ROBUST ADAPTIVE BEAMFORMING USING DATA DEPENDENT CONSTRAINTS

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ABSTRACT

An adaptive beamformer which is robust to uncertainty in source DOA is derived. The beamformer is a weighted sum of minimum variance distortionless response (MVDR) beamformers pointed at a set of candidate DOAs, where the relative contribution of each MVDR beamformer is determined from a combination of observed data and prior knowledge about the DOA. When SNR is high, the MVDR beamformer whose look direction is closest to the source dominates, and nearly optimal performance is obtained. When SNR is low, the weighted combination of beamformers has a wider main beam which is robust to DOA uncertainty.

1. INTRODUCTION

Adaptive beamforming is a method for estimating a desired signal impinging on an array of sensors. When the direction-of-arrival (DOA) of the source is known, the minimum variance distortionless response (MVDR) beamformer [1] provides a distortionless version of the signal while suppressing noise and interference. However, if there is a mismatch between the source DOA and the look direction of the beamformer, there can be significant degradation in performance, particularly at high signal-to-noise ratio (SNR) [2].

Numerous methods have been proposed to improve robustness to pointing errors. Data driven approaches use observed data to correct the constraint subspace to match the actual subspace of the desired signal. This works well when good subspace estimates can be obtained, i.e. for high SNR and/or long observation time. However, there can be significant mismatch when reliable estimates cannot be obtained. Data independent approaches such as linearly constrained minumum variance (LCMV) beamforming reduce degradation by imposing additional linear constraints [3] (see also [4] and the references therein). Robustness to pointing error is achieved at the expense of sub-optimal performance under ideal conditions.

We develop an adaptive beamformer which combines the responsiveness of the data driven approach with the robustness of the data independent approach. Like other data driven beamformers, the proposed beamformer tends to point in the direction of the desired signal when SNR is high and DOA can be estimated reliably. When SNR is low, the beampattern has wider main beam which is robust to DOA uncertainty.

2. BACKGROUND

Consider the problem of estimating the waveform of a narrowband planewave signal incident on an array of N sensors from DOA θ_o in the presence of noise and interference. The $N \times 1$ vector of received signals is given by:

$$\mathbf{x}(t) = \mathbf{a}(\theta_o)s_o(t) + \sum_{i=1}^{d} \mathbf{a}(\theta_i)s_i(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{a}(\theta_i)$ is the $N \times 1$ steering vector in the direction θ_i, $s_o(t)$ is the desired signal, $s_i(t), i = 1, \ldots, d$, are interference signals, and $\mathbf{n}(t)$ is the $N \times 1$ vector of additive noise.

The output of the beamformer is a linear estimate of the desired signal, i.e.,

$$y(t) = \mathbf{w}^H \mathbf{x}(t). \tag{2}$$

In MVDR beamforming, the weights $\mathbf{w}$ are chosen to minimize the output power of the beamformer, $E\left\{|y(t)|^2\right\}$, while maintaining a distortionless response in the direction of the desired signal. The weights are found from the solution to

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R_x} \mathbf{w} \quad \text{subject to} \quad \mathbf{a}(\theta)^H \mathbf{w} = 1, \tag{3}$$

where $\mathbf{R_x}$ is the data correlation matrix

$$\mathbf{R_x} = E\left\{\mathbf{x}(t)\mathbf{x}(t)^H\right\}. \tag{4}$$

The MVDR weights are given by

$$\mathbf{w}_{MV} = \frac{\mathbf{R_x}^{-1}\mathbf{a}(\theta_o)}{\mathbf{a}(\theta_o)^H \mathbf{R_x}^{-1}\mathbf{a}(\theta_o)}. \tag{5}$$

In practice, the data correlation matrix $\mathbf{R_x}$ is not known, and the beamformer weights in (5) are implemented by substituting an estimate of $\mathbf{R_x}$ such as the sample correlation matrix obtained from K snapshots of the data

$$\hat{\mathbf{R}}_K = \frac{1}{K}\sum_{i=1}^{K} \mathbf{x}(t_i)\mathbf{x}(t_i)^H. \tag{6}$$

