)$, with quantizer step size Δ_i, and finally, the quantizer indexes are coded by an entropy coder, $E_i(\cdot)$, matched to the class statistics.

Ideal entropy coders are assumed. This is not an unrealistic assumption since there exist arithmetic coders which perform very close to the first order entropy of memoryless sources [14].

[1] In practice, the variance is estimated by computing the mean square value of blocks of subband samples, whereby each block is classified according to its variance estimate.

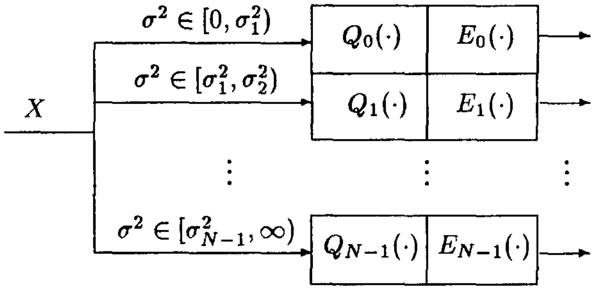

Figure 1. Multiple entropy coder system based on classification and uniform threshold quantization.

Let H_N be the first order overall average entropy of the multiple entropy coder system after quantization, and D_N the corresponding mean square error (MSE) distortion using N quantizers.

Thus, the coding problem for the system in Figure 1 can be formulated as follows:

$$\text{minimize } H_N(\sigma_1^2, \sigma_2^2, \ldots \sigma_{N-1}^2, \Delta_0, \Delta_1, \ldots, \Delta_{N-1}),$$
$$\text{subject to} \tag{2}$$
$$D_N(\sigma_1^2, \sigma_2^2, \ldots, \sigma_{N-1}^2, \Delta_0, \Delta_1, \ldots, \Delta_{N-1}) = D_c,$$

where $\sigma_0^2 = 0$, $\sigma_N^2 = \infty$, and D_c is the target distortion. The optimum performance is found by solving the problem in Equation 2 *jointly* for the optimal quantizer step sizes, Δ_i, and the variance decision levels, σ_i^2.

The overall average entropy is found as the expected entropy of the N classes, and is given by:

$$H_N = \sum_{i=0}^{N-1} \int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) R_i(\sigma^2) d\sigma^2, \tag{3}$$

where $R_i(\sigma^2)$ is the rate of the output of quantizer $Q_i(\cdot)$ with an input pdf $p_{X|\Sigma^2}(x|\sigma^2)$, and is given by:

$$R_i(\sigma^2) = \sum_{k=-\infty}^{\infty} \left\{ \int_{\frac{2k-1}{2}\Delta_i}^{\frac{2k+1}{2}\Delta_i} p_{X|\Sigma^2}(x|\sigma^2) dx \cdot \right. \tag{4}$$
$$\left. \log_2 \frac{\int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) d\sigma^2}{\int_{\frac{2k-1}{2}\Delta_i}^{\frac{2k+1}{2}\Delta_i} \int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{X,\Sigma^2}(x, \sigma^2) d\sigma^2 dx} \right\}.$$

Quantization of the infinite Gaussian mixture distribution with the system shown in Figure 1, gives a MSE distortion, D_N, given by:

$$D_N = \sum_{i=0}^{N-1} \left\{ \int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) \cdot \right. \tag{5}$$
$$\left. \sum_{k=-\infty}^{\infty} \int_{\frac{2k-1}{2}\Delta_i}^{\frac{2k+1}{2}\Delta_i} (x - r_{k,i})^2 p_{X|\Sigma^2}(x|\sigma^2) \, dx \, d\sigma^2 \right\},$$

where $r_{k,i}$ is the kth representation level, and is chosen as the centroid of the pdf of class i, $p_i(x)$. Hence, $r_{k,i}$ is given by:

$$r_{k,i} = \frac{\int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) \int_{\frac{2k-1}{2}\Delta_i}^{\frac{2k+1}{2}\Delta_i} x \cdot p_{X|\Sigma^2}(x|\sigma^2) \, dx \, d\sigma^2}{\int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) \int_{\frac{2k-1}{2}\Delta_i}^{\frac{2k+1}{2}\Delta_i} p_{X|\Sigma^2}(x|\sigma^2) \, dx \, d\sigma^2}, \tag{6}$$

and $p_i(x)$ by:

$$p_i(x) = \frac{\int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{X,\Sigma^2}(x, \sigma^2) d\sigma^2}{\int_{\sigma_i^2}^{\sigma_{i+1}^2} p_{\Sigma^2}(\sigma^2) d\sigma^2}. \tag{7}$$

