TRACKING MOVING SOURCES USING SUBSPACE-BASED ADAPTIVE LINEAR METHODS

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ABSTRACT

Several works reported in the literature show that the subspace-based linear methods are computationally much more interesting than the eigendecomposition-based techniques and only slightly less accurate from the statistical point of view. They therefore have a clear potential for real time applications. We here retain the basic ideas behind this class of methods and we formulate the subspace tracking problem as a classical adaptive least squares (LS) one. Solving this adaptive LS problem results in subspace tracking algorithms of computational complexity linearly proportional to the sample vector dimension. We suggest a possible implementation for tracking the direction-of-arrival (DOA) of slowly moving sources using the LS approach. The problem of estimating crossing targets is also discussed and we propose an efficient strategy to deal with it.

1. INTRODUCTION

In high resolution signal parameter estimation, the subspace-based methods have been of great interest in recent years [1]. Those based on batch eigenvalue decomposition of the sample covariance matrix have particularly been investigated and several techniques proposed in the literature are the outcome of this research effort. However, the intensive computational implementation of such an approach prevents its use in real-time applications.

In an attempt to overcome this limitation, various adaptive algorithms for subspace tracking have been proposed and reported in the literature [2]-[5]. The main motivation of our work is the research of subspace-based methods which do not need any eigendecomposition operation. Representative members of these so-called "linear" methods (LMs) are BEWE, SWEDE, and the propagator method (PM) (see [10] and the references therein). These methods estimate the subspaces using only linear operations on the collected data. Thus, the LMs can easily be implemented in subspace-tracking applications by recursively updating the sample covariance matrix using an appropriate technique. The resulting algorithms form by themselves, a family of subspace tracking algorithms of complexity varying from $O(MK)$ to $O(M^2K)$ operations where M is the input vector dimension and K is the number of source signals. Unfortunately, in our computer simulations, we observed that these algorithms offer poor performance when the sources are closely spaced or when some of them cross. The largest portion of this paper will be devoted to a different approach. Indeed, we show that the problem of tracking the subspaces using the LMs can also be treated as a classical adaptive least squares problem. Based on this observation, we derive a gradient-based and an RLS adaptive algorithm of complexity $O(MK)$. Accordingly, this approach will be referred to as linear approach for subspace tracking (LAST). The main interest of LAST relies on the fact that the subspaces can be expressed as an explicit fonction of the DOA's. Keeping this in mind, we suggest a target angle tracking algorithm based on the prediction of the DOA's and LAST. The performance of the resulting tracking algorithm is illustrated in situations concerning closely spaced and crossing targets.

2. THE SUBSPACE-BASED LINEAR METHODS

Consider the samples $\boldsymbol{x}(t)$, recorded during the observation time on the sensor outputs satisfying the following model:

$$\boldsymbol{x}(t) = \boldsymbol{A}(\theta)\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{x}(t) \in \boldsymbol{C}^{M\text{X}1}$ is the data vector, $\boldsymbol{s}(t) \in \boldsymbol{C}^{K\text{X}1}$ is the vector of signal amplitudes, $\boldsymbol{n}(t) \in \boldsymbol{C}^{M\text{X}1}$ is an additive noise, and $\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1), ..., \boldsymbol{a}(\theta_K)] \in \boldsymbol{C}^{M\text{X}K}$ is the matrix of the steering vectors $\boldsymbol{a}(\theta_j)$ and θ_j, $j = 1, ..., K$ is the DOA of source j, measured with respect to the normal of the array. It is assumed that $\boldsymbol{a}(\theta_j)$ is a smooth fonction of θ_j and that its form is known (i.e. the array is calibrated). Under the common assumptions that the complex amplitudes $\boldsymbol{s}_j(t)$ are stationary random variables and that the components of the noise vector $\boldsymbol{n}(t)$ are zeromean stationary random variables uncorrelated with $\boldsymbol{s}_j(t)$, the covariance matrix of the received signals can be defined by

$$\boldsymbol{R} = E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \boldsymbol{R}_n \tag{2}$$

where $\boldsymbol{S} = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$ is the signal covariance matrix assumed to be nonsingular ($^{"H"}$ denotes Hermitian transposition) and $\boldsymbol{R}_n$ is the noise covariance matrix. The common underlying assumption for the introduced LMs is that the steering matrix $\boldsymbol{A}$, being of full rank, has at least one set of K rows which forms a nonsingular matrix. The LMs then differ from one to another in the way of exploiting this assumption and in other additional considerations [10].

