A NEW MULTICHANNEL BLIND EQUALIZATION CRITERIUM BASED ON A GENERALIZED RAYLEIGH QUOTIENT

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ABSTRACT

Recent works have presented novel techniques that exploit cyclostationarity for channel identification (equalization) in data communication systems using only second order statistics. In particular, it has been shown the feasibility of blind identification based on the 'forward shift' structure of the correlation matrices of the source. In this paper we propose an alternative algorithm based on this property but with an improved choice of the autocorrelation matrices to be considered. This new representation of the equalization problem provide a cost function formulated as a generalized Rayleigh quotient, which may be efficiently implemented by using conjugate gradient techniques. Several simulations over different data transmission constellations support our theoretical analysis.

1. INTRODUCTION

Intersymbol interference (ISI) is the main impairment introduced through radio channel transmission by multipath propagation. To achieve high-speed reliable communication, channel identification or equalization becomes necessary to overcome the limiting effects of the ISI. Equalization is traditionally achieved by introducing a cooperation phase between transmitter and receiver when an a priori known sequence is transmitted in order to update the equalizer set [1]. However, in many applications as mobile communications, a self-adaptive (blind) equalizer could be very desirable to increase the transmission efficiency. The existing methods for blind equalization at baud rate involve in several ways the use of higher-order statistics of the observation [2].

In some recent works it has been pointed out the feasibility of blind identification of possibly non minimum phase channels using second order information, provided that several measurements per symbol could be achieved by either multiple spatial sensors, temporal oversampling or both of them. Exploiting the cyclostationarity of the received signal, some new schemes provide a more accurate estimation with a smaller sample size using only second order statistics [3 and references therein]. At this moment there exist three main approaches for the identification of multichannel filters. On one hand, several works by *Tong et al* [4 and references therein] have shown an identification (and equalization) method based on the knowledge of the 'forward shift' structure of the correlation matrices of the source. The change in the rank of these correlation matrices provide enough information to identify the multichannel matrix. However, this method requires two SVD for the estimation, and also it is shown that the performance decreases rapidly as SNR decreases. On the other hand, *Moulines et al* [5 and references therein] have proposed another approach based on the concept of noise subspace with a method similar to MUSIC for DOA estimation. The main idea here exploited is the orthogonality between the linear space spanned by the columns of the filtering matrix and any vector in the noise subspace. Only one SVD is needed but also a constrained (linear or quadratic) minimization must be included to avoid the trivial solution. Finally, Slock [6] was probably the first who realized a polyphase representation for the stationary signal-vector showing the minimum phase character of the filtering matrix: therefore, a new identification scheme is proposed based on a multichannel linear predictor. Several SVD decomposition and a pseudoinverse are needed in the calculations, therefore increasing the complexity of the scheme.

Our proposal is based on the works by *Tong et al*, but introducing a different perspective: we are not concerned with the identification but with the problem of multichannel blind equalization: that is, our goal is the proposal of a new cost function for blind multichannel equalization. As we will show in the sequel, our design can be formulated as a generalized

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Rayleigh quotient; this fact will also provide a fast implementation with low computational complexity through conjugate gradient techniques

2. PROBLEM FORMULATION

First, let us recall the problem formulation, showing the classical block diagram in fig.1:

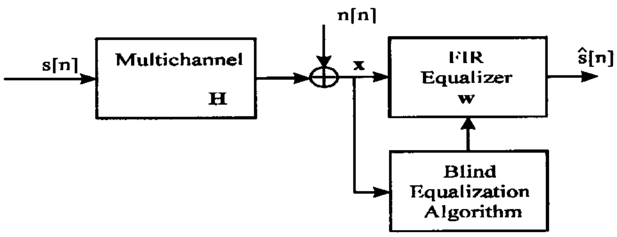

Fig.1. Block diagram of multichannel transmission and equalization scheme

An independent sequence $s[n]$ of a modulated signal is transmitted through a multichannel obtained either by an array receiver, either oversampling or both of them. In this situation, the received sequence is cyclostationary, and therefore a second order algorithm would be enough for identifying or equalizing a possible non minimum phase channel. It is well known that any cyclostationary signal could be considered as a stationary vector obtained by filtering by the full column rank matrix **H**. Sequence $n[n]$ is a Gaussian white noise independent of the transmitted sequence. The model usually assumed, establishes a linear model stated as follows:

$$\mathbf{x}(i) = \mathbf{H}\mathbf{s}(i) + \mathbf{n}(i) \quad (1)$$

Neglecting the noise effect, it has been shown the condition for the identifiability of **H**, up to a constant inherent to the problem, by using the rank properties of autocorrelation matrices $\mathbf{R}_x(0)$, $\mathbf{R}_x(1)$ defined as follows:

$$\begin{aligned} \mathbf{R}_x(0) &= E\{\mathbf{x}(i)\mathbf{x}^H(i)\} = \mathbf{H}\mathbf{H}^H \\ \mathbf{R}_x(1) &= E\{\mathbf{x}(i)\mathbf{x}^H(i-1)\} = \mathbf{H}\mathbf{J}\mathbf{H}^H \end{aligned} \quad (2)$$

where $(\ )^H$ means hermitian and **J** is a shifting matrix [4].

