BLIND SDMA WITH A SIGNAL SPACE GEOMETRY MATCHED ALGORITHM

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ABSTRACT

In this paper we address the problem of the blind reconstruction of binary sequences transmitted by P sources, sharing simultaneously the same frequency channel. This is the situation in Space Division Multiple Access (SDMA) systems, where an array receiver is used to discriminate the spatial signatures of the sources. We exploit the geometry induced by the finite nature of the sources alphabet in order to derive a blind multichannel inversion algorithm. This algorithm minimizes a generalized constant modulus (GCM) cost function, subject to the constraints imposed by the signal space geometry (geometric priors). We define the GCM cost function for the case of additive Gaussian noise channels. We show that, except for a sign ambiguity, the minimization of the GCM criterion has a unique solution. Therefore, convergence to the global minimum is guaranteed. Computer simulations illustrate the performance achieved with the proposed method.

1. INTRODUCTION

During the last few years, results of research on array signal processing techniques for multiuser wireless communication systems have been published in different scientifique journals. The idea relies on the concept of SDMA systems, where the spatial dimension is exploited by an antenna array to discriminate the distinct radio channels (spatial signatures) used by each source, thus tacking to its advantage the complexity of the radio channel, in particular, the existence of multipath propagation. According to this concept, several sources using the same carrier frequency can also share the same time slot, provided that their respective spatial signatures are different. This kind of medium access technique has an obvious impact on the increase of the capacity of wireless communication systems.

Several techniques have been developed to implement blind SDMA shemes. In [2, 3] modified versions of the popular constant modulus algorithm are presented. The major problem with this kind approach is that, when using descent gradient optimization algorithms, convergence to the global minima of the specified cost functions is not guaranteed. Moreover, for the case of noisy data, which is the case of interest, the minima of those costs functions depend on the signal to noise ratio (SNR). As a result the estimate of the inverse channel is not accurate enough. Another approach is considered in [4]. This is based on the available previous knowledge of the sources' alphabet and of its geometric representation: a hypercube as represented in Figure 1(a). However, the algorithm that is described there tries to estimate the geometry of the channel distorted version of that hypercube, i.e., the parallelipiped represented in Figure 1(b). This is too complicated because the processing scheme has to deal with data whose structure is completely unknown. Here, we exploit the concept of signal space, where the nice geometry in the alphabet space can be recovered up to a rotation, see Figure 1(c). The problem is then reduced to the estimation of that rotation, suffered by the hypercube representing the alphabet.

The paper is organized as follows. Section 2. dicusses the determination of the array data representation in signal space. In section 3., we introduce the signal space geometry matched (SSGM) algorithm. This is based on a GCM cost function, which specifies a criterion for the estimation of the data space hypercube. It is shown that this cost function is invariant with SNR, and that its minimization provides an unique solution. Techniques for the removal of the inter-symbol interference, and for data reconstruction are also described. The performance of the SSGM algorithm is evaluated in section 4..

2. SIGNAL SPACE GEOMETRY

Consider the baseband complex vector

$$\mathbf{x}_k = \mathbf{H}\mathbf{a}_k + \mathbf{w}_k, \qquad (1)$$

$\mathbf{H}$ being an $(M \times D)$, $M > D$, rank D complex matrix. In general, the number of rows of $\mathbf{H}$ is given by $M = NL$, where N and L are the number of array elements and the number of time samples per symbol period, respectively. For simplicity, and without any lost of generality of the method introduced in the paper, we take $L = 1$. Also, we define $D = P + P_{isi}$, where P is the number of sources and P_{isi} is the number of delayed replicas (intersymbol interference-isi). We assume that data transmission is based on binary phase shift keying (BPSK) modulation, thus $\mathbf{a}_k$ is a D-vector with binary entries (± 1). Both $\mathbf{H}$ and $\mathbf{a}_k$ can be partitioned as follows: $\mathbf{H} = [\mathbf{H}_s \ \mathbf{H}_{isi}]$ and $\mathbf{a}_k^T = \begin{bmatrix} \mathbf{s}_k^T \ \text{isi}_k^T \end{bmatrix}$. Along the paper, we will use $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$, respectively for transpose, complex conjugate, and complex conjugate transpose. Here $\mathbf{s}_k$ includes the P source symbols arriving through the direct propagation path, and isi_k collects the interfering symbols arriving from secondary paths. All other channel effects, such as propagation delays and attenuations, spreading and reflection losses, are imbedded in $\mathbf{H}$.

