

A NEW MODEL FOR NON-RAYLEIGH CLUTTER: SPACE-TIME ADAPTIVE PROCESSING IN STABLE IMPULSIVE INTERFERENCE

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ABSTRACT

This paper studies methods for Space-Time Adaptive Processing-based (STAP-based) parameter estimation in the presence of *impulsive noise backgrounds*. Towards this goal, the theory of alpha-stable random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive radar clutter. Our goal is to develop joint target angle and Doppler, maximum likelihood-based estimation techniques from radar measurements retrieved in the presence of severe clutter modeled as an alpha-stable, complex random process. We derive the Cramér-Rao bounds for the additive Cauchy interference scenario to assess the best-case estimation accuracy which can be achieved. The results are of great importance in the study of space-time adaptive processing (STAP) for airborne pulse Doppler radar arrays operating in impulsive interference environments.

1. INTRODUCTION

Future advanced airborne radar systems must be able to detect, identify, and estimate the parameters of a target in severe interference backgrounds. As a result, the problem of clutter and jamming suppression has been the focus of considerable research in the radar engineering community [1, 2, 3]. It is recognized that effective clutter suppression can be achieved only on the basis of appropriate statistical modeling. Most of the theoretical work in detection and estimation for radar applications has focused on the case where clutter is assumed to follow the Gaussian model. The Gaussian assumption is frequently motivated by the physics of the problem and it often leads to mathematically tractable solutions. Recently, experimental results have been reported where clutter returns are impulsive in nature [4]. In addition, a statistical model of impulsive interference has been proposed, which is based on

the theory of symmetric alpha-stable ($S\alpha S$) random processes [5]. The model is of a statistical-physical nature and has been shown to arise under very general assumptions and to describe a broad class of impulsive interference. In addition, the theory of *complex isotropic alpha-stable* random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive radar clutter.

Space Time Adaptive Processing (STAP) has been introduced as a generalization of displaced-phase-center antenna (DPCA) processing and has been recognized as the technology which will enable long-range detection of increasingly smaller targets in the presence of severe clutter and jamming. STAP refers to adaptive antenna processors that simultaneously combine the signals received on multiple elements of an antenna array and from multiple pulse repetition periods of a radar coherent processing interval [6]. In other words, the STAP processor can be viewed as a two-dimensional (2-D) filter which performs both beamforming (spatial filtering) and Doppler (temporal) filtering to suppress interference and achieve target detection and parameter estimation. As a result, several researchers have concentrated on the theoretical advancement and understanding of the STAP discipline [6, 7, 8, 9].

As mentioned in [6], much of the work reported for radar systems has concentrated on *target detection* in Gaussian or non-Gaussian backgrounds [2, 10, 11, 12