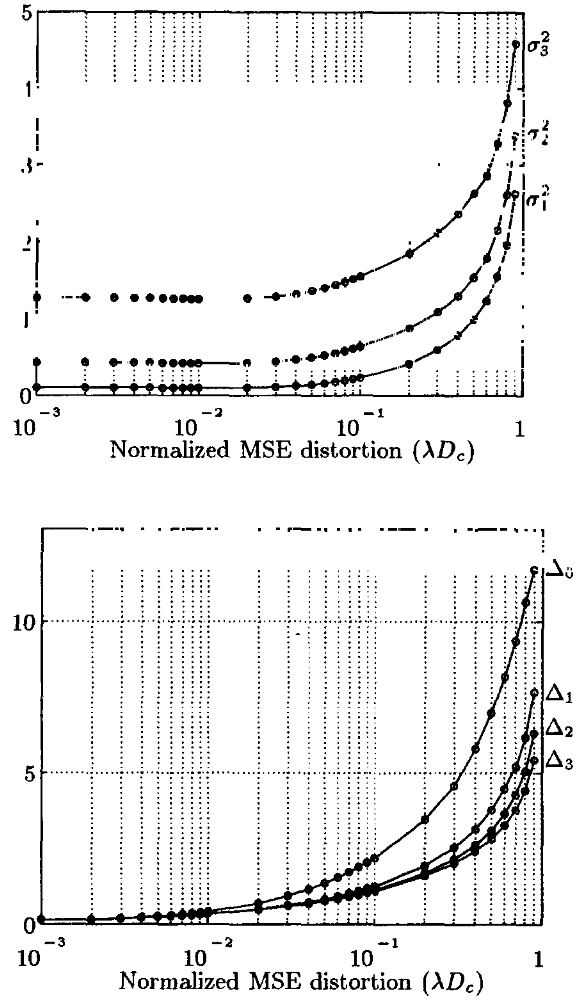

Figure 2. Top: Optimal variance decision levels (σ_i^2) as a function of the normalized MSE distortion. Bottom: Optimal quantizer step sizes (Δ_i) as a function of the normalized MSE distortion. In both figures the number of entropy coders equals $N = 4$.

4 THEORETICAL SOLUTION

The problem stated in Equation (2) was solved by an iterative numerical algorithm. The optimal variance decision levels, σ_i^2, for $N = 4$ are given in Figure 2 (top), and the corresponding optimal quantizer step sizes, Δ_i, are shown

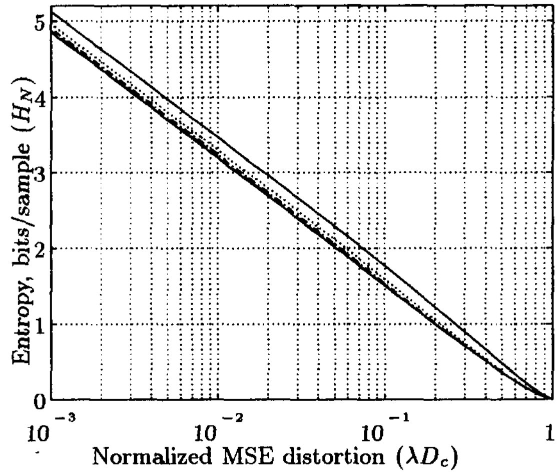

Figure 3. Theoretical rate distortion performance. $N=1$ (——), $N=2$ (······), $N=3$ (–·–·–), $N=4$ (– – –), and $N=5$ (——) entropy coders.

in Figure 2 (bottom). The results show that $\Delta_i \geq \Delta_{i+1}$ for a constant distortion. Thus, samples in classes with a large variance should be quantized with a higher resolution than samples with a lower variance. From the figure, notice that the quantizer step sizes are approximately equal for all quantizers at low distortions. This is in accordance with the theoretical proof given in [15].

