BLIND SOURCE SEPARATION USING TIME-FREQUENCY DISTRIBUTIONS: ALGORITHM AND ASYMPTOTIC PERFORMANCE.

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ABSTRACT

This paper addresses the problem of the blind source separation which consists of recovering a set of signals of which only instantaneous linear mixtures are observed. A blind source separation approach exploiting the difference in the time-frequency (t-f) signatures of the sources is considered. The approach is based on the diagonalization of a combined set of 'spatial time-frequency distributions'. Asymptotic performance analysis of the proposed method is performed. Numerical simulations are provided to demonstrate the effectiveness of our approach and to validate the theoretical expression of the asymptotic performance.

1. INTRODUCTION

Blind source separation consists of recovering a set of signals of which only instantaneous linear mixtures are observed. The first solution to this problem was based on the cancellation of higher order moments assuming non-Gaussian and i.i.d. source signals [1]. Since then, other criteria based on minimizations of cost functions, such as the sum of square fourth order cumulants [2, 3], contrast functions [2] or likelihood function [4], have been used by several researchers. In the case of non i.i.d. source signals and even Gaussian sources, solutions based on second order statistics are possible [5, 6]. Matsuaka et al. have shown that the problem of the separation of nonstationary signals can be solved using second order decorrelation only [7]. They implicitly use the nonstationarity of the signal via a neural net approach. Herein, we propose to take advantage explicitly of the nonstationarity property of the signals to be separated. This is done by resorting to the powerful tool of time frequency signal representations.

In this paper, we develop an approach based on a joint diagonalization of a combined set of spatial time-frequency distributions. This approach exploits the difference between the t-f signatures of the sources. In contrast to existing methods, the proposed approach allows the separation of Gaussian sources with identical spectra shape but with different time-frequency localization properties. Moreover, the effects of spreading the noise power while localizing the source energy in the time-frequency domain amounts to increase the robustness of the proposed approach with respect to noise.

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2. PROBLEM FORMULATION

Consider m sensors receiving an instantaneous linear mixture of signals emitted from n sources. The $m \times 1$ vector $\mathbf{x}(t)$ denotes the output of the sensors at time instant t which may be corrupted by an additive noise $\mathbf{n}(t)$. Hence, the linear data model is given by:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where the $m \times n$ matrix $\mathbf{A}$ is called the 'mixing matrix'. The n source signals are collected in a $n \times 1$ vector denoted $\mathbf{s}(t)$ which is referred to as the source signal vector. The sources are assumed to have different structures and localization properties in the time frequency domain. The mixing matrix $\mathbf{A}$ is full column rank but is otherwise unknown. In contrast with traditional parametric methods, no specific structure of the mixture matrix is assumed.

The problem of blind source separation has two inherent ambiguities. First, it is not possible to know the original labeling of the sources, hence any permutation of the estimated sources is also a satisfactory solution. The second ambiguity is that it is inherently impossible to uniquely identify the source signals. We take advantage of the second indeterminacy by treating the source signals as if they have *unit power*. This normalization still leaves undetermined the ordering and the phases of the columns of $\mathbf{A}$. Hence, the blind source separation is a technique for the identification of the mixing matrix and/or the recovering of the source signals up to a fixed permutation and some complex factors.

3. SPATIAL TIME-FREQUENCY DISTRIBUTIONS

The discrete-time form of the Cohen's class of time-frequency distributions (TFD), for signal $x(t)$, is given by [8]

$$D_{xx}(t,f) = \sum_{l,m=-\infty}^{\infty} \phi(m,l)x(t+m+l)x^*(t+m-l)e^{-j4\pi fl} \tag{2}$$

where t and f represent the time index and the frequency index, respectively. The kernel $\phi(m,l)$ characterizes the distribution and is a function of both the time and lag variables. The cross-TFD of two signals $x_1(t)$ and $x_2(t)$ is defined by

$$D_{x_1x_2}(t,f) = \sum_{l,m=-\infty}^{\infty} \phi(m,l)x_1(t+m+l)x_2^*(t+m-l)e^{-j4\pi fl} \quad (3)$$

Expressions (2) and (3) are now used to define the following data *spatial time-frequency distribution* (STFD) *matrix*,

$$\mathbf{D_{xx}}(t,f) = \sum_{l,m=-\infty}^{\infty} \phi(m,l)\mathbf{x}(t+m+l)\mathbf{x}^*(t+m-l)e^{-j4\pi fl} \quad (4)$$

where $[\mathbf{D_{xx}}(t,f)]_{ij} = D_{x_ix_j}(t,f)$, for $i,j = 1,\cdots,n$.