2.1. Subspace estimation using the LMs

The estimation of the subspaces in a typical off-line application using the LMs can be achieved in two steps. First, the

covariance matrix is entirely or partially estimated from N snapshots of the array output $\{\boldsymbol{x}(t)\}_{t=1}^{N}$. Then, the noise or signal subspace is computed by applying simple linear operations on the corresponding block-covariance matrices. The statistical performance of the LMs for estimating the direction-of-arrival of multiple -fixed- targets were investigated and compared with MUSIC (see [9] and the references therein). All these methods were shown to be computationally much more interesting than MUSIC and only slightly less accurate from the statistical point of view. For details on the off-line implementation of the discussed LMs, we refer to [10] where the number of operations is also given.

2.2. On-line implementation: First approach

Based on the aforementioned off-line estimation procedure, the LMs can easily be implemented in the on-line case by using a time-varying data covariance matrix. This matrix can then be updated by using, for instance, exponential windowing

$$\widehat{\boldsymbol{R}}(t) = \beta\widehat{\boldsymbol{R}}(t-1) + (1-\beta)\boldsymbol{x}(t)\boldsymbol{x}^H(t) \tag{3}$$

where $0 < \beta \leq 1$ is a forgetting factor to be suitably chosen. This technique has been applied to SWEDE in [6]. The applicability of this first approach to BEWE and the PM for tracking the DOA's of moving sources has been verified in our simulations. Implementing the resulting subspace tracking algorithms demands an order of operations of $O(MK)$ for BEWE, $O(MK^2)$ for SWEDE and $O(M^2K)$ for the PM. We found however that these tracking algorithms perform poorly when the targets are closely spaced or when some of them cross. In order to overcome this problem, we propose to focus on a different approach, the so-called linear approach for subspace tracking (LAST).

3. LINEAR APPROACH FOR SUBSPACE TRACKING

BEWE and SWEDE may be viewed as special cases of the PM which was shown to be the most performant from the statistical point of view but also computationally the most requiring [9, 10]. LAST retains the key idea of the PM while reducing its computational implementation. Consider the following partition of $\boldsymbol{A}$

$$\boldsymbol{A}^H = \begin{array}{c} \overset{K}{[\boldsymbol{A}_1^H} \quad \overset{M-K}{\boldsymbol{A}_2^H]} \end{array} \tag{4}$$

where we assume that $M > K$. Under the assumption that $\boldsymbol{A}_1$ is nonsingular, the PM defines a unique linear operator $\boldsymbol{P} \in C^{K\mathrm{x}(M-K)}$, such that

$$\boldsymbol{P} \triangleq \boldsymbol{A}_1^{-H}\boldsymbol{A}_2^H, \qquad \boldsymbol{A}^H \begin{bmatrix} \boldsymbol{P} \\ -\boldsymbol{I} \end{bmatrix} = \boldsymbol{A}^H\boldsymbol{Q} = \boldsymbol{0} \tag{5}$$