In the sequel, the following hypothesis are assumed: (i) the P sources are jointly statistically independent, each one generating independent and equally like binary symbols, (ii) $\mathbf{w}_k$ is a complex Gaussian noise vector with zero

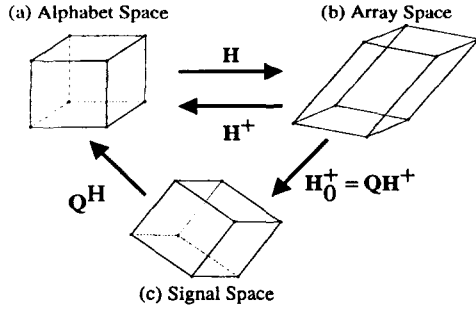**Figure 1. Signal geometries in BPSK/ SDMA systems ($D = 3$)**

mean and covariance matrix $\sigma^2 I_M$, I_M denoting the M dimensional identity matrix, (iii) a_k and w_k are jointly independent random vectors, and (iv) the channel matrix H is invariant along K data samples. Notice that we do not assume any prior knowledge about the parameters D, P, and σ^2, neither about the channel matrix H. The only prior that we have is the geometry of the sources' alphabet. In fact, a_k thakes values on a finite alphabet with cardinality 2^D, which specifies a size 2 hypercube in the alphabet space, as represented in Figure 1(a). The channel matrix H maps this hypercube into an unknown hyper-parallelipiped in array space, see Figure 1(b). In [4], an algorithm that exploits the geometry of this parallelipiped is presented. It is derived for the noise free version of (1) and it provides joint estimates of H and the symbol time sequences. As in [1], we propose to preprocess the array data, mapping it into the signal space where the basic geometry of the sources alphabet is recovered. This is also illustrated in Figure 1(c). In signal space, the noisy data is clustered around the vertices of a size 2 hypercube. To perform the data transformation, we use an eigen analysis of the array data covariance matrix (or, in practice, the sample covariance matrix estimated from K data samples)

$$R = HH^H + \sigma^2 I_M, \qquad (2)$$

yielding an estimate of H given by

$$H_0 = VD.$$

Here, V is the matrix formed by the signal subspace eigenvectors of R, and D is a D-dimensional diagonal matrix whose entries are the singular values of H. As it is well known, this procedure also provides estimates of D and σ^2, as σ^2 equals any of the $M - D$ smallest eigenvalues of R. Clearly,

$$H = H_0 Q, \qquad (3)$$

where Q is a unitary matrix. We define the transformation

$$z_k = H_0^+ x_k,$$

and using eq(1) and eq(3) we get

$$z_k = Q a_k + \eta_k, \qquad (4)$$

where the noise covariance matrix is $R_\eta = \sigma^2 D^{-2}$. This means that the noise spherical simmetry is lost but, more important than this, is the noise reduction achieved by cancelling the noise components in the orthogonal space of the columns of H. Therefore, in signal space the SNR is greater than in array space. Moreover, while in the array space,

channel inversion is an unconstrained problem, in the signal space we just have to estimate the unitary matrix Q, whose columns specify the orientation of the size 2 hypercube determined by the centroids of the data clusters, see Figure 3(a) as an example. In the following section, we present the Signal Space Matched Geometry (SSMG) algorithm, which provides estimates of the columns of Q.

3. THE SSGM ALGORITHM

In [1], we present a solution for the problem of estimating the columns of Q. This is based on a nearest neighborhood clustering algorithm that estimates the centroids of $D + 1$ adjacent clusters, thus enabling to determine the D principal directions of the signal space hypercube. Here, we take a different approach consisting on estimating directly the columns $q_1, \ldots, q_D$ of Q using a generalized constant modulus (GCM) criterion. This criterion is specified by the cost function

$$J(w) = E\left\{ \left[\Re^2 \left\{ w^H z_k \right\} - 1 \right]^2 + \Im^2 \left\{ w^H z_k \right\} \right\} - J_\eta(w), \qquad (5)$$