In minimum mean square error (MMSE) beamforming, the mean square error betwen the desired signal and the beamformer output, $E\left\{|y(t) - s_o(t)|^2\right\}$, is minimized. In this case, the weights are given by the Wiener filter,

$$\mathbf{w}_{MS} = \mathbf{R_x}^{-1}\mathbf{a}(\theta_o)\sigma_s^2 \tag{7}$$

where $\sigma_s^2 = E\{|s_o(t)|^2\}$ is the desired signal power. Although derived under different considerations, the MMSE weight vector (7) is just a scaled version of the MVDR weight vector (5). In practice, the presumed DOA θ_s is used in place of the true DOA θ_o, and $\mathbf{R_x}$ is replaced by $\hat{\mathbf{R}}_K$. If σ_s^2 is unknown, it must also be replaced by a suitable estimate. One method for estimating σ_s^2 is to use the value of the MVDR spatial spectum estimate at θ_s, $\hat{\sigma}_s^2 = \left(\mathbf{a}(\theta_s)^H\hat{\mathbf{R}}_K^{-1}\mathbf{a}(\theta_s)\right)^{-1}$. In this case, the MMSE and MVDR beamformers are identical and have the form

$$\mathbf{w}_{MS} = \mathbf{w}_{MV} = \frac{\hat{\mathbf{R}}_K^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}(\theta_s)^H\hat{\mathbf{R}}_K^{-1}\mathbf{a}(\theta_s)}. \quad (8)$$

The MVDR beamformer can suffer significant performance degradation when the steering direction θ_s is not the same as the source DOA θ_o. This is because the beamformer preserves all energy in the one dimensional steering vector subspace, and minimizes the remaining energy in the $N-1$ dimensional orthogonal subspace. When the true DOA of the signal does not match the steering vector, only the portion of the signal which is in the constraint subspace is preserved. The remaining portion acts like interference, and the beamformer attempts to minimize it along with the rest of the noise and interference.

Numerous techniques have been proposed for improving pointing accuracy. In the maximum energy approach [1], the MVDR beamformer is scanned over the range of possible DOAs, and the weights which maximize the output power are chosen. This is equivalent to using the MVDR direction finding (DF) algorithm to estimate $\hat{\theta}_s$, and then substituting the estimated DOA for the look direction. In general, other DF techniques can also be used to estimate the DOA. This technique works well when the observed data is sufficient to yield a good DOA estimate. However, if the DOA estimate is poor, the beamformer will not necessarily point to the desired signal, and the mismatch can even be worse than when using the presumed steering vector θ_s.

Eigenspace (ES) beamformers [6] correct the presumed steering vector by projecting it onto an estimated signal plus interference subspace. Letting $\hat{\mathbf{E}}_{S+I}$ denote the signal plus interference eigenvectors of $\hat{\mathbf{R}}_K$, the resulting constraint vector is $\mathbf{a}_s = \hat{\mathbf{E}}_{S+I}\hat{\mathbf{E}}_{S+I}^H\mathbf{a}(\theta_s)$. Like the DF approach, this method works well when good estimates of the signal plus interference subspace can be obtained, but performs poorly otherwise.

In the more conservative and robust approach of LCMV beamforming, additional constraints of the form $\mathbf{C}^H\mathbf{w} = \mathbf{f}$ are added to protect the signal over a wider range of DOAs. The LCMV weights are given by

$$\mathbf{w} = \mathbf{R_x}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R_x}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} \quad (9)$$

where $\mathbf{C}$ is the $N \times P$ matrix of constraint vectors and $\mathbf{f}$ is the $P \times 1$ vector of constraint values. Many types of constraints have been proposed in the context of LCMV beamforming. Some examples are directional constraints, derivative constraints, eigenvector constraints, and quiescent pattern constraints. The additional constraints protect the desired signal but reduce the adaptive degrees of freedom used for noise and interference suppression. As a consequence, degradation due to mismatch is greatly reduced at the expense of sub-optimal performance under ideal conditions. Quadratic constraints, which are equivalent to artificial noise injection, also reduce sensitivity [5].