Clearly, matrix $\boldsymbol{Q}$ in (5) spans the noise subspace. Furthermore, it is quite easy to see directly that $\boldsymbol{Q}_s^H = [\boldsymbol{I} \quad \boldsymbol{P}]$ satisfies $\boldsymbol{Q}_s^H\boldsymbol{Q} = \boldsymbol{0}$. Consequently, the problem of estimating the noise and signal subspaces using the PM can be reduced to that of finding the operator $\boldsymbol{P}$. In most of the works reported in the literature concerning the PM, the operator $\boldsymbol{P}$ is estimated by applying linear operations to the sample covariance matrix [7, 8]. In order to avoid computing the whole covariance matrix, we propose to minimize the following scalar fonction

$$J(\boldsymbol{P}) = E|\boldsymbol{x}_2 - \boldsymbol{P}^H\boldsymbol{x}_1|^2 \tag{6}$$

where $|.|$ is the Euclidean vector norm and where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are vectors made from the elements of $\boldsymbol{x}(t)$ corresponding to the rows of $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, respectively. Note that, in the no-noise case, the definition (5) of the operator $\boldsymbol{P}$ still holds in (6). Note also that, in the presence of noise, we are dealing with a classical adaptive least squares problem and, as such, a number of optimization techniques can be developed (see e.g. [12]).

3.1. Optimization by a gradient technique

It is straightforward to apply a gradient technique to (6) which results in a subspace tracking algorithm of computational complexity $3MK + O(K^2)$

$$\begin{cases} \boldsymbol{P}(t) = & \boldsymbol{P}(t-1) + \mu\boldsymbol{x}_1(t)\boldsymbol{e}^H(t) \\ \boldsymbol{e}(t) = & \boldsymbol{x}_2(t) - \boldsymbol{P}^H(t-1)\boldsymbol{x}_1(t) \end{cases} \tag{7}$$

where $\mu > 0$ is the step size. This algorithm was derived in a different way in [11] and its performance for estimating the DOA's of slowly moving sources was investigated.

3.2. Optimization by an RLS technique

Using an exponentially weighted least squares technique, we minimize the cost function

$$J(\boldsymbol{P}) = \sum_{i=1}^{t} \beta^{t-i}|\boldsymbol{x}_2(i) - \boldsymbol{P}^H(t)\boldsymbol{x}_1(i)|^2 \tag{8}$$

This function attains its minimum value if

$$\boldsymbol{P}(t) = \boldsymbol{G}^{-1}(t)\boldsymbol{H}(t) \tag{9}$$

$$\boldsymbol{G}(t) = \beta\boldsymbol{G}(t-1) + \boldsymbol{x}_1(t)\boldsymbol{x}_1^H(t) \tag{10}$$

$$\boldsymbol{H}(t) = \beta\boldsymbol{H}(t-1) + \boldsymbol{x}_1(t)\boldsymbol{x}_2^H(t) \tag{11}$$

A more numerically efficient way to compute the inverse of $\boldsymbol{G}$ is to apply the matrix inversion lemma to the recursive equation (10). The resulting algorithm for updating $\boldsymbol{P}(t)$ is given below without derivations.

$$\begin{cases} \boldsymbol{h}(t) = & \boldsymbol{C}(t-1)\boldsymbol{x}_1(t)[\beta + \boldsymbol{x}_1^H(t)\boldsymbol{C}(t-1)\boldsymbol{x}_1(t)]^{-1} \\ \boldsymbol{C}(t) = & \beta^{-1}\left[\boldsymbol{C}(t-1) - \boldsymbol{h}(t)\boldsymbol{x}_1^H(t)\boldsymbol{C}(t-1)\right] \\ \boldsymbol{e}(t) = & \boldsymbol{x}_2(t) - \boldsymbol{P}^H(t-1)\boldsymbol{x}_1(t) \\ \boldsymbol{P}(t) = & \boldsymbol{P}(t-1) + \boldsymbol{h}(t)\boldsymbol{e}^H(t) \end{cases} \tag{12}$$

Note that the signal subspace which results from the minimization of (6) may be estimated by using the approach outlined in section 2. The main interest of using LAST relies on the fact that $\boldsymbol{P}$ (and consequently the noise and signal subspaces) is an explicit fonction of θ. This will be the topic of the following section.