subject to

$$WW^H = W^H W = I_D, \qquad (6)$$

the columns of W being D minimizers of (5). We define

$$J_\eta(w) = \frac{3}{2} \left[w^H w + \Re(w^H E\{z_k z_k^T\} w^*) \right] w^H R_\eta w - \frac{1}{2} w^H R_\eta w + \frac{3}{4} (w^H R_\eta w)^2, \qquad (7)$$

where $E\{z_k z_k^T\} = QQ^T$ and $R_\eta = \sigma^2 D^{-2}$. This later quantity is estimated when the transformation that maps the array space into the signal space is determined (see section 2.), and the former can be estimated from the data using the sample mean operator on the available K data samples. It can be shown, after some persistent algebraic work, that (5) is given by

$$J(w) = E\left\{ \left[\Re^2 \left(w^H Q a_k \right) - 1 \right]^2 + \Im^2 \left[w^H Q a_k \right] \right\}. \qquad (8)$$

Notice that, when compared with conventional CM approaches [3, 2], the GCM criterion (5), or its equivalent (8): (i) instead of trying to recover the magnitude of the symbols, it enables to reconstruct the sources alphabet and (ii) it accounts for the noise statistics through $J_\eta(\cdot)$. The result of using this term appears in (8), which is theoretically free of the noise effects. Moreover, (iii) the constraint in (6) matches the optimization criterion to the available prior knowledge of the signal space geometry. Although these are interesting characteristics of the GCM cost function here introduced, its most important property is given by the following fact.

Fact 1: Let a_k be a zero mean stationary random vector with D binary entries and covariance $R_a = I_D$, and $Q = [q_1, q_2, \ldots q_D]$ a complex D dimensional unitary matrix. Then (8) (or (5)) has precisely $2D$ equal minima corresponding to the minimizers $\pm q_1, \pm q_2, \ldots \pm q_D$. Additionally, if the constraint (6) is superimposed, then the minimization of (8) (or (5)) yields D minimizers which are the columns of Q with a sign ambiguity, that is

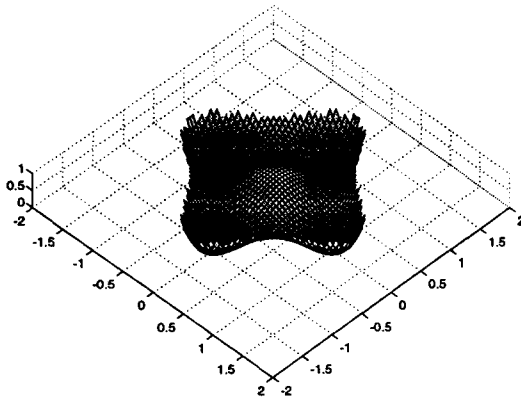$$\arg \min_{W_{i=1,\ldots D}} J(w_{i=1,\ldots D}) = QT,$$

Figure 2. Example of a GCM surface $(D = 2)$.

where T is an arbitrary D dimensional permutation matrix with ± 1 entries.

The proof of this fact is not trivial and, due to the space constraints of this paper, it will be presented elsewhere. In any case, we show in Figure 2 a typical surface generated by (8) for the case $D = 2$, where it is clear the presence of D pairs of symmetric global minima. We emphasize that the sign ambiguity associated with the estimates of the columns of Q has no impact on symbol reconstruction, provided that a differential source encoding scheme is used. In Figure 3(b), we illustrate through an example the ability of the SSGM algorithm to estimate the hypercube specified by Q. Notice that, in spite of the moderately large dispersion of the signal space data points, the estimated hypercube is practically coincident with the actual one.

Removal of the ISI. As discussed in section 2. for H and a_k, the matrix $Q = [Q_s \; Q_{isi}]$ where Q_s is the $(D \times P)$ channel matrix in signal space associated with the P-vector symbols s_k transmitted by the P sources. To estimate these symbols it is necessary to remove the ISI components of the transformed data (4). To do this, we begin by noting that being

$$\mathbf{R_Z}(1) = E\left\{z_k z_{k-1}^H\right\},$$

the lag 1 covariance matrix of the signal space data, then

$$\mathbf{A} = \mathbf{R_Z}(1)\mathbf{R_Z}^H(1) = Q_{isi}Q_{isi}^H. \qquad (9)$$

It is obvious that any column of the unitary matrix Q verifies

$$q_i^H \mathbf{A} q_i = \begin{cases} 1 & \text{if } q_i \text{ is a column of } Q_{isi} \\ 0 & \text{if } q_i \text{ is a column of } Q_s \end{cases} \qquad (10)$$