3. BAYESIAN BEAMFORMER

In this paper, an adaptive beamformer which is robust to uncertainty in source DOA is derived using a Bayesian approach similar to [7]. We assume that θ is a random parameter with a priori probability density function (pdf) $p(\theta)$, which reflects the level of uncertainty in the source DOA. For computational simplicity, it is assumed that $p(\theta)$ is defined only on a discrete set of M points, $\Theta = \{\theta_1 \cdots \theta_M\}$, in the a priori parameter space. Let $\mathbf{x}_L$ denote L snapshots of the received data vector taken at times $t_1, \ldots, t_L$. The MMSE estimate of the desired signal is the conditional mean of $s_o(t)$ given $\mathbf{x}_L$,

$$\begin{aligned} y(t) &= E\{s_o(t)|\mathbf{x}_L\} = E\{E\{s_o(t)|\mathbf{x}_L,\theta\}\} \quad (10) \\ &= \sum_{i=1}^{M} p(\theta_i|\mathbf{x}_L)E\{s_o(t)|\mathbf{x}_L,\theta_i\}, \quad (11) \end{aligned}$$

where $p(\theta_i|\mathbf{x}_L)$ is the a posteriori pdf of θ given the observations,

$$p(\theta_i|\mathbf{x}_L) = \frac{p(\theta_i)p(\mathbf{x}_L|\theta_i)}{\sum_{k=1}^{M} p(\theta_k)p(\mathbf{x}_L|\theta_k)}. \quad (12)$$

When the desired signal and observations are jointly Gaussian given θ, the conditional mean $E\{s_o(t)|\mathbf{x}_L,\theta_i\}$ is the output of the MMSE beamformer (7) pointed at θ_i. Under these assumptions, (11) becomes

$$y(t) = \sum_{i=1}^{M} p(\theta_i|\mathbf{x}_L)\mathbf{w}_{MS}^H(\theta_i)\mathbf{x}(t). \quad (13)$$

As in the known DOA case, we substitute $\hat{\mathbf{R}}_K$ for $\mathbf{R_x}$, and at each θ_i, $\hat{\sigma}_s^2(\theta_i) = \left(\mathbf{a}(\theta_i)^H\hat{\mathbf{R}}_K^{-1}\mathbf{a}(\theta_i)\right)^{-1}$. The beamformer weights are then a linear combination of MVDR beamformers, weighted by the a posteriori pdf.

Under the Gaussian assumption, $p(\mathbf{x}_L|\theta_i)$ is a Gaussian density with zero mean and covariance $\mathbf{R_x}(\theta_i)$ given by

$$\mathbf{R_x}(\theta_i) = \sigma_s^2\mathbf{a}(\theta_i)\mathbf{a}(\theta_i)^H + \mathbf{R_n}, \quad (14)$$

where $\mathbf{R_n}$ is the interference plus noise correlation matrix. When there are no interferers $\mathbf{R_n} = \sigma_n^2\mathbf{I}$, and the a posteriori pdf has the form

$$p(\theta_i|\mathbf{x}_L) = \frac{p(\theta_i)\exp\left\{\beta L\mathbf{a}(\theta_i)^H\hat{\mathbf{R}}_L\mathbf{a}(\theta_i)\right\}}{\sum_{k=1}^{P} p(\theta_k)\exp\left\{\beta L\mathbf{a}(\theta_k)^H\hat{\mathbf{R}}_L\mathbf{a}(\theta_k)\right\}}. \quad (15)$$

where $\hat{\mathbf{R}}_L$ is the sample correlation matrix of $\mathbf{x}_L$ and β is a monotonically increasing function of SNR (σ_s^2/σ_n^2). The denominator in (15) is a normalization constant and the numerator is a monotonic function of the conventional beamformer spatial spectrum estimate $\mathbf{a}(\theta_i)^H\hat{\mathbf{R}}_L\mathbf{a}(\theta_i)$. At high SNR it will tend to have a peak near the source DOA, and at low SNR it will be relatively flat over all DOAs. As a consequence, $p(\theta_i|\mathbf{x}_L)$ will have the same behavior, magnified by the exponential function. When SNR is high, the a posteriori pdf will be sharply peaked near the true DOA, and the Bayesian beamformer will reduce to an MVDR beamformer pointed to the θ_i closest to the true DOA θ_s. When SNR is low, the a posteriori pdf will revert to the a priori pdf, and the beampattern will have a wide mainbeam over the a priori parameter space. This case was studied in [8].

