ROBUST IMPULSE NOISE SUPPRESSION USING ADAPTIVE WAVELET DE-NOISING

S. Burley and M. Darnell

Department of Electronic and Electrical Engineering
University of Leeds, Leeds, LS2 9JT, UK
eensimjb@electeng.leeds.ac.uk, m.darnell@elec-eng.leeds.ac.uk

ABSTRACT

It is widely acknowledged that the effect of impulsive noise is a major source of performance degradation within a wide range of communication systems. This is due to the fact that non-Gaussian interference is neglected within the system design philosophy for reasons of complexity and tractability. In this paper, we directly address this problem using a novel 'de-noising' technique in which significant performance gains are achieved with low-complexity.

1. INTRODUCTION

The performance evaluation of communication systems has traditionally relied on the assumption of an additive Gaussian noise channel. However, in numerous circumstances this assumption is not always justifiable and the communication medium can be more accurately modelled by heavy-tailed, non-Gaussian distributions. One process which is not adequately described in terms of the Gaussian assumption is the process that generates impulsive noise bursts. For example, noise experienced on radio channels typically comprises infrequent, high amplitude pulses associated with either man-made or natural sources, superimposed on a more homogeneous (Gaussian) background. Consequently, the presence of impulsive noise is a major source of performance degradation when discrete-time, linear detection schemes such as the matched filter are used; this is largely due to the existence of non-Gaussian interference being neglected in the design philosophy.

A technique commonly used to suppress impulsive interference involves passing the received data samples through a memoryless nonlinearity. Typical nonlinear functions are the hard-limiter and the Gaussian-tailed nonlinearity. These suppress large excursions from the wanted signal level by weighting the received data samples prior to matched filter detection. Although this approach to noise suppression is not based on any optimal criteria, it is justified in that an increased signal-to-noise ratio usually results when a suitable threshold is chosen. In addition, limiting techniques incur no excessive computational complexity.

Synthesis of the optimum receiver, based on the Bayesian theory of signal detection, requires *a priori* knowledge of the underlying noise process. Unfortunately, this information is generally not available in any realistic application. Furthermore, impulsive noise is highly dependent on the physical environment and is also non-stationary. Consequently, the process of obtaining an accurate statistical model proves difficult and renders the optimum solution to the signal detection problem impossible/unattainable. Hence, the main objective of this paper is to present a detector design which is robust in the presence of impulsive noise, and can be implemented in a low-complexity real-time architecture.

During recent years, the signal processing community has developed a renewed interest in the design of structured bases for the linear expansion of signals. In particular, the subject of wavelets and time-scale analysis has received increasing interest as a new method of expanding functions onto a set of self-similar, orthonormal basis functions [1]. This is largely due to the fact that such techniques offer increased flexibility over more traditional transform methods, combined with the existence of efficient computational structures, in the form of multirate filter banks, which allow rapid calculation of the expansion coefficients.

Recent advances in the application of wavelets include Donoho and Johnstone's novel approach to signal recovery in additive white Gaussian noise (AWGN) [2]. Here, a simple thresholding technique involving a 'keep (shrink) or kill' policy is applied to the expansion coefficients. Both hard and soft-thresholding procedures have been examined which kill the wavelet coefficients corresponding to AWGN whilst retaining the larger coefficients corresponding to signal features. In this paper, we present a novel de-noising technique which differs from that in [2] by focusing on the suppression of non-Gaussian, impulsive noise. Furthermore, when incorporated within a digital radio receiver, considerable performance gains over a variety of non-Gaussian radio channels are obtained.

The paper is organised as follows: in Section 2, the noise model is discussed; in Section 3, the proposed detector based on a de-noising technique specific to non-Gaussian interference is explained in detail; in Section 4, simulation results are given; finally, in Section 5, conclusions are drawn from the work described in this paper.

Notation: A convenient way of analysing multirate filter banks is in terms of a time-domain operator. Here, for example, we have adopted the following notation; A column vector $\mathbf{x}$ of length n is denoted by $\mathbf{x}^{(n)}$, while filtering with $h_i(n)$, $i \in \{0,1\}$ followed by two-fold subsampling is denoted by $\mathbf{H}_i^{(m,n)}$, $i \in \{0,1\}$, where $\mathbf{H}_i^{(m,n)}$, $i \in \{0,1\}$ is a matrix of dimension $m \times n$ whose row entries are even shifted versions of the filter impulse response $h_i(n)$, $i \in \{0,1\}$. We consider the case when the filter coefficients are purely real, and when the filter bank computes an orthonormal discrete wavelet transform. Now for a signal containing $N = 2^n$ samples, the filter bank structure comprises $J = \log_2(N) - 1$ stages, and the wavelet expansion coefficients at the j^{th} level are given by

$$\alpha_j^{(N/2^j)} = \mathbf{H}_1^{(N/2^j, N/2^{j-1})} \left[\prod_{\substack{l=j-1 \\ j>1}}^{1} \mathbf{H}_0^{(N/2^l, N/2^{l-1})} \right] \mathbf{x}^{(N)}, \quad j = 1, \ldots, J,$$

while the approximation coefficients are computed as

$$\beta_j^{(N/2^j)} = \left[\prod_{l=j}^{1} \mathbf{H}_0^{(N/2^l, N/2^{l-1})} \right] \mathbf{x}^{(N)}, \quad j = 1, \ldots, J.$$

