A NEW LEAST SQUARES APPROACH TO BLIND BEAMFORMING

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ABSTRACT

We present a new two-step approach to blind beamforming based on the least squares criterion. The first step consists of "whitening" the array received vector, i.e., transforming its response matrix to some unknown unitary matrix. The second step consists of estimating the unitary matrix from the fourth order cumulants by a least squares criterion. In contrast to the corresponding "joint diagonalization" step of the JADE algorithm, our second step exploits all the structural information in the problem and consequently yields better performance. Simulation results demonstrating the improved performance over the JADE algorithm are included.

I. INTRODUCTION

Blind beamforming is aimed at estimating the array directional response matrix without a priori knowledge of the array manifold. Many different schemes have been recently proposed for this task, see [1]-[9] and the references therein.

An efficient two-step scheme, referred to as Joint Approximate Diagonalization of Eigenmatrices (JADE), was proposed by Cardoso and Souloumiac [1]. Their first step consists of "whitening" the array received vector, i.e., transforming its response matrix to some unknown unitary matrix. Their second step consists of estimating the unitary matrix by "joint diagonalization" of the whole set of fourth order cumulant matrices of the whitened process. Though the JADE criterion was based solely on intuitive grounds, it was shown in [10] that this criterion is in fact the least squares solution to the joint diagonalization problem, with certain structural information on the diagonal matrices *being ignored.*

In this paper we present a new least squares criterion to the estimation of the unitary matrix. In contrast to the JADE criterion, this criterion exploits *all* the structural information in the problem and consequently yields better performance.

II. PROBLEM FORMULATION

Consider an array composed of p sensors with arbitrary locations and arbitrary directional characteristics. Assume that q narrowband sources, centered around a known frequency impinge on the array from distinct directions.

Using complex envelope representation, the $p \times 1$ vector received by the array can be expressed by

$$\mathbf{x}(t) = \sum_{i=1}^{q} \mathbf{a}_i s_i(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{a}_i$ is the $p \times 1$ response vector to the i-th source, $s_i(t)$ is the signal of the i-th source as received at the reference point, and $\mathbf{n}(t)$ is the $p \times 1$ vector of the noise at the sensors.

In matrix notation this becomes

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{A}$ is the $p \times q$ matrix

$$\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_q]. \tag{3}$$

Suppose we are given M samples of the array vector $\{\mathbf{x}(t_i)\}_{i=1}^{M}$. The blind beamforming problem amounts to estimating the the array response matrix $\mathbf{A}$ from the sampled data $\{\mathbf{x}(t_i)\}_{i=1}^{M}$.

To solve this problem we assume the following:

A1 : *The matrix* $\mathbf{A}$ *is full column rank.*

A2 : *The signals* $\{s_i(t)\}$ *are statistically independent with zero mean and covariance matrix* $\mathbf{I}$.

A3 : *The noise* $\{\mathbf{n}(t_i)\}$ *are Gaussian random vectors, independent of the signals and independent of each other with zero mean and covariance matrix* $\sigma^2\mathbf{I}$.

Suppose also that we have "whitened" the array response matrix, that is performed the transformation

$$\mathbf{z}(t) = \mathbf{W}\mathbf{x}(t), \tag{4}$$

where $\mathbf{W}$ is a $q \times p$ whitening matrix satisfying

$$\mathbf{WA} = \mathbf{U}, \tag{5}$$

with $\mathbf{U}$ denoting an *unknown* $q \times q$ unitary matrix.

The resulting $q \times 1$ vector $\mathbf{z}(t)$ can be expressed as

$$\mathbf{z}(t) = \mathbf{Us}(t) + \tilde{\mathbf{n}}(t), \tag{6}$$

where $\tilde{\mathbf{n}}(t) = \mathbf{Wn}(t)$, being a linear transformation of a Gaussian vector, is also Gaussian.

Now, let $\mathbf{C}$ denote the $q^2 \times q^2$ matrix built from the fourth-order cumulants of $\mathbf{z}(t)$,

$$\mathbf{C} = cum\left[\left(\mathbf{z}^*(t) \bigotimes \mathbf{z}(t)\right)\left(\mathbf{z}^*(t) \bigotimes \mathbf{z}(t)\right)^H\right], \tag{7}$$

where $\bigotimes$ denotes the Kroneker product and $*$ denotes the complex conjugate.