Figure 3 displays the theoretical rate distortion performance of the system using $N = 1, 2, 3, 4$, or 5 entropy coders. A substantial rate improvement is obtained by using multiple entropy coders. For example at a normalized MSE distortion of 0.001^2, a bit rate reduction of 0.27 bits per sample is experienced by using five entropy coders instead of only one.

4.1 Theoretical comparison with one quantizer

In [15] a similar theoretical model was investigated, where the optimization problem was solved using a high rate approximation, i.e. one common uniform threshold quantizer was used for all classes. It was proved that this was optimal for high rates. Table 1 shows the difference of the rate distortion performance of the reference system and the proposed system.

Number of quantizers	Normalized distortion λD_c				
	0.0004	0.04	0.4	0.7	0.9
	Rate (bits/sample)				
1	3.86	2.22	0.55	0.17	0.037
5	3.86	2.18	0.51	0.15	0.033

Table 1. Rate distortion performance of the systems when using one common quantizer and five different quantizers. $N = 5$ classes are used in all results.

From Table 1, it is clear that for very low rates the rate is reduced by 11–12% for a fixed distortion D by allowing different quantizers for each class. For high rates there is nothing to gain by using different quantizers in each class.

5 SUBBAND IMAGE CODER

5.1 Implementation of the subband coder

The theoretical results were adopted in a subband image coder. The images were first decomposed by a parallel 8×8 FIR filter bank, jointly optimized to maximize the coding gain and to minimize ringing and blocking artifacts [16]. The lowest frequency subband was coded by a fixed rate DPCM coder using 6 bits per sample. All the other frequency bands were coded by the method described in Section 3. The variance estimates were found by computing the mean square value of 4×4 blocks. Each block was classified according to the optimal variance decision levels, σ_i^2, $i \in \{0, 1, \ldots, N\}$ and applied to a quantizer with step size Δ_i. Finally, the quantizer indexes were coded class-wise in arithmetic coders optimized for the class statistics. The classification information, telling the decoder which class each block belongs to, was sent as side information. All the necessary side information was included in the simulation results. The variance decision levels and quantizer step sizes were scaled according to the total variance and total standard deviation of the image, respectively.

[2] A normalized distortion of 0.001 is equivalent to a signal-to-noise ratio (SNR) of $10 \cdot \log_{10}((1/\lambda)/D_c) = 30$ dB.

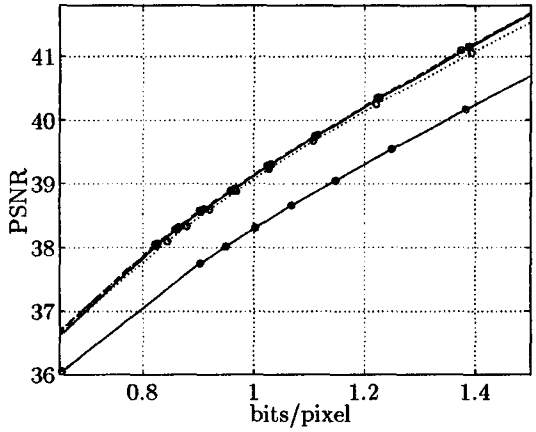

Figure 4. Simulation results. Image: Luminance component of *Lenna*. $N=1$ (——), $N=2$ (······), $N=3$ (–·–·–), $N=4$ (– – –), and $N=5$ (——) entropy coders.