When interferers are present, $p(\theta_i|\mathbf{x}_L)$ is a function of $\mathbf{R_n}$, which is not known. Rather than try to approximate the a posteriori pdf, we use the intuition gained from the white noise case to define a data dependent weighting function $q(\theta|\mathbf{x}_L)$ as follows:

$$q(\theta_i|\mathbf{x}_L) = \alpha\, p(\theta_i) \exp\left\{\beta L \left(\mathbf{a}(\theta_i)^H \hat{\mathbf{R}}_L^{-1} \mathbf{a}(\theta_i)\right)^{-1}\right\}, \quad (16)$$

where α is a normalization constant. The form of $q(\theta_i|\mathbf{x}_L)$ is the same as (15), with the MVDR spatial spectrum estimate $\mathbf{a}(\theta_i)^H \hat{\mathbf{R}}_K^{-1} \mathbf{a}(\theta_i)$ replacing the conventional beamformer spatial spectrum estimate. This weighting function provides better performance than (15) when interferers are present. In (15) β was a function of the SNR, which is not usually known, but β can be viewed as a variable which may be adjusted to tune the responsiveness of beamformer to the source SNR, just as the number of snapshots L can be chosen to tune temporal responsiveness. Note that the number of snapshots, K, used in estimating $\hat{\mathbf{R}}_K$ and the number of snapshots, L, used in calculating $q(\theta_i|\mathbf{x}_L)$ need not be the same. The beamformer is updated in two steps. First the weighting function is found from (16), then the weights are calculated from

$$\mathbf{w}_B = \sum_{i=1}^{M} q(\theta_i|\mathbf{x}_L)\mathbf{w}_{MV}(\theta_i). \quad (17)$$

4. EXAMPLE

In Figures 1-4, typical performance of the proposed Bayesian beamformer is compared to an LCMV beamformer using quiescent pattern constraints [9], a DF-based beamformer, and an eigenspace beamformer. The array is a uniform linear array (ULA) with half-wavelength spacing and $N = 20$ elements. The a priori uncertainty in the DOA is over the region $u = \sin(\theta) \in [-0.2, 0.2]$. For an 20-element array, this interval is twice the null-to-null beamwidth of the conventional beampattern. For the LCMV beamformer, a total of 5 constraints were used. The DF beamformer used the MUSIC algorithm for DOA estimation. In the Bayesian beamformer, the set Θ is composed of $M = 33$ evenly spaced points on the interval $[-0.2, 0.2]$. In all of the beamformers, artificial noise injection at a level of 0 dB was used for improved sidelobe control.

Performance is compared for a scenario with a desired source from DOA $u_s = 0.14$, and two uncorrelated interferers with DOAs $u_{i1} = -0.5$ and $u_{i2} = 0.6$. The interference to noise ratio (INR) was 20 dB. For the desired signal, a high SNR (0 dB) and low SNR (-20 dB) case are shown.

Figures 1 and 3 show the weighting function $q(u_i|\mathbf{x}_L)$ and typical beampatterns for a single trial in the two cases, and Figures 2 and 4 show a histogram of array gain for the different beamformers obtained from 500 trials. In the high SNR case, $q(u_i|\mathbf{x}_L)$ is sharply peaked near the true DOA. The Bayesian beamformer, as well as the DF and ES beamformers all point to the source while nulling the interference. The ES beamformer is somewhat superior in both sidelobe control and array gain. The array gain for all three beamformers is relatively stable over all trials and close to the optimal value of 36 dB. The LCMV beamformer has a lower array gain, close to 30 dB. At low SNR, $q(u_i|\mathbf{x}_L)$ is nearly constant, implying that the observations provide little information about the source DOA. As a consequence, neither the source DOA nor the signal plus interference subspace can be accurately estimated. The DF beamformer does not always point at the desired signal, and the ES beamformer has a random pattern. The histogram of array gain values for the DF-based beamformer shows that the DOA estimate is accurate enough to provide optimal performance only about half of the time, and can be so inaccurate as to reduce array gain to as low as 0 dB. The ES beamformer has even worse performance. Our Bayesian beamformer is now more robust, with a wide beam covering the entire a priori interval. The array gain is stable near 30 dB, which is about the same as the LCMV processor.

5. REFERENCES

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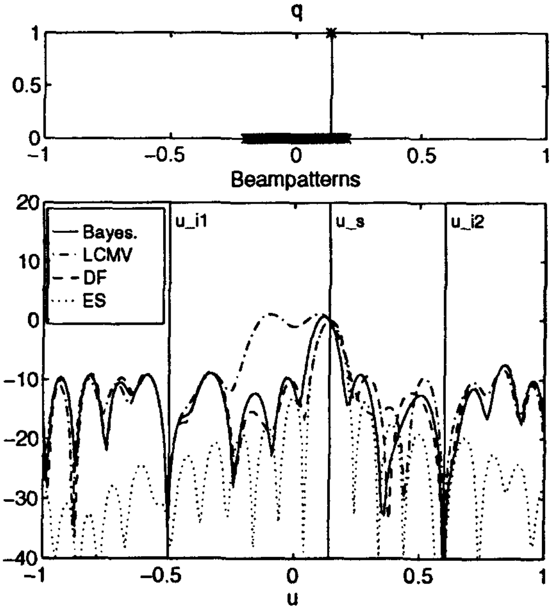