From (6), using the well-known properties of cumulants, we get

$$\mathbf{C} = \sum_{i=1}^{q} (\mathbf{u}_i^* \bigotimes \mathbf{u}_i) k_i (\mathbf{u}_i^* \bigotimes \mathbf{u}_i)^H, \tag{8}$$

where k_i is the kurtosis of the i-th element of $s_i(t)$, given by

$$k_i = cum(s_i(t), s_i^*(t), s_i(t), s_i^*(t)). \tag{9}$$

This may be written as

$$\mathbf{C} = \tilde{\mathbf{U}}\mathbf{K}\tilde{\mathbf{U}}^H, \tag{10}$$

where $\tilde{\mathbf{U}}$ is the $q^2 \times q$ matrix

$$\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1 \ldots, \tilde{\mathbf{u}}_q] = [\mathbf{u}_1^* \bigotimes \mathbf{u}_1, \ldots, \mathbf{u}_q^* \bigotimes \mathbf{u}_q], \tag{11}$$

with $\mathbf{u}_i$ denoting the i-th column of $\mathbf{U}$, and $\mathbf{K}$ is the $q \times q$ diagonal matrix

$$\mathbf{K} = diag(k_1, \ldots, k_q). \tag{12}$$

Let $\hat{\mathbf{C}}$ denote the sample-estimate of $\mathbf{C}$ obtained from a finite batch of M samples $\{\mathbf{z}(t_i)\}_{i=1}^{M}$. The problem we address in this paper can be stated as follows. Given $\hat{\mathbf{C}}$, estimate the unitary matrix $\mathbf{U}$.

III. THE LEAST SQUARES SOLUTION

Least Squares is a natural and common estimation criterion. In the problem at hand this criterion amounts to the minimization of the Frobenius norm of the difference between sample-cumulant matrix $\hat{\mathbf{C}}$ and the true-cumulant matrix $\mathbf{C}$, given by (10), i.e.,

$$\min_{\mathbf{U},\mathbf{K}} \| \hat{\mathbf{C}} - \tilde{\mathbf{U}}\mathbf{K}\tilde{\mathbf{U}}^H \|_F^2 . \tag{13}$$

Comparing this criterion with the least squares criterion for the joint diagonalization problem, [10], it should be pointed out that while the latter ignores the dependence of the elements of the diagonal matrices on the elements of the matrix $\mathbf{U}$, our new criterion fully exploits all the structural information in the problem and hence yields a better fit to sample-cumulant data $\hat{\mathbf{C}}$.

To derive the least squares estimator of $\mathbf{U}$ we first minimize (13) with respect to the diagonal matrix $\mathbf{K}$, with the matrix $\tilde{\mathbf{U}}$ held fixed,

$$\min_{\mathbf{K}} \| \hat{\mathbf{C}} - \tilde{\mathbf{U}}\mathbf{K}\tilde{\mathbf{U}}^H \|_F^2 = \min_{\{k_i\}} \| \hat{\mathbf{C}} - \sum_{i=1}^{q} k_i \tilde{\mathbf{u}}_i \tilde{\mathbf{u}}_i^H \|_F^2 . \tag{14}$$

Now, using the vectorizing operator, $vec(\cdot)$, the minimization turns into the following standard least squares problem

$$\min_{\mathbf{k}} \| vec(\hat{\mathbf{C}}) - \mathbf{Hk} \|^2, \tag{15}$$

where $\mathbf{H}$ is the $q^2 \times q$ matrix

$$\mathbf{H} = \left[vec(\tilde{\mathbf{u}}_1 \tilde{\mathbf{u}}_1^H), \ldots, vec(\tilde{\mathbf{u}}_q \tilde{\mathbf{u}}_q^H)\right], \tag{16}$$

and $\mathbf{k}$ is the $q \times 1$ vector

$$\mathbf{k} = [k_1, \ldots, k_q]^T. \tag{17}$$

The solution is given by the well-known expression

$$\hat{\mathbf{k}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1} \mathbf{H}^H vec(\hat{\mathbf{C}}), \tag{18}$$

which when substituted into (15) and then into (13) yields the following estimation criterion

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \| \mathbf{H} \left(\mathbf{H}^H\mathbf{H}\right)^{-1} \mathbf{H}^H vec(\hat{\mathbf{C}}) \|^2 . \tag{19}$$

Exploiting the readily provable relation

$$\mathbf{H}^H\mathbf{H} = \mathbf{I}, \tag{20}$$

we get

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \| \mathbf{H}^H vec(\hat{\mathbf{C}}) \|^2 . \tag{21}$$