Under the linear data model of equation (1) and assuming noise-free environment, the STFD matrix takes the following simple structure:

$$\mathbf{D_{xx}}(t,f) = \mathbf{A}\mathbf{D_{ss}}(t,f)\mathbf{A}^H \quad (5)$$

where $\mathbf{D_{ss}}(t,f)$ is the signal TFD matrix whose entries are the auto- and cross-TFDs of the sources. We note that $\mathbf{D_{xx}}(t,f)$ is of dimension $m \times m$, whereas $\mathbf{D_{ss}}(t,f)$ is of $n \times n$ dimension. For narrowband array signal processing applications, matrix $\mathbf{A}$ holds the spatial information and maps the auto- and cross-TFDs of the sources into auto- and cross-TFDs of the data.

Since the off-diagonal elements of $\mathbf{D_{ss}}(t,f)$ are cross-terms, then this matrix is diagonal for each time-frequency (t-f) point which corresponds to a true power concentration, i.e. signal auto-term. In the sequel, we consider the t-f points which satisfy this property. In practice, to simplify the selection of auto-terms, we apply a smoothing kernel $\phi(m,l)$ that significantly decreases the contribution of the cross-terms in the t-f plane. This kernel can be a member of the reduced interference distribution (RID) introduced in [9] or signal-dependent which matches the underlying signal characteristics [10].

4. PROPOSED ALGORITHM

Let $\mathbf{W}$ denotes a $m \times n$ matrix, such that $(\mathbf{WA})(\mathbf{WA})^H = \mathbf{UU}^H = \mathbf{I}$, i.e. $\mathbf{WA}$ is a $m \times m$ unitary matrix (this matrix is referred to as a whitening matrix, since it whitens the signal part of the observations). Pre- and post-multiplying the TFD-matrices $\mathbf{D_{xx}}(t,f)$ by $\mathbf{W}$, we then define the *whitened TFD-matrices* as:

$$\underline{\mathbf{D}}_{\mathbf{xx}}(t,f) = \mathbf{W}\mathbf{D_{xx}}(t,f)\mathbf{W}^H \quad (6)$$

From the definition of $\mathbf{W}$ and Eq.(5), we may expressed $\underline{\mathbf{D}}_{\mathbf{xx}}(t,f)$ as

$$\underline{\mathbf{D}}_{\mathbf{xx}}(t,f) = \mathbf{U}\mathbf{D_{ss}}(t,f)\mathbf{U}^H \quad (7)$$

Since the matrix $\mathbf{U}$ is unitary and $\mathbf{D_{ss}}(t,f)$ is diagonal, expression (7) shows that any whitened data STFD-matrix is diagonal in the basis of the columns of the matrix $\mathbf{U}$ (the eigenvalues of $\underline{\mathbf{D}}_{\mathbf{xx}}(t,f)$ being the diagonal entries of $\mathbf{D_{ss}}(t,f)$).

If, for the (t_a,f_a) point, the diagonal elements of $\mathbf{D_{ss}}(t_a,f_a)$ are all distinct, the missing unitary matrix $\mathbf{U}$ may be 'uniquely' (i.e. up to permutation and phase shifts) retrieved by computing the eigendecomposition of $\mathbf{D_{zz}}(t_a,f_a)$. However, when the t-f signatures of the different signals are not highly overlapping or frequently intersecting, which is likely to be the case, the selected (t_a,f_a) point often corresponds to a single signal auto-term, rendering matrix $\mathbf{D_{ss}}(t_a,f_a)$ defficient. That is, only one diagonal element of $\mathbf{D_{ss}}(t_a,f_a)$ is different from zero. It follows that the determination of the matrix $\mathbf{U}$ from the eigendecomposition of a single whitened data STFD-matrix is no longer 'unique' in the sense defined above. The situation is more favorable when considering *simultaneous diagonalization* of a combined set $\{\mathbf{D_{zz}}(t_i,f_i)|i = 1,\cdots,p\}$ of p STFD matrices. This amounts to incorporating several time-frequency points in the source separation problem. It is noteworthy that two source signals with identical t-f signatures can not be separated even with the inclusion of all information in the t-f plane.