4. DOA ESTIMATION USING LAST

By using an efficient technique to predict $\{\theta_j\}_{j=1}^K$ at time t, it may be possible to predict the operator $\boldsymbol{P}(t)$ according to its definition in (5). Then, with the $\boldsymbol{P}(t)$ predicted in this way, we can force any of the LAST algorithms to converge to the "true" $\boldsymbol{P}(t)$. We propose to track the DOA's using the approximate Newton algorithm (see e.g. [6]) which results from the minimization of the cost function $f(\theta) = \boldsymbol{a}^H(\theta)\boldsymbol{\Pi}\boldsymbol{a}(\theta)$ where $\boldsymbol{\Pi}$ is the orthogonal projector onto the noise subspace. However, convergence to the global minimum can be guaranteed only if initialized in the vicinity of the global minimum. Assuming that θ is a slow function of time, we use some dynamic parameters such as the bearing velocities $\dot{\theta}(t)$ and accelerations $\ddot{\theta}(t)$ to predict it and hence to have initial guesses for the true values of $\boldsymbol{P}$ and θ.

4.1. The tracking algorithm

In this subsection we give a more detailed specification of a possible implementation of the proposed tracking algorithm. When a new data sample $\boldsymbol{x}(t)$ is available, we use $\widetilde{\theta}(t-1)$, the predicted DOA at time $t-1$, to obtain $\widetilde{\boldsymbol{P}}(t)$ according to (5). $\widetilde{\boldsymbol{P}}(t)$ is then inserted in the RLS algorithm (12) and estimates $\widehat{\boldsymbol{P}}(t)$ and $\widehat{\boldsymbol{Q}}_s(t)$ are obtained. The DOA's are estimated via the approximative Newton algorithm initialized with $\widetilde{\theta}(t-1)$ and $\widehat{\boldsymbol{Q}}_s(t)$. Finally, $\widehat{\theta}(t)$ is predicted using $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ previously updated with a moving rectangular window of length L.

FOR $t = 1, 2, \ldots$ **DO**

$\boldsymbol{A}(\widetilde{\theta}) = \textbf{Update\_A}[\widetilde{\theta}(t-1)]$;

$\widetilde{\boldsymbol{P}}(t) = \boldsymbol{A}_1^{-H}(\widetilde{\theta})\boldsymbol{A}_2^H(\widetilde{\theta})$;

$\boldsymbol{h}(t) = \boldsymbol{C}(t-1)\boldsymbol{x}_1(t)[\beta + \boldsymbol{x}_1^H(t)\boldsymbol{C}(t-1)\boldsymbol{x}_1(t)]^{-1}$;

$\boldsymbol{C}(t) = \beta^{-1}\left[\boldsymbol{C}(t-1) - \boldsymbol{h}(t)\boldsymbol{x}_1^H(t)\boldsymbol{C}(t-1)\right]$;

$\widehat{\boldsymbol{P}}(t) = \widetilde{\boldsymbol{P}}(t) + \boldsymbol{h}(t)[\boldsymbol{x}_2^H(t) - \boldsymbol{x}_1^H(t)\widetilde{\boldsymbol{P}}(t)]$;

$\widehat{\boldsymbol{Q}}_s(t) = [\boldsymbol{I} \quad \widehat{\boldsymbol{P}}(t)]^H$; $\quad \widehat{\boldsymbol{Q}}_s(t) = \text{orth}(\widehat{\boldsymbol{Q}}_s(t))$;

$\widehat{\boldsymbol{\Pi}}(t) = \boldsymbol{I}_M - \widehat{\boldsymbol{Q}}_s(t)\widehat{\boldsymbol{Q}}_s^H(t)$;

$$\widehat{\theta}_j(t) = \widetilde{\theta}_j(t-1) - \frac{\text{Re}[\boldsymbol{d}^H(\theta)\widehat{\boldsymbol{\Pi}}(t)\boldsymbol{a}(\theta)]}{\boldsymbol{d}^H(\theta)\widehat{\boldsymbol{\Pi}}(t)\boldsymbol{d}(\theta)}\Big|_{\theta=\widetilde{\theta}_j(t-1)}$$

$\dot{\theta}_j(t) = L^{-1}[(L-1)\dot{\theta}_j(t-1) + \widehat{\theta}_j(t) - \widehat{\theta}_j(t-1)]$;