Figure 1: $p(u)$ and beampatterns of adaptive beamformers for N=20 element ULA. The desired signal is at $u_s = 0.14$ with SNR = 0 dB and the interferers are at u_{i1} = -0.5 and u_{i2} = 0.6 with INR = 20 dB. A priori interval = [-0.2,0.2]. Number of snapshots $K = L = 100$.

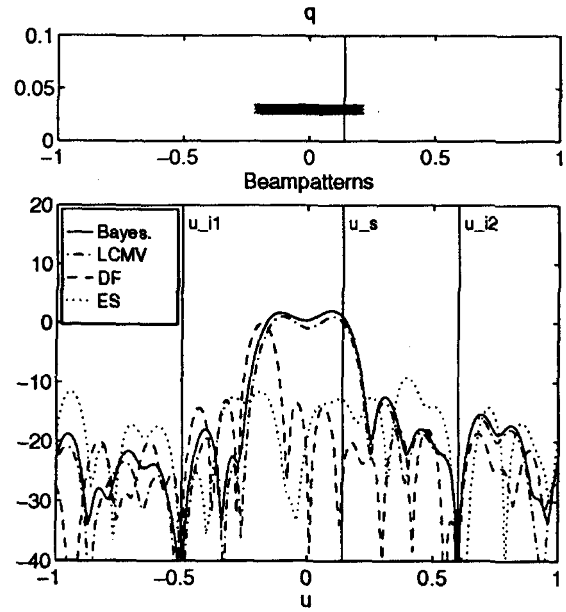

Figure 3: Same as Figure 1 with SNR = -20 dB.

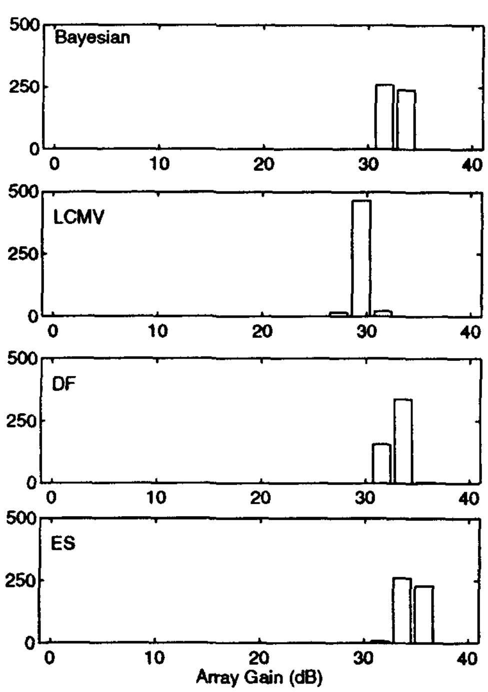

Figure 2: Histogram of array gain for beamformers from 500 trials for SNR = 0 dB.

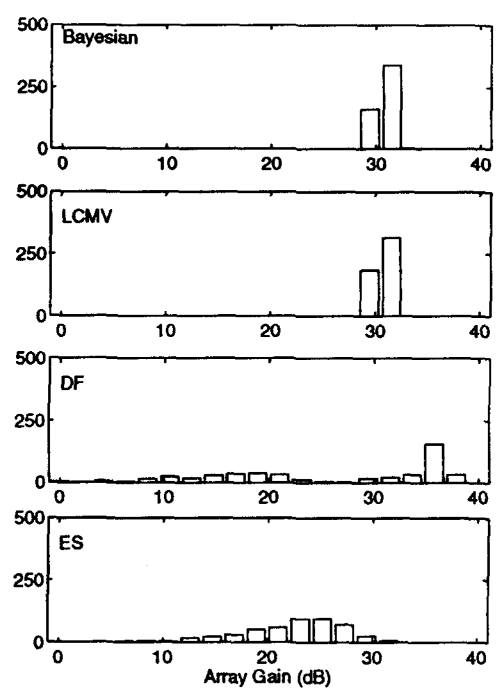

Figure 4: Same as Figure 2 with SNR = -20 dB.