Joint diagonalization: The *joint diagonalization* [6] can be explained by first noting that the problem of the diagonalization of a single $n \times n$ normal matrix M is equivalent to the minimization of the criterion [11]

$$C(\mathbf{M},\mathbf{V}) \stackrel{\text{def}}{=} -\sum_i |\mathbf{v}_i^*\mathbf{M}\mathbf{v}_i|^2 \quad (8)$$

over the set of unitary matrices $\mathbf{V} = [\mathbf{v}_1,\cdots,\mathbf{v}_n]$. Hence, the joint diagonalization of a set $\{\mathbf{M}_k|k = 1..K\}$ of K arbitrary $n \times n$ matrices is defined as the minimization of the following JD criterion:

$$C(\mathbf{V}) \stackrel{\text{def}}{=} -\sum_k C(\mathbf{M}_k,\mathbf{V}) = -\sum_{ki} |\mathbf{v}_i^*\mathbf{M}_k\mathbf{v}_i|^2 \quad (9)$$

under the same unitary constraint. An efficient joint approximate diagonalization algorithm exists in [6] and it is a generalization of the Jacobi technique [11] for the exact diagonalization of a single normal matrix.

Identification Procedure: Equations (5-9) constitute the blind source separation approach based on TFD which is summarized by the following steps

- Determine the whitening matrix $\hat{\mathbf{W}}$ from the eigendecomposition of an estimate of the covariance matrix of the data (see [6] for more details).
- Determine the unitary matrix $\hat{\mathbf{U}}$ by minimizing the joint approximate diagonalization criterion for a specific set of whitened TFD matrices $\{\underline{\mathbf{D}}_{\mathbf{xx}}(t_i,f_i)|i = 1,\cdots,p\}$,
- Obtain an estimate of the mixture matrix $\hat{\mathbf{A}}$ as $\hat{\mathbf{A}} = \hat{\mathbf{W}}^{\#}\hat{\mathbf{U}}$, where the superscript # denotes the pseudo-inverse, and an estimate of the source signals $\hat{\mathbf{s}}(t)$ as $\hat{\mathbf{s}}(t) = \hat{\mathbf{U}}^H\mathbf{W}\mathbf{x}(t)$.

5. ASYMPTOTIC PERFORMANCE

The performance is characterized in terms of signal rejection. After identification of the matrix $\mathbf{A}$, the estimated source signals may be obtained as $\hat{\mathbf{s}}(t) = \hat{\mathbf{A}}^{\#}\mathbf{x}(t) = \hat{\mathbf{A}}^{\#}\mathbf{A}\mathbf{s}(t) + \hat{\mathbf{A}}^{\#}\mathbf{n}(t)$.

The matrix $\hat{\mathbf{P}}$ defined by $\hat{\mathbf{P}} = \hat{\mathbf{A}}^{\#}\mathbf{A}$ should be close to some matrix $\mathbf{P}$ with only one zero phase term in each row and each column (phase and permutation indeterminacies). For convenience, we assume that $\hat{\mathbf{P}}$ is close to a diagonal rather than to some other permutation matrix. The p-th estimated source signal is

$$\hat{\mathbf{s}}_p(t) = \sum_{q=1}^{n} \hat{P}_{pq}\mathbf{s}_q(t) + (\mathbf{A}^{\#}\mathbf{n}(t))_p \qquad (10)$$

The power of the q-th source signal residual (interference) in the p-th estimated source signal is: $\mathcal{I}_{pq} = E|\hat{\mathbf{P}}_{pq}|^2$ (since the sources have unit power, this quantity is nothing but the interference to signal ratio (ISR) for the q and p-th source). As a global measure of performance, we use the overall rejection level defined as the sum of all the interferences

$$\mathcal{I}_{perf} \stackrel{\text{def}}{=} \sum_{q \neq p} E|\hat{\mathbf{P}}_{pq}|^2 = \sum_{q \neq p} \mathcal{I}_{pq} \qquad (11)$$

In the case of Gaussian noise and deterministic source signals, we have derived closed form expressions of the rejection index at the limit of large snapshots. Details of the calculation are presented in [12].

$$\mathcal{I}_{pq} = \mathcal{I}_{pq}^0 + \sigma^2\mathcal{I}_{pq}^1 + \sigma^4\mathcal{I}_{pq}^2 \qquad (12)$$

where the coefficients of the expansion are

$$\mathcal{I}_{pq}^0 = \frac{1}{4}\Bigg[\alpha_{pq}^2|r_{Tpq}|^2 - \sum_{k=1}^{K}\alpha_{pq}\alpha_{pqk}\left[r_{Tpq}D_{s_qs_p}(t_k,f_k) + r_{Tqp}D_{s_ps_q}(t_k,f_k)\right] + \sum_{k,l=1}^{K}\alpha_{pqk}\alpha_{pql}D_{s_ps_q}(t_k,f_k)D_{s_qs_p}(t_l,f_l)\Bigg]$$