$\ddot{\theta}_j(t) = L^{-1}[(L-1)\ddot{\theta}_j(t-1) + \dot{\theta}_j(t) - \dot{\theta}_j(t-1)]$;

$\widetilde{\theta}_j(t) = \widehat{\theta}_j(t) + \dot{\theta}_j(t) + \ddot{\theta}_j(t)/2$;

END_FOR

where $\text{orth}(\widehat{\boldsymbol{Q}}_s)$ returns an orthonormal basis of the columns of $\widehat{\boldsymbol{Q}}_s$, $\boldsymbol{d}(\theta_j) = d\boldsymbol{a}(\theta_j)/d\theta_j$ and $\textbf{Update\_A}(\theta)$ is a procedure for computing the steering matrix using θ. The implementation of the proposed DOA tracking algorithm is $O(MK^2) + 3MK + O(K^2)$ complex multiplications. It is important to note that according to this DOA tracking scheme, the data association problem does not arise: the association problem is embedded in the problem of estimating the subspaces and the DOA's. More elaborated DOA tracking algorithms involving motion dynamics are described in [13, 14]. Note also that a similar technique based on the predicted DOA's has recently been discussed in [15].

4.2. Tracking crossing targets

In the definition of $\boldsymbol{P}$, the inverse of $\boldsymbol{A}_1$ can be a source of numerical instability. It occurs when the DOA's tend to be equal. We propose some modifications to take into account situations where the targets may cross. Consider the following criterion

$$|\theta_i(t) - \theta_j(t)| \geq \tau, \quad j \neq i \tag{13}$$

where τ is a chosen threshold value. If some targets in (13) are close to each other, we update $\boldsymbol{A}$ by considering only one of the targets within a distance of τ. During the crossing time period, the tracks of the remaining targets are estimated using the most recent estimates of θ, $\dot{\theta}$ and $\ddot{\theta}$ available: $\widehat{\theta}_j(t) = \widehat{\theta}_j(t-1) + \dot{\theta}_j(t-1) + \ddot{\theta}_j(t-1)/2$. Obviously, the rank of the resulting steering matrix $\boldsymbol{A}'$ will be less than K and, hence, the inverse of $\boldsymbol{A}_1$ should be replaced by its pseudo-inverse such that $\boldsymbol{P} = \boldsymbol{A}_1'(\boldsymbol{A}_1'^H\boldsymbol{A}_1')^{-1}\boldsymbol{A}_2'^H$ where $\boldsymbol{A}_1'$ and $\boldsymbol{A}_2'$ are obtained from the partition (4) of $\boldsymbol{A}'$.

5. NUMERICAL EXAMPLES

We here present some simulation results illustrating the properties of the discussed subspace-based linear methods for tracking slowly moving sources. In the first example we estimate the DOA's of four uncorrelated signals impinging on a uniform linear array of 10 sensors. The sensor separation is half a wavelength. They have an SNR of 0 dB, 5 dB, 4 dB and 4 dB, respectively. We apply BEWE and the PM after having updated the sample covariance matrix via (3) with a forgetting factor $\beta = .97$. After each subspace update, we apply TLS-ESPRIT to compute the DOA's from the signal subspace update. Note that SWEDE cannot be used in this scenario because it requires $M \geq 3K$ [6]. Figures 1(a) and (b) exhibit the simulation results. In a second example, the same scenario is considered except that the sources are closely spaced. We again apply the PM and the proposed LAST algorithm with predicted DOA's developed in Subsection 4.1. The length of the moving rectangular window is $L = 50$ and $\beta = .97$. Figures 1(c) and (d) depict the obtained results. In the last experiment, we investigate the performance of LAST with predicted DOA's for tracking sources with crossing angles. For this purpose, we assume that there are one source with constant DOA and SNR=0 dB and four moving sources with 4 dB each. We set $\tau = 2°$, $\beta = 0.97$ and $L = 80$. Fig. 2(a) displays the LAST DOA estimates. As a comparison, we use the exact eigenvalue decomposition. The sample covariance matrix is updated using (3) and $\beta = 0.97$. Then, we apply TLS-ESPRIT to compute both the signal subspace and the DOA's. The results are shown in plot (b) of Fig. 2.