2. THE NOISE MODEL

The model describing impulsive noise is based on the expanded noise model in [3]. Here, the background noise component is described statistically by a zero-mean, Gaussian random process having variance σ_w^2, while the non-Gaussian, impulsive component is described by a train of randomly arriving delta functions whose amplitude is governed by a heavy-tailed density. Hence, the n^{th} sample at the receiver front-end, $y(n)$, can be expressed as

$$y(n) = x(n) + w(n) + i(n), \tag{1}$$

where $x(n)$ is the n^{th} transmitted sample, $w(n)$ is AWGN and $i(n)$ is the impulsive noise. Here, all components of $y(n)$ are assumed to be mutually uncorrelated. The impulsive noise component $i(n)$ is modelled as

$$i(n) = s(n)\,h(n), \tag{2}$$

where $h(n)$ is an ever present impulse whose amplitude is governed by a Laplacian density, and $s(n)$ is a switching process of ones and zeros. If $s(n)$ is a one (zero) then an impulse is (is not) present. The switching mechanism is chosen to be a Poisson random process, where the mean time of impulse arrival is governed by the variance λ_p.

Unless otherwise stated, we will adopt a Laplacian density having zero-mean and variance 100, while the mean time of impulse arrival is 30ms.

3. THE PROPOSED DETECTOR

Donoho and Johnstone have recently developed a powerful noise reduction technique [2] in which they adopt a thresholding strategy in conjunction with the discrete wavelet transform. The technique has been applied successfully to both 1 and 2-dimensional data, and is proven to be near-optimal for a wide class of signals corrupted by AWGN. Summarising their results, they consider a discretised signal $\mathbf{x}$ of length N, which is corrupted by zero-mean AWGN having variance σ^2. Hence, using the matrix notation defined previously, we can formulate the problem more explicitly as

$$\mathbf{y}^{(N)} = \mathbf{x}^{(N)} + \sigma\,\mathbf{w}^{(N)}, \tag{3}$$

where the main objective is to recover the signal vector $\mathbf{x}$ from the noisy observations $\mathbf{y}$. On computing the wavelet expansion coefficients of $\mathbf{y}$ using

$$\alpha_j^{(N/2^j)} = \mathbf{H}_1^{(N/2^j, N/2^{j-1})} \left[\prod_{\substack{l=j-1 \\ j>1}}^{1} \mathbf{H}_0^{(N/2^l, N/2^{l-1})} \right] \mathbf{y}^{(N)}, \quad j = 1, \ldots, \log_2(N) - 1, \tag{4}$$

a 'keep (shrink) or kill' policy is then applied to the individual wavelet coefficients prior to computing the inverse transform.

Donoho and Johnstone consider two thresholding strategies; hard-thresholding in which α_j is kept if it is above some threshold T, else it is set to zero, i.e.

$$\hat{\alpha}_j^{(k)} = T_H\left(\alpha_j^{(k)}, T\right) = \begin{cases} \alpha_j^{(k)}, & \left|\alpha_j^{(k)}\right| \ge T \\ 0, & \left|\alpha_j^{(k)}\right| < T \end{cases}, \quad j = 1, \ldots, J, \quad k = 1, \ldots, N/2^j; \tag{5}$$

and soft-thresholding which additionally 'shrinks' those values of α_j by T which are not set to zero, i.e.

$$\hat{\alpha}_j^{(k)} = T_I\left(\alpha_j^{(k)}, T\right) = \begin{cases} \mathrm{sgn}\left(\alpha_j^{(k)}\right)\left(\left|\alpha_j^{(k)}\right| - T\right), & \left|\alpha_j^{(k)}\right| \ge T \\ 0, & \left|\alpha_j^{(k)}\right| < T \end{cases}, \quad j = 1, \ldots, J, \quad k = 1, \ldots, N/2^j. \tag{6}$$

The threshold T is chosen as $T = \sigma\sqrt{2\ln(N)/N}$, where an estimate of the noise standard deviation is derived from the wavelet expansion coefficients. Furthermore, soft-thresholding is usually chosen in favour of its hard-thresholding counterpart, in order to avoid the generation of spurious oscillations when computing the inverse transform.