$$\mathcal{I}_{pq}^1 = \frac{1}{4}\Bigg[\frac{\alpha_{pq}^2}{T}(r_{Tpp}J_{qq} + r_{Tqq}J_{pp}) - \sum_{k=1}^{K}\alpha_{pq}\alpha_{pqk}\Big[r_{Tpq}J_{qp} + r_{Tqp}J_{pq}\frac{2}{T}(D_{s_ps_p}(t_k,f_k)J_{qq} + D_{s_qs_q}(t_k,f_k)J_{pp})\Big] + \sum_{k,l=1}^{K}\alpha_{pqk}\alpha_{pql}(D_{s_ps_q}(t_k,f_k)J_{qp} + D_{s_qs_p}(t_l,f_l)J_{pq} + F_{s_ps_p}^{k,l}J_{qq} + F_{s_qs_q}^{k,l}J_{pp})\Bigg]$$

$$\mathcal{I}_{pq}^2 = \frac{1}{4}\Bigg[\frac{1}{T}\Bigg[\alpha_{pq}^2(J_{qq}J_{pp} + \frac{|J_{pq}|^2}{m-n}) - 2\sum_{k=1}^{K}\alpha_{pq}\alpha_{pqk}J_{qq}J_{pp}\Bigg] + \sum_{k,l=1}^{K}\alpha_{pqk}\alpha_{pql}(|J_{pq}|^2 + \phi_{kl}J_{pp}J_{qq})\Bigg]$$

with

$$\alpha_{pq} = 1 + \frac{|\mathbf{d}_p|^2 - |\mathbf{d}_q|^2}{|\mathbf{d}_p - \mathbf{d}_q|^2}$$

$$\mathbf{d}_r = [D_{s_rs_r}(t_1,f_1),\ldots,D_{s_rs_r}(t_K,f_K)]^T$$

$$\alpha_{pqk} = \frac{D_{s_ps_p}^*(t_k,f_k) - D_{s_qs_q}^*(t_k,f_k)}{|\mathbf{d}_p - \mathbf{d}_q|^2}$$

$$r_{Tpq} = \frac{1}{T}\sum_{t=1}^{T}s_p(t)s_q^*(t)$$

$$J_{pq} = (\mathbf{A}^H\mathbf{A})_{pq}^{-1}$$

$$F_{s_ps_p}^{k,l} = \sum_{v',v,m=-\infty}^{+\infty}\phi(m,v)\phi(m-v-v'+t_k-t_l,v')\, s_p(t_k+m+v)s_p^*(t_k+m-v-2v')e^{-j4\pi f_k v}e^{-j4\pi f_l v'}$$

$$\phi_{kl} = \sum_{v,m=-\infty}^{+\infty}\phi(m,v)\phi^*(m+(t_k-t_l),v)e^{-j4\pi(f_k-f_l)v}$$

For high signal to noise ratio, the expansion (12) is dominated by the first term $\mathcal{I}_{pq}^0$. Below, some comments on this term are given:

- If the sources p and q have identical t-f signatures over the chosen t-f points (i.e. $\mathbf{d}_p = \mathbf{d}_q$), the corresponding ISR $\mathcal{I}_{pq} \rightarrow \infty$.
- As the correlation function r_{Tpq} of the sources p and q and the cross-terms $D_{s_ps_q}(t_k,f_k)$ vanish, the corresponding ISR given by $\mathcal{I}_{pq}$ also vanishes, yielding a perfect separation.
- $\mathcal{I}_{pq}^0$ is independent of the mixing matrix. In the array processing context, it means that performance in terms of interference rejection are unaffected by the array geometry. The performance depends only on the sample size and the t-f signatures of the sources.

6. PERFORMANCE EVALUATION

Numerical experiments: we consider a uniform linear array of three sensors having half wavelength spacing and receiving signals from two sources in the presence of white Gaussian noise. The sources arrive from different directions $\phi_1 = 0$ and $\phi_2 = 20$ degrees. The source signals are generated by filtering a complex circular white Gaussian processes by an AR model of order one with coefficient $a_1 = 0.85\exp(j2\pi f_1(t))$ and $a_2 = 0.85\exp(j2\pi f_2(t))$, where we have:

$$f_1(t) = \begin{cases} 0.0625 & \text{for } t = 1:400 \\ 0.1250 & \text{for } t = 401:450 \\ 0.3750 & \text{for } t = 451:850 \end{cases}$$

$$f_2(t) = \begin{cases} 0.3750 & \text{for } t = 1:400 \\ 0.1250 + \delta f & \text{for } t = 401:450 \\ 0.0625 & \text{for } t = 451:850 \end{cases}$$

The signal to noise ratio (SNR) is set at 5 dB. The kernel used for the computation of the TFDs is the Choi-Williams kernel [8] , which provides a good reduction of the cross-terms. Eight TFD matrices are considered. The corresponding t-f points are those of the highest power in the t-f domain. The mean rejection level is evaluated over 500 Monte-Carlo runs.