3. OUR PROPOSAL

Our proposal is based on Bezout's theorem which states that if **H** is full column rank there exist one vector **w** that [7]:

$$\mathbf{w}^H\mathbf{H} = [\mathbf{0}\ \mathbf{1}\ \mathbf{0}] \quad (3)$$

Observe that vector **w** could be considered as any row of the equalization matrix given by:

$$\mathbf{W} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \quad (4)$$

provided that **H** is full column rank (this hypothesis is stated always as an identifiability condition feasible when the virtual channels have no common zero [3]).

Therefore, in order to deconvolve the transmitted sequence, our goal is to design an appropriate cost function to find vector **w** uniquely. In this approach, the information we want to use is only based on the estimation of several matrices $\mathbf{R}_x(k)$; as we will show later, the choice of index k will be critical in our design. The main features of our approach can be summarized in the following steps:

Step 1. Let us generalize matrix **J** in equation (2) defining an auxiliary matrix $\mathbf{G}(k)$ as a symmetric both-sides k-shifting matrix:

$$\mathbf{G}(k) = \mathbf{J}^k + \left(\mathbf{J}^H\right)^k \quad (5)$$

where k is a degree of freedom and the symmetric property is a constraint imposed by the Rayleigh quotient formulation to be developed later.

Step 2. Recall that equalizer vector **w** which verifies equation (3) will also verify the following equations:

$$\begin{aligned} \mathbf{w}^H\mathbf{H}\mathbf{H}^H\mathbf{w} &= 1 \\ \mathbf{w}^H\mathbf{H}\mathbf{G}(k)\mathbf{H}^H\mathbf{w} &= 0 \end{aligned} \quad (6)$$

These equations provide a first approach to the formulation of our proposal of cost function. However, the question to be answered is whether both equations allow the computation of **w** in a unique way. Let us express (6) in a different manner by using the following linear transformation (remenber that in our formulation **H** is full column rank and therefore it allows us to interchange the basis analysis):

$$\mathbf{z} = \mathbf{H}^H\mathbf{w} \quad (7)$$

In this new space, (6) yield as:

$$\begin{aligned} \mathbf{z}^H\mathbf{z} &= 1 \\ \mathbf{z}^H\mathbf{G}(k)\mathbf{z} &= 0 \end{aligned} \quad (8)$$

It is known that (8) can be formulated as a Rayleigh quotient (**G** is Hermitian by definition [5]) [8].

$$\rho_G(\mathbf{z}) = \frac{\mathbf{z}^H\mathbf{G}(k)\mathbf{z}}{\mathbf{z}^H\mathbf{z}} \quad (9)$$

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In our formulation, the solution of (8) implies that vector **z** must belong to the null subspace of matrix **G**(k). Therefore, a first constraint must be imposed in that sense: we require a rank one null-subspace to assure a unique global solution. Regarding (7) and (3): we realized that we are interested in a very particular eigenvector, which deconvolves properly the transmitted sequence, that is:

$$\mathbf{z} = \mathbf{H}^H \mathbf{w} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \tag{10}$$

Therefore, collecting the main ideas described in this section, the auxiliary matrix **G**(k) must be:

A. Symmetric
B. Related with matrix **J** by (5)
C. Rank-one null-subspace as is shown in (10).

Combining these constraints, we provide a simple choice for matrix **G**(k) (NxN, assumed odd): index k= (N+1)/2, because it is a N-1 rank matrix where all the columns, except the central one, form a basis; moreover, by forcing the orthogonality to the image space and imposing that any vector belonging to the null subspace should be as is required in (10), the central column must be a zero vector; i.e., in the following example for dimension N=5 should be as follows:

$$\mathbf{G}(k=3) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

Step 3. Once we have chosen this auxiliary matrix **G**(k), consider that both matrices in (6) should be easily estimated from the incoming data: (5) provides a simple relationship with the data autocorrelation matrices as follows:

$$\begin{aligned} \mathbf{HH}^H &= \mathbf{R}_x(0) \\ \mathbf{H}\mathbf{G}(k)\mathbf{H}^H &= \mathbf{R}_x(k) + \mathbf{R}_x(-k) \end{aligned} \tag{12}$$

4. PRACTICAL IMPLEMENTATION

Equations (6) provide an equalization algorithm based on the following optimization criterium:

$$\min_{\mathbf{w}} \left(\mathbf{w}^H \mathbf{HG}(k) \mathbf{H}^H \mathbf{w} \right) \quad \text{constrained to} \quad \mathbf{w}^H \mathbf{HH}^H \mathbf{w} = 1 \tag{13}$$

i.e., the minimization of a quadratic function with a quadratic constraint. It is well known [8] that (13) can be formulated as a generalized eigenvalue problem. In order to achieve a competitive computational complexity and at the same time, achieving fast convergence of the recursice version, we have recalled that eigenproblems are efficiently supported by using conjugate gradient techniques [9]. Also, in [10] it is remarked the nice property of the Rayleigh quotient with singular points at which the gradient is zero are either unstable saddle points or the global minimum.