The de-noising technique presented so far applies specifically to the problem of signal recovery in AWGN. However, we are concerned with the design of a low-complexity receiver, which is robust in the presence of impulsive noise typically encountered on radio channels. Hence, the work presented here differs from that in [2] by considering a thresholding strategy which operates on large wavelet expansion coefficients, generated by impulsive phenomena. The advantages of this method include improved receiver performance in non-Gaussian conditions, along with robustness to a wide variety of contaminating densities giving rise to outliers, and varying degrees of impulsivity.

We first assume that the detector has a reliable estimate of the background noise variance σ_w^2, which is derived from a real-time channel evaluation (RTCE) procedure [4]. Following this, a running estimate is computed by separating the received data into Gaussian and impulsive components using a threshold test, based on the current noise variance estimate. It is important to remember that this adaptive estimation process is based on the assumption that the noise is independent, and that the receiver has *a priori* knowledge of the information bearing waveform's variance.

The proposed detector, in the first instance, reads in $N = 2^{J+1}$ samples, and performs the following test

$$\left\| \mathbf{y}^{(N)} \right\|_\infty \geq T, \qquad (7)$$

where T is a threshold derived from the estimate of the background noise variance. If the test fails, no further action is taken. However, if the test holds, then a contaminating density giving rise to outliers is present. The block of received data samples is then transformed using an orthonormal wavelet basis. Here, the tiling of the time-frequency plane differs from that induced by the traditional discrete wavelet transform, by iterating off the highpass component of the filter bank structure, i.e. the wavelet expansion coefficients are computed as

$$\boldsymbol{\alpha}_j^{(N/2^j)} = \mathbf{H}_0^{(N/2^j, N/2^{j-1})} \left[\prod_{\substack{l=j-1 \\ j>1}}^{1} \mathbf{H}_1^{(N/2^l, N/2^{l-1})} \right] \mathbf{y}^{(N)}, \quad j = 1, \ldots, \log_2(N) - 1. \qquad (8)$$

Hence, if we now consider a band-pass signal (whose normalised frequency lies between zero and $\pi/2$) and a single Dirac function, then we arrive at the time-frequency representation shown in Figure 1. The reasoning behind such an approach is that greater signal preservation in the time-frequency plane is achieved when adopting any form of thresholding to suppress the effects of impulsive noise.

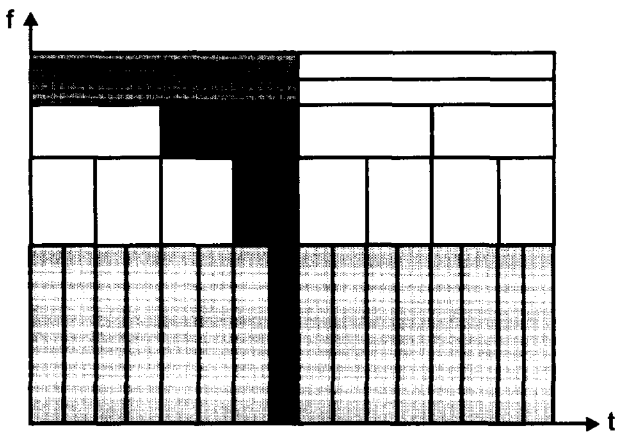

Figure 1 Modified Tiling of the Time-Frequency Plane.

Since the energy associated with impulsive noise will reside in all levels of the decomposition, a thresholding procedure is performed across all scales which 'shrinks' excessively large expansion coefficients, corresponding to impulsive noise, i.e.

$$\hat{\alpha}_j^{(k)} = T_l\left(\alpha_j^{(k)}, T_1\right) = \begin{cases} \operatorname{sgn}\left(\alpha_j^{(k)}\right)\left(\gamma\, \sigma \sqrt{2 \ln(N)/N}\right), & \left|\alpha_j^{(k)}\right| \geq T \\ \alpha_j^{(k)}, & \left|\alpha_j^{(k)}\right| < T \end{cases},$$
$$j = 1, \ldots, J, \qquad k = 1, \ldots, N/2^j, \qquad (9)$$

where $\gamma \in \mathrm{R}^+$. Hence, corrupted coefficients are simply replaced by a scaled version of Donoho and Johnstone's universal threshold. Selection of the threshold T is chosen on the grounds that $\pm 3\sigma_w$ encompasses all but 0.3% of the Gaussian density's support. Hence, if any coefficients exceed $\pm 3.3\sigma_w$, then it is assumed that a contaminating density giving rise to outliers is present.

Finally, the inverse transform is computed. On removing the impulsive component, the preprocessed signal is then assumed to be contaminated by AWGN only. Based on this assumption, the problem of detection in the impulsive environment is reduced to detection in a Gaussian environment. Consequently, the preprocessed signal is demodulated using matched filter detection.