Now from (16), using the well known properties of Kroneker products, we get

$$\| \mathbf{H}^H vec(\hat{\mathbf{C}}) \|^2 = \sum_{i=1}^{q} \| \tilde{\mathbf{u}}_i^H \hat{\mathbf{C}} \tilde{\mathbf{u}}_i \|^2, \quad (22)$$

which when substituted into (21) yields

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q} \left(\tilde{\mathbf{u}}_i^H \hat{\mathbf{C}} \tilde{\mathbf{u}}_i \right)^2. \quad (23)$$

To simplify this expression, let $\lambda_1 \geq \lambda_2 \geq, \ldots, \geq \lambda_{q^2}$ and $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{q^2}$ denote the eigenvalues and the corresponding eigenvectors of the sample-cumulants matrix $\hat{\mathbf{C}}$,

$$\hat{\mathbf{C}} = \sum_{j=1}^{q^2} \lambda_j \mathbf{v}_j \mathbf{v}_j^H. \quad (24)$$

Substituting this expression into (23), we get

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q} \left(\sum_{j=1}^{q^2} \lambda_j \| \tilde{\mathbf{u}}_i^H \mathbf{v}_j \|^2 \right)^2. \quad (25)$$

Now, using (11), we get

$$\tilde{\mathbf{u}}_i^H \mathbf{v}_j = (\mathbf{u}_i^* \bigotimes \mathbf{u}_i)^H vec(\mathbf{V}_j) = \mathbf{u}_i \mathbf{V}_j \mathbf{u}_i, \quad (26)$$

where $\mathbf{V}_j$ is the $q \times q$ matrix formed from the $q^2 \times 1$ vector $\mathbf{v}_j$ by the $unvec(\cdot)$ operation.

Substiuting (26) into (25), we finally get

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q} \sum_{j=1}^{q^2} \left(\lambda_j \| \mathbf{u}_i^H \mathbf{V}_j \mathbf{u}_i \|^2 \right)^2. \quad (27)$$

This criterion should be compared with the JADE criterion of Cardoso and Souloumaic, [1], which can be written as

$$\hat{\hat{\mathbf{U}}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q^2} \sum_{j=1}^{q} |\lambda_j|^2 \| \mathbf{u}_i^H \mathbf{V}_j \mathbf{u}_i \|^2. \quad (28)$$

Notice that though both criteria are expressed in terms of the quadratic forms $\{\mathbf{u}_i^H \mathbf{V}_j \mathbf{u}_i\}$, the two criteria are functionally different. In this respect, notice that $\{\lambda_j\}$, being the eigenvalues of an Hermitian but not necessarily positive definite matrix, are *real* but not necessarily positive.

It follows from (10) that asymptotically, as the number of samples grows to infinity, $\lambda_j \rightarrow k_j; j = 1 \ldots, q$ and $\lambda_j \rightarrow 0; j = q+1 \ldots, q^2$. This implies that the $q^2 - q$ "small" eigenvalues of $\hat{\mathbf{C}}$ are essentially "noise" and thus their contribution to (27) can be neglected without imparing significantly the estimation accuracy. Making this simplifying approximation, the estimation criterion reduces to

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q} \sum_{j=1}^{q} \left(\lambda_j \| \mathbf{u}_i^H \mathbf{V}_j \mathbf{u}_i \|^2 \right)^2. \quad (29)$$

This should be compared with the corresponding version of the JADE criterion, given by

$$\hat{\hat{\mathbf{U}}} = \arg\max_{\mathbf{U}} \sum_{i=1}^{q} \sum_{j=1}^{q} |\lambda_j|^2 \| \mathbf{u}_i^H \mathbf{V}_j \mathbf{u}_i \|^2. \quad (30)$$

IV. SIMULATION RESULTS

To demonstrate the performance of the proposed algorithm we comared it with the JADE algorithm in several simulated experiments.

In these experiments four uncorrelated sources impinged from directions $-26^0, 0^0, 26^0$, and 51^0 on a 5 element uniform linear array with inter-element spacing of 0.4λ. In each experiment the Signal-to-Noise Ratio (SNR) was fixed while the number of samples M was a parameter. For each value of M we performed 100 Monte Carlo runs and computed the Signal-to-Interference-plus- Noise Ratio (SINR) of the sources at the beamformer output.

