Blind Equalization of Nonlinear Communication Channels Using Recurrent Wavelet Neural Networks*

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Abstract ***This paper investigates the application of a Recurrent Wavelet Neural Network(RWNN) to the blind equalization of nonlinear communication channels. We propose a RWNN based structure and a novel training approach for blind equalization, and we evaluate its performance via computer simulations for nonlinear communication channel model. It is shown that the RWNN blind equalizer performs much better than the linear CMA and the RRBF blind equalizers in nonlinear channel case. The small size and high performance of the RWNN equalizer make it suitable for high speed channel blind equalization.***

I. Introduction

The performance of digital communication system is largely affected by an ability to overcome the channel impairments during signal propagation. Traditional techniques for communication channel equalization are based on linear transversal equalizers(LTE's), whose coefficients are being adjusted to match the channel characteristics. Depending on whether the equalizer knows an originally transmitted sequence or not, it is characterized as trained adaptation or blind equalizer respectively. Blind equalization is a particularly useful and difficult type of equalization, as for example in the case of multipoint communication networks. Blind equalization schemes such as the Constant Modulus Algorithm(CMA)[1], the Tricepstrum Equalization Algorithm(TEA)[2], Recurrent Radial Basis Function(RRBF) Networks based blind equalizer[3] et al. have been developed for linear channels. The use of these schemes with nonlinear unknown channels is questionable.

Blind equalization is however an inherently nonlinear problem and it is desired to incorporate some nonlinearity in the equalizer structure. A Recurrent Wavelet Neural Network(RWNN) being essentially an IIR nonlinear filter, can be trained to have desired dynamical behavior, using a stochastic gradient approach via the Real Time Recurrent Learning(RTRL) algorithm. In this paper we propose the use of a RWNN equalizer for the blind equalization of nonlinear channels. A novel training approach using only a partly set of statistics of the transmitted signal is introduced. It is shown that the RWNNs of reasonable size have the ability to accurately model the inverse of a communication channel with a performance superior than that of the traditional equalization algorithms. The properties of a RWNN make it attractive for the blind equalization of nonlinear channels.

II. Recurrent Wavelet Neural Networks

Some recent works relating neural networks and wavelets appeared in the literature[4, 5], the classical approximator feedforward neural network with sigmoidal units is replaced by a wavelet neural network with the Morlet wavelet as activation function.

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Recurrent Wavelet Neural Networks, in which every unit is connected to every other unit, are highly nonlinear dynamical systems that exhibit a rich and complex dynamical behavior. In contrast to the wavelet network introduced in [4, 5], the RWNN is well suited for use in real time adaptive signal processing. Furthermore, the RWNN has the advantage that a priori information of the underlying system need not be known, the dynamics of the system are configured in the recurrent connections and the network approximates the system over time.

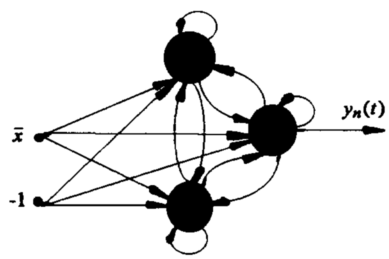

Fig. 1 Recurrent wavelet neural network

A RWNN depicted in Fig.1 has n units and m external input(including the bias input). The output node in general is a linear element so that the entire dynamic range of the system can be captured. Rest of the nodes take a multidimensional wavelet $\Psi(\bar{x})$. Let $\bar{y}(t)$ denote the n-tuple of outputs of the units in the network and let $\bar{x}(t)$ denote the m-tuple of external inputs to the network at time t. It will be convenient in what follows to define $\bar{z}(t)$ to be (m+n)-tuple obtained by concatenating $\bar{x}(t)$ and $\bar{y}(t)$. To distinguish the components of $\bar{z}(t)$ representing the unit outputs from those representing external input values where necessary, let U denote the set of indices k such that z_k, the k th component of $\bar{z}$, is the output of a unit in the network, and let I denote the set of indices k for which z_k is an external input, i.e.

The output of the network is

$$y_n(t+1) = \bar{W}^T \bar{z} \qquad \bar{z} = [\bar{x}\,\bar{y}]^T \tag{2}$$

where $\bar{W}$ is the weight vector between the output unit and the remaining nodes in the network. The outputs of the rest of the nodes is given by

$$y_k(t+1) = \prod_{i=1}^{m+n} \psi\left(\frac{z(i) - t_{ki}}{s_{ki}}\right) \tag{3}$$

where $k \in \{1, 2, \cdots, n-1\}$, t_{ki} and s_{ki} are the translation and dilation parameters which are independent of each other. Here we consider a multidimensional wavelet as the direct product of one dimensional wavelets.

The instantaneous error at time $t+1$ is given by

$$E(t+1) = \frac{1}{2}[d_n(t+1) - y_n(t+1)]^2 = \frac{1}{2}e_n^2 \tag{4}$$

where $d_n(t+1)$ is the desired output of the network. Our objective is to minimize $E(t+1)$ in the parameter space spanned by $\bar{W}$, $\bar{t}$ and $\bar{s}$. For this we compute the gradient of $E(t+1)$ in the parameter space, the update rule is

$$\Delta w_i = -\eta_1 \frac{\partial E(t+1)}{\partial w_i} = \eta_1 e_n \frac{\partial y_n(t+1)}{\partial w_i} \tag{5a}$$

$$\Delta t_{ij} = -\eta_2 \frac{\partial E(t+1)}{\partial t_{ij}} = \eta_2 e_n \frac{\partial y_n(t+1)}{\partial t_{ij}} \tag{5b}$$

$$\Delta s_{ij} = -\eta_3 \frac{\partial E(t+1)}{\partial s_{ij}} = \eta_3 e_n \frac{\partial y_n(t+1)}{\partial s_{ij}} \tag{5c}$$

where η_1, η_2 and η_3 are the learning rates.