Table 1 shows the mean rejection level in dB versus the 'spectral shift' δf both for SOBI algorithm [6] and the new algorithm. Note that for $\delta f = 0$, the two *Gaussian* source signals have *identical spectra shape.* In this case, while SOBI fails[1] in separating the two sources, the proposed algorithm succeed.

[1] We admit that a source separation algorithm fails when the mean rejection level is greater than -10 dB.

Spectral shift (δf)	Mean Rejection level in dB	
	SOBI	TFS
0.000	-8.86	-12.22
0.002	-10.01	-12.21
0.010	-10.18	-12.34
0.050	-11.09	-12.53
0.200	-12.92	-12.54

Table 1. Performance of SOBI and TFS algorithms vs δf

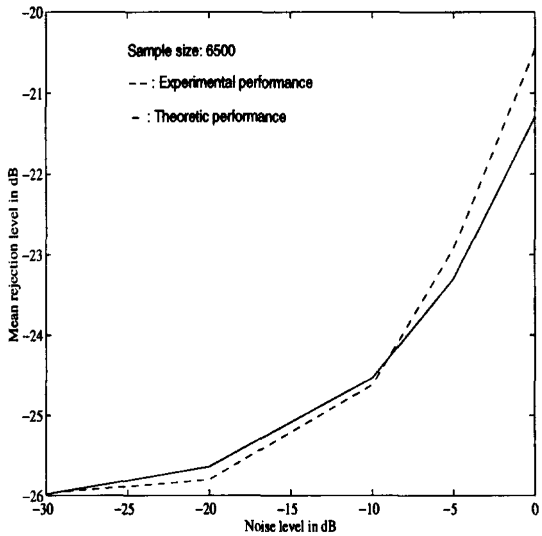

Figure 1. Performance validation vs σ^2.

Validation of the asymptotic performance: Herein, the evaluation of the domain of validity of the first-order performance approximation (12) is considered. The previous settings are used with the exception of the source signals which are deterministic sinusoids at frequencies $f_1 = 0.4375$ and $f_2 = 0.0625$. The TFDs are computed using windowed Wigner distribution. The chosen window width is $M = 2L+1$, with $L = 32$. The identification is performed using $\frac{T}{M}$ STFD matrices spaced in time by M samples (T being the sample size). The overall rejection level is evaluated over 500 independent runs.

In Fig.1, the rejection level $\mathcal{I}_{perf}$ is plotted in dB as a function of the noise power σ^2 (also expressed in dB). In Fig.2, the rejection level $\mathcal{I}_{perf}$ is plotted in dB as against sample size. Both figures 1 and 2 show that the approximation is better at high SNR and for large sample size. This means that the asymptotic conditions are reached faster in this range of parameters.

7. CONCLUSION

In this paper, the problem of blind separation of linear spatial mixture of non-stationary source signal based on time frequency distributions has been investigated. A solution based on the diagonalization of a combined set of spatial time frequency distribution matrices has been proposed. A closed form expression for the performance criterion of the method has been developed. Numerical simulations have been provided to support the theoretical claims.

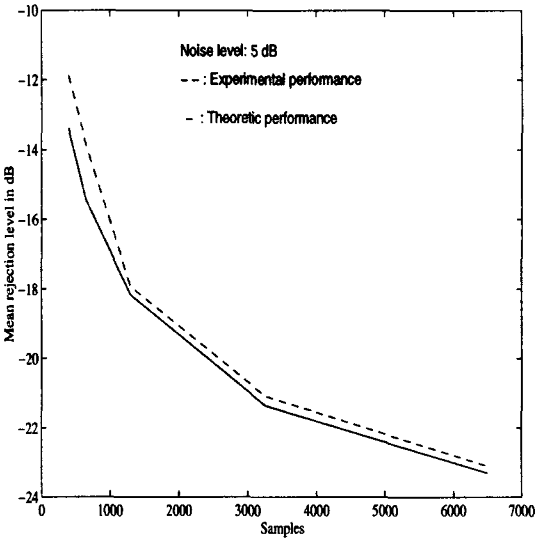

Figure 2. Performance validation vs samples size (T).

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