Let us denote the generalized eigenvalue problem as follows:

$$\mathbf{Aw} = \lambda \mathbf{Bw} \quad \text{where} \quad \mathbf{A} = \mathbf{HG}(k)\mathbf{H}^H, \quad \mathbf{B} = \mathbf{HH}^H \tag{14}$$

where **A**, **B** are Hermitian matrices and **B** is definite positive. However, we must consider that in our formulation **A** is indefinite, and therefore a minimization process applied to equation (12) will lead us to the minimum (negative) eigenvalue instead of the desirable minimum absolute eigenvalue (see eq. (8)). In this situation, only by several iterations could be determined the lowest absolute eigenvalue by conjugate gradient techniques [8]. Therefore, in order to increase the algorithm efficiency, we have overcome this fact just considering and alternative implementation based on a modified criterium:

$$\mathbf{AA}^H \mathbf{w} = \lambda \mathbf{Bw} \tag{15}$$

where $\mathbf{AA}^H$ is now positive definite; although this equation provides different solutions than (14), observe that the eigenvector related with the minimun eigenvalue also holds condition (3).

5. COMPUTER SIMULATION RESULTS

To show the performance and convergence properties of our equalization algorithm, we have considered the scenario given in [5], where several independent sequences are transmitted through four virtual channels given in table 1. White Gaussian noise is added to the

$\mathbf{h}_0$	-0.049+j 0.359	0.482-j 0.569	-0.556+j 0.587	1	-0.171+j 0.061
$\mathbf{h}_1$	0.443-j 0.0364	1	0.921-j 0.194	0.189-j 0.208	-0.087-j 0.054
$\mathbf{h}_2$	-0.211-j 0.322	-0.199+j 0.918	1	-0.284-j 0.524	0.136-j 0.19
$\mathbf{h}_3$	0.417+j 0.030	1	0.873+j 0.145	0.285+j 0.309	-0.049+j 0.161

Table 1. Scenario considered for simulations. Four virtual complex channels

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output and a Signal to Noise Ratio (SNR) of 40 dB is considered.

Also, we want to point out that in practical situations, the ensemble average of the signal correlation matrices are not known. Therefore, in the implementation of equation (15), the true matrices have been replaced by their recursive estimation.

In Fig.2 we show the deconvolved constellations for 4QAM, 16QAM, and 8PSK data transmission. The convergence speed could be very competitive to standard algorithms, also providing a simple implementation by conjugate gradient techniques: Fig.2a) for 16QAM at 1000 samples, and Fig. 2b), 2c) for 8PSK and 4QAM at 300 samples; it must be remarked that the different convergence speed is uniquely related with the constellation dimension in order to obtain a proper matrices estimation.

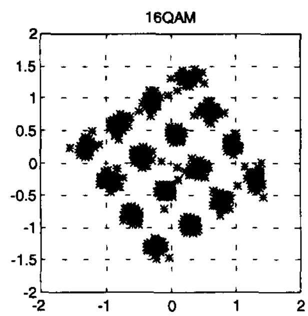

Fig.2a. Deconvolved sequence for 16QAM at 1000 samples

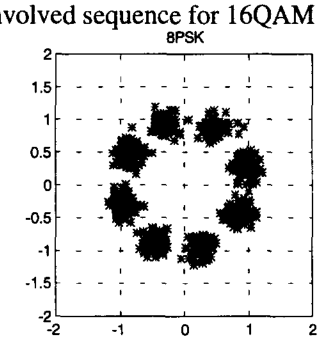

Fig2b. Deconvolved sequence for 8PSK at 300 samples

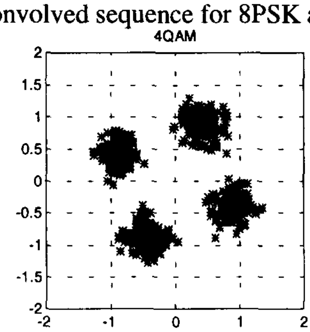

Fig.2c. Deconvolved sequence for 4QAM at 300 samples

5. CONCLUDING REMARKS

In this paper we have dealt with a new approach to the blind multichannel equalization problem. Exploiting the cyclostationary characteristic of oversampled or array received transmitted data, a SIMO (Single Input - Multiple output) identification problem has arised. Our proposal is based specifically on the shifting property of the autocorrelation matrices of the sources. A detailed analysis of the effect of their shifting index has provided a nice optimization criterium for proper deconvolution, formulated as a generalized Rayleigh quotient. This fact achieve a fast implementation based on conjugate gradient techniques. Several simulations support our proposal and its feasibility as an iterative deconvolution scheme.

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