Figure 4 shows the performance of the practical image coder using $N = 1, 2, 3, 4$, and 5 classes for coding the 512×512 luminance component of *Lenna*. The performance difference by using different numbers of entropy coders is approximately equal to the theoretical results, however, the main difference is that in the practical image coder the side information has to be accounted for. When the number of classes increases, the rate distortion performance increases up to a certain point. Beyond this point the performance decreases when the number of classes are increased further. In Figure 4 the performance is higher by using four entropy coders than five entropy coders.

In the theoretical formulation, the side information was not accounted for. Furthermore, it is emphasized the statistical model given in Section 2 is not perfect, and especially the exponential mixing density assumption is not entirely correct. It is clear from the results that the proposed classification scheme gives a significant improvement of the image coder system. The rate is reduced by 15–20% for a fixed PSNR by using four classes instead of one.

5.2 Comparison with one quantizer

Figure 5 shows a comparison between an image coder using one common quantizer and a coder which uses four quantizers.

With one entropy coder the two systems are identical. With more than one entropy coder the systems have different performance. In Figure 5, four entropy coders are used. From the figure it is seen at a bit rate of 1.4 bits/pixel

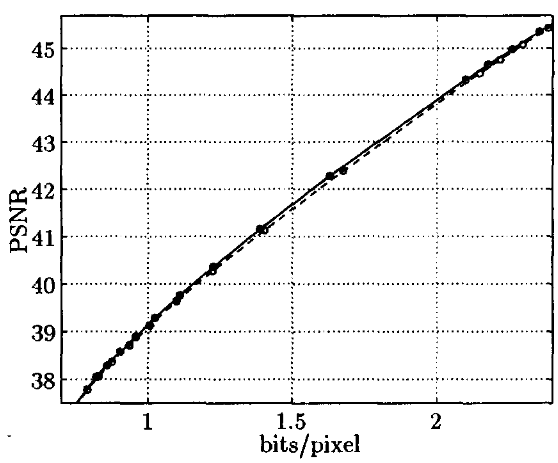

Figure 5. Simulated results using one quantizer (– –) and one quantizer for each class (——). In each case $N = 4$ classes are used.

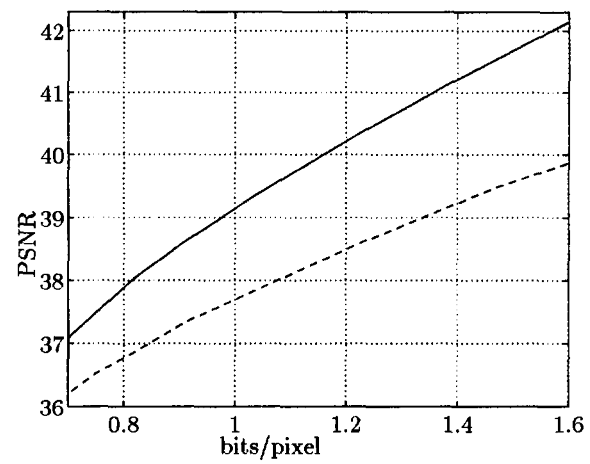

Figure 6. Comparison with a JPEG coder for coding the luminance component of the *Lenna* image. The proposed coder (——) and JPEG coder (– –).

approximately 0.1 dB is gained by allowing different quantizers for each class instead of using one common quantizer. At low and high rates the absolute PSNR gain is lower. This is in accordance with the theoretical results found in Section 4.1.

5.3 Comparison with JPEG

Figure 6 shows the rate distortion performance of the proposed image coder compared with JPEG [17].

Figure 6 shows that the proposed image coder performs better than the JPEG coder. At 1.4 bits/pixel the proposed image coder has 2.0 dB higher PSNR than the JPEG coder, for the image *Lenna*.

6 CONCLUSIONS

A method which jointly optimizes the classification, quantization, and entropy coding of an infinite Gaussian mixture distribution source was developed. No high rate approximations have been used.

The theoretical results showed that by allowing different quantizers in each class a rate reduction of 11–12% was obtained at low rates using five entropy coders. At high rates there was no gain by using different quantizers in each class. This is in accordance with [15].

The proposed coding method gives a good rate distortion performance at a low complexity.

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