We make a few observations from these figures.

- In the example involving well-separated moving sources (Fig. 1(a) and (b)), BEWE and the PM perform similarly and quite well.

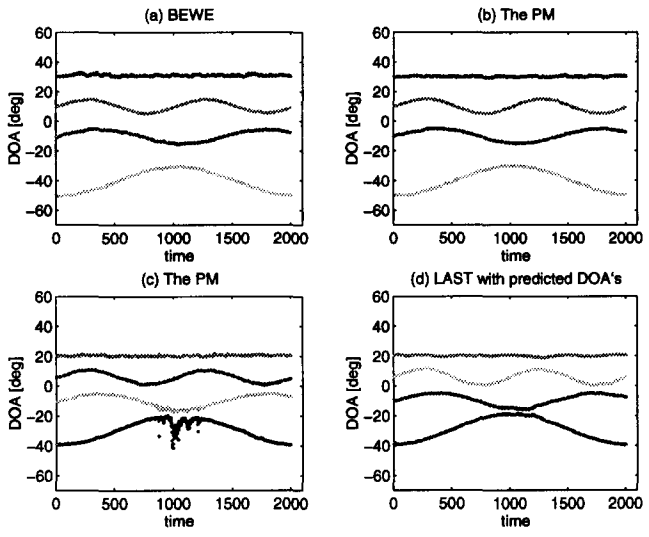

Figure 1: DOA estimates for well-separated ((a) and (b)), and closely-spaced ((c) and (d)) signal sources.

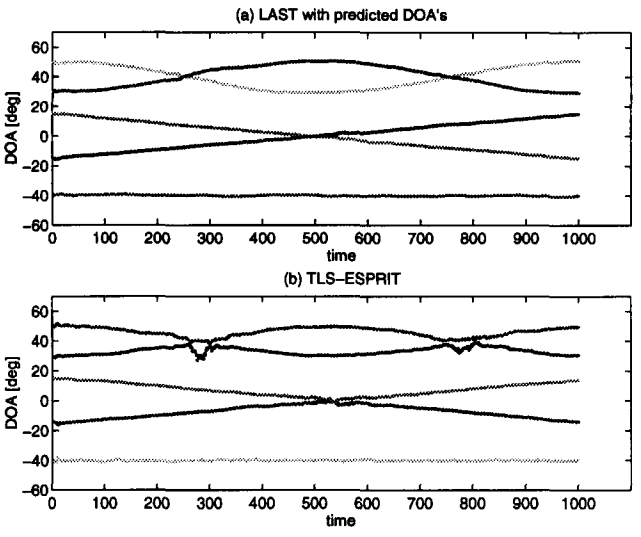

Figure 2: DOA estimates with crossing targets.

- In Fig.1(c), while the PM fails to work when two targets are closely spaced, LAST performs satisfactorily. Fig.1(d) shows that the prediction of the DOA's improves the robustness of this algorithm in resolving closely spaced signals.
- In the case where some of the targets cross (Fig. 2(a) and (b)), the performances of TLS-ESPRIT are degraded. Moreover, the estimates are not correctly associated with previous estimates when the targets temporarily overlap. On the contrary, the proposed LAST algorithm with predicted DOA's shows excellent tracking performance.

6. CONCLUDING REMARKS

In this paper we have discussed the applicability of the so-called subspace-based linear methods for tracking the subspaces recursively. We retained the basic ideas of one of this methods and we showed that the subspaces can be obtained as a solution of an unconstrained minimization problem. The main interest of this interpretation relies on the possibility of using predicted DOA's. We suggested an algorithm for tracking the DOA's and we give some computer simulations demonstrating the success of this approach. Future works will be a theoretical analysis of the convergence property and a comparative study with other subspace tracking algorithms.

7. REFERENCES

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