The RTRL algorithm consists of computing at each time step t, the parameter changes given by equation (5a, b, c).

III. The RWNN Blind Equalizer

The digital communication system considered is illustrated in Fig. 2, where a binary sequence $s(t)$ is transmitted through a nonlinear channel and then corrupted by additive white Gaussian noise. The transmitted signal $s(t)$ is assumed to be an independent sequence taking values of either 1 or -1 with equal probability. If the output of the equalizer(RWNN in our case) is exactly the same as the transmitted signal (with a possible time delay, and/or phase shift) then it should have the same moments as the transmitted signal.

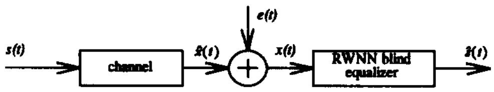

Fig. 2. The block diagram of the communication system with RWNN blind equalizer

We would like to minimize, at time $t+1$, the objective function:

$$E(t+1)=\sum_{k=1}^{4}\alpha_k e_k^2(t+1)$$
$$=\sum_{k=1}^{4}\alpha_k(E_{t+1}\{\hat{s}^k\}-E\{s^k\})^2 \qquad (6)$$

where E_{t+1} denote the estimated mean value using the $t+1$ output of the RWNN and α_k are positive constants that define the weight of the corresponding term e_k in the objective function (6). We would like to derive a way to update the parameters of the RWNN, depending on the output at time $t+1$, namely $\hat{s}(t+1)$, so that the objective function (6) gets minimized.

Each of the four mean values in (6) can be computed recursively using averaging as:

$$E_{t+1}\{\hat{s}^k\}=\frac{1}{t+1}(tE_t\{\hat{s}^k\}+\hat{s}^k(t+1)) \qquad (7)$$

Differentiating $E(t+1)$ with respect to the current weights w_i, translation and dilation parameter t_{ki}, s_{ki} can be computed recursively. Therefore, the algorithm for the minimization of the objective function (6) with a RWNN via the RTRL becomes:

1) Initialize the estimates for $E_0\{\hat{s}^k\}$ to zero for $k=1,2,3,4$.

2) Present a new sample of the channel output to the RWNN input. Compute the RWNN $\hat{s}(t+1)$ using (2).

3) Update the moment estimates $E_{t+1}\{\hat{s}^k\},\ k=1,2,3,4$ using (7).

4) Update the weights, translation and dilation parameters in the direction of the steepest descent with learning rate η_1, η_2 and η_3 respectively.

5) Go to Step 2, unless the objective function has been sufficiently minimized.

IV. RWNN Blind Equalization Simulations

In our simulations we evaluated the performance of the proposed RWNN blind equalizer for nonlinear communication channel and the results were compared to those obtained with a RRBF blind equalizer and a linear CMA equalizer based on the Godard criterion. The model of the nonlinear channel used for the simulations is

$$\hat{x}(t)=C(s(t))$$
$$=s(t)+0.5s(t-1)-0.9[s(t)+0.5s(t-1)]^3 \qquad (8)$$

and white Gaussian noise $e(t)$ with $E\{e^2(t)\}=0.2$.

The RWNN blind equalizer has three units, one input and one output. The scalar mother wavelet $\psi(x)=-xe^{-\frac{1}{2}x^2}$ to which the universal approximation theorem described in [4, 5] applies. For our multidimensional node we take the direct product of the above one-dimension wavelet. The values of learning rates η_1, η_2 and η_3 were chosen to be equal to 0.25. The values of the coefficients α_k in the objective function (6) were: $\alpha_1=2$, $\alpha_2=10$, $\alpha_3=0$ and $\alpha_4=10$.

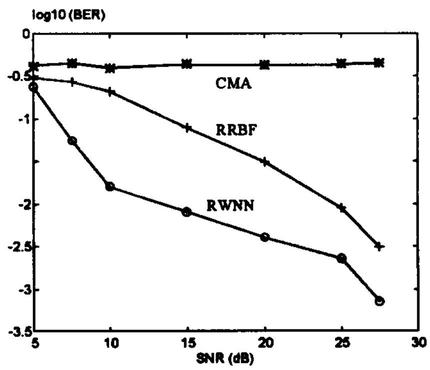

Fig. 3 BER comparison of the RWNN, CMA and RRBF blind equalizers for nonlinear channel

As we can see from Fig. 3 in which we plot the Bit Error Rate(BER) curves for these three types of blind equalizers, the linear CMA equalizer exhibits an almost constant high error rate for the range of SNR shown. The RWNN blind equalizer succeeds to open the eye-pattern in less than 18 iterations for SNR=25dB. The fast convergence of the RWNN blind equalizer make it suitable for use in environments where small initial adaptation delays are desired (such as mobile communications, cellular telephony).

V. Conclusion

This paper introduced a new blind equalizer based on a RWNN structure and a novel training approach, Which is capable of compensating the nonlinear channel distortion. Since RWNNs essentially model nonlinear infinite memory filters, they can accurately realize, with a relatively small number of parameters, the inverse of finite memory systems and thus compensate effectively for the nonlinear channel introduced interferences. Computer simulation results show that small size RWNN blind equalizer performs much better than the linear CMA and the RRBF blind equalizers. The small size and high performance of the RWNN blind equalizer make it attractive for high speed channel equalization.

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