In practice, since the lag 1 sample covariance matrix is used, we have to select as columns of Q_s the P minimizers $q_i^H \mathbf{A} q_i$. The ISI free signal space data can be obtained from (4) using the transformation

$$\begin{aligned} y_k &= Q_s^H z_k \\ &= s_k + u_k \end{aligned} \qquad (11)$$

where the zero mean complex noise process u_k has an already estimated covariance matrix $\mathbf{R_u} = \sigma^2 Q_s^H \mathbf{D}^{-2} Q_s$.
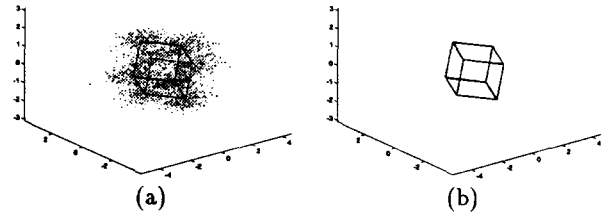

Figure 3. Signal geometries in BPSK/SDMA systems $(D = 3)$

Symbol reconstruction. Exploiting the finite nature of the known sources' alphabet $\mathcal{A}$, i. e., the set of 2^P binary code vectors c_p where s_k takes values, and being known the statistics of the Gaussian noise process u_k, the optimum symbol reconstruction strategy is provided by the maximum a posteriori (MAP) criterion. Thus, for each time instant the selected symbol is given by

$$s_k = \arg\min_{c_p \in \mathcal{A}} \left[y_k^H - c_p^H\right]\mathbf{R_u}^{-1}\left[y_k - c_p\right]. \qquad (12)$$

In this section we described the SSGM algorithm which, estimates the signal space data geometry. A method to remove the ISI and the MAP symbol detector were also presented. In the following section, the performance of our algorithm is evaluated based on computer simulations. We also compare our results with those obtained with the optimum receiver, where the channel matrix H and the noise statistics in (1) are known.

4. COMPUTER SIMULATIONS

In the simulations we used the channel matrix H specified in tables 1 and 2. The number of sources is $P = 3$. The two first columns of H correspond to source 1 (one ISI replica), and the sixth column corresponds to source 3. To evaluate the performance of the SSGM algorithm, we present the results obtained with a Monte Carlo simulation consisting of 100 independent runs of the channel. In each run, a block of $K = 2000$ array samples was generated. Figure 4 shows the Frobenius norm of the error $Q_s - \widehat{Q_s}$ associated with the estimate $\widehat{Q_s}$, as function of SNR

$$\mathrm{SNR} = \frac{E\left\{\|\mathbf{H}a_k\|^2\right\}}{E\left\{\|w_k\|^2\right\}}.$$

It can be seen that the SSGM algorithm provides accurate estimates of Q_s even for moderate values of SNR. Notice that the definition of SNR is a global measure accounting for the power contributions of all arrivals. As an example, we show in Figure 5 the ability of our method to remove the ISI components in the available data. This corresponds to a situation where SNR= 10 dB and, as we can see, the values of the minima and of the maxima fit with those in (10).

In Figure 6, we present the bit error rates (ber) obtained for each source. The solid lines correspond to the optimum receiver, i.e., that designed based on previous knowledge of the cahnnel matrix, and the dashed lines were obtained from the Monte Carlo simulation. Its clear that, as expected, the ber strongly depends on the channel used by each source. On the other hand, the experimental results are very close to the optimum bers. As a final comment, we can say that the preliminary simulation results here presented show the efficiency of our algorithm to blindly discriminate the present sources, even in the presence of ISI.