For comparison, the estimation of the unitary matrix was carried using the two versions of the prposed solution, i.e., the full version (27) and the simplified version (29), as well as the corresponding versions of the JADE algorithm, i.e., (28) and (30). The SINR results obtained by the proposed solution are presented by solid lines while those obtained by the JADE algorithm are presented by dashed lines. The upper line in each pair represents the full algorithm while the lower line represents the simplified algorithm.

In the first experiment the sources were FM modulated with SNR of $10dB$, $15dB$, $20dB$ and $5dB$, respectively. The results corresponding to the source at 26^0 are presented in Figure 1. Notice that the performance gain between the full versions increased in this case to more than $5dB$ at $M = 20$.

The scenario in the second experiment was identical to the first except that the sources were QAM16 modulated. The results corresponding to the source at boresight are presented in Figure 2. Notice that the performance gain between the full versions varies from $4dB$ at $M = 40$ to $2dB$ at $M = 100$. Also, notice that in contrast to the FM modulated signals in the third experiment, the QAM16 signals require more samples for the same performance level.

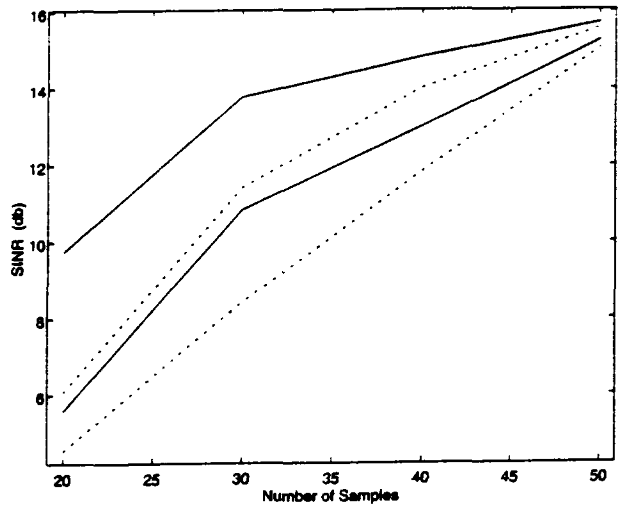

Figure 1: Four equipower uncorrelated sources impinge from directions $-26^0, 0^0, 26^0$, and 51^0 on a 5 element uniform linear array with inter-element spacing of 0.4λ. The signals are FM modulated with SNR of *10dB*, *15dB*, *20dB* and *5dB*. The figure presents the SINR of the source at 26^0. The two solid lines represent the full and the simplified versions of the proposed algorithm, while the two dashed lines represent the corresponding versions of the JADE algorithm.

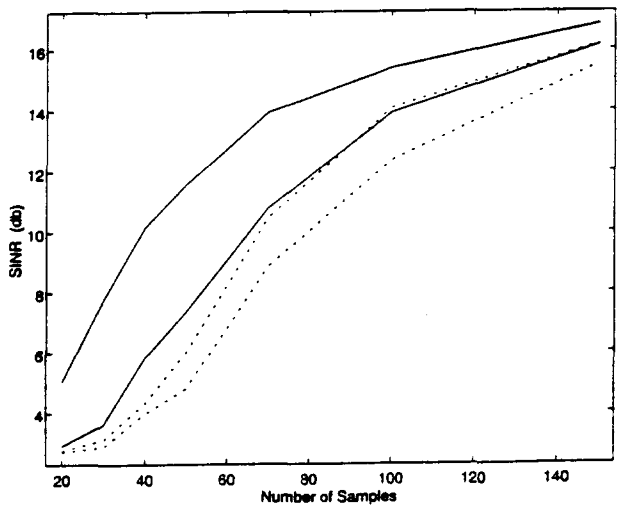

Figure 2: The scenario is identical to that in Figure 1 except that the modulation of the signals is QAM16. The figure presents the SINR of the source at boresight.

V. CONCLUDING REMARKS

We have presented a new approach to blind identification of the array response matrix. The approach is based on two steps. The first step consists of "whitening" the array received vector, i.e., transforming its response matrix to some unknown unitary matrix. The second step consists of estimating the unitary matrix from the fourth-order cumulants by a least squares criterion. While the first step is identical to that performed in the JADE algorithm, the second step differs from the corresponding "joint diagonalization" step of the JADE algorithm in that it fully exploits all the structural information in the cumulant matrix. The performance gain over the JADE algorithm was demonstrated by simulations, and was shown to be especially conspicuous when the number of samples is small.

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