4. SIMULATION RESULTS

We now test the proposed detector by simply observing the noise statistics at the input and output of the de-noising algorithm. Here, we have adopted the Haar basis as a result of its time-localisation properties; furthermore, minimal computational complexity is incurred. Figure 2 contains the probability density function (pdf) of simulated impulsive noise, where the background noise level results in $E_b/N_0 = 10\,\mathrm{dB}$, while Figure 3 contains the resulting noise pdf after de-noising. Overlaid on each of these is the pdf of Gaussian noise. From Figures 2 and 3, it can be seen that while the pdf of impulsive noise is heavier in the tails compared with Gaussian noise, after de-noising the resulting pdf shows a good fit.

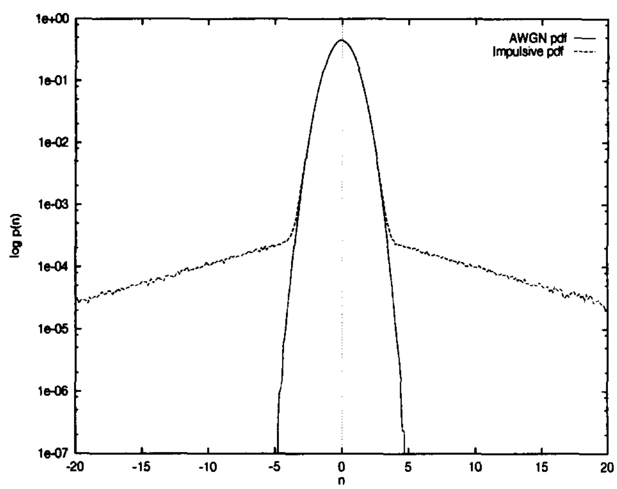

Figure 2 Simulated pdf of impulsive noise.

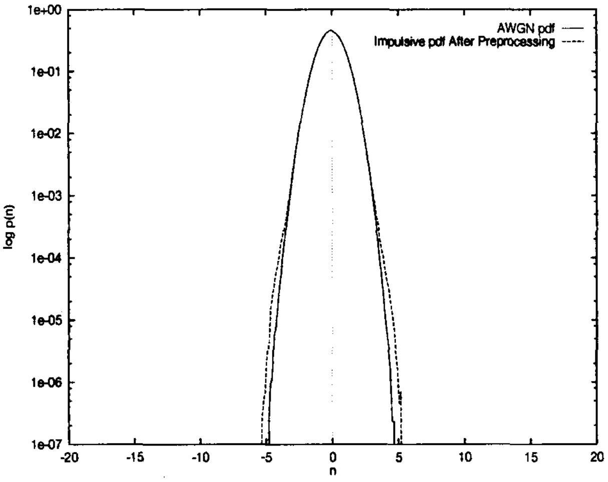

Figure 3 Simulated pdf of impulsive noise after de-noising.

Figure 4 shows the simulated bit error rate (BER) performance of 2-PSK in the presence of impulsive noise. The benefits gained when employing the new detection procedure, compared with a variety of limiting schemes, are evident (the deviation from theoretical performance is due to the increased tail probability of the noise entering the matched filter). Further results show that the detector is robust to varying degrees of impulsivity and underlying noise distributions. In addition, the curve labelled 'AWGN Performance of Proposed Detector' quantitatively agrees with the theoretical performance for AWGN over the range of BER simulated, and demonstrates that there is no loss in performance in the absence of impulsive noise.

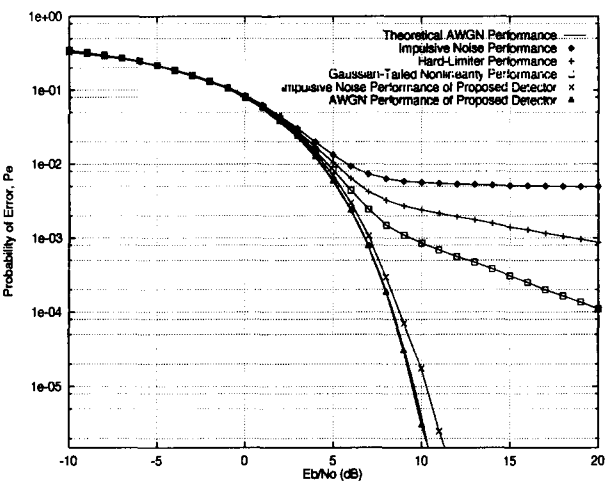

Figure 4 Comparative Performance Assessment of Proposed Detector.

5. CONCLUSIONS

A novel de-noising algorithm for suppressing impulsive noise has been presented. The resulting algorithm serves as a preprocessing unit to enhance matched filter performance and is modulation independent. The algorithm has been incorporated within a digital radio receiver, and simulation results show that extended matched filter performance is possible when the noise process of the communicating medium is non-Gaussian.

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