$\mathbf{h_1}$	$\mathbf{h_2}$	$\mathbf{h_3}$
$1.615 - 0.012i$	$0.948 - 0.753i$	$1.336 - 0.009i$
$-0.676 + 0.376i$	$-0.323 + 0.737i$	$0.825 + 0.261i$
$0.987 - 0.305i$	$0.785 - 0.389i$	$0.057 + 2.044i$
$-0.125 + 0.951i$	$-0.857 + 1.126i$	$-1.425 - 0.554i$
$-0.087 - 0.209i$	$0.257 - 0.916i$	$0.480 - 0.101i$
$0.349 + 0.716i$	$-0.794 + 0.723i$	$0.613 - 0.250i$
$-0.996 + 0.209i$	$0.628 - 1.442i$	$0.255 + 2.075i$
$0.531 - 0.208i$	$-0.113 + 1.053i$	$-1.774 - 0.132i$
$-1.215 + 0.744i$	$0.676 - 1.027i$	$-0.370 - 0.261i$
$0.351 - 1.306i$	$-0.291 + 1.653i$	$0.257 - 1.071i$

Table 1. First 3 columns of H

$\mathbf{h_4}$	$\mathbf{h_5}$	$\mathbf{h_6}$
$-0.861 + 1.506i$	$-0.802 + 0.966i$	$-0.052 - 0.273i$
$0.858 - 0.450i$	$1.106 + 0.986i$	$-0.728 - 1.641i$
$-0.691 + 0.354i$	$0.805 + 0.495i$	$-1.195 + 0.335i$
$0.091 + 0.253i$	$1.888 - 0.720i$	$-1.209 - 0.262i$
$0.828 + 0.949i$	$-0.116 - 1.165i$	$-0.806 + 1.562i$
$0.637 - 1.678i$	$0.176 - 1.107i$	$-0.069 + 0.696i$
$-0.982 + 0.551i$	$-1.460 - 0.078i$	$0.527 + 1.352i$
$0.711 - 0.389i$	$0.377 + 0.435i$	$0.895 + 0.055i$
$-0.357 - 0.289i$	$-0.172 + 0.559i$	$0.710 + 0.017i$
$-0.856 - 0.505i$	$1.668 - 0.407i$	$0.335 - 0.613i$

Table 2. Last 3 columns of H

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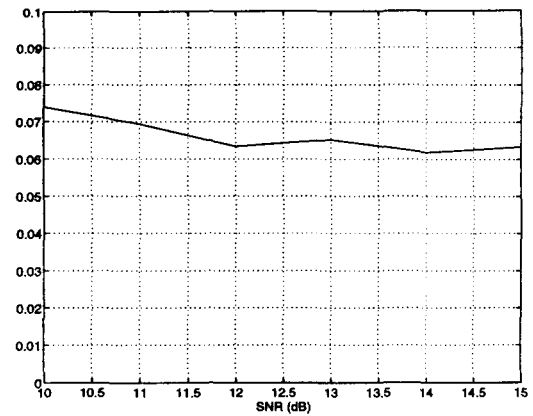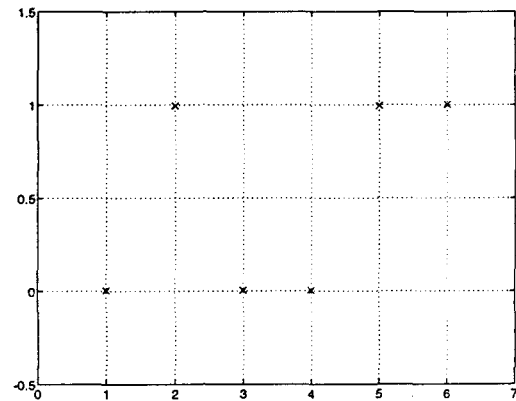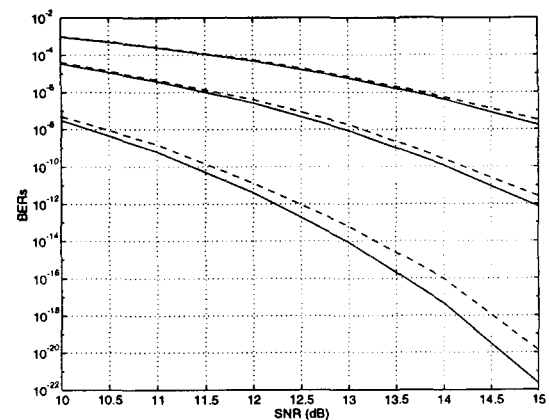
Figure 4. Estimation error of $\mathbf{Q}_s$

Figure 5. Removal of ISI

Figure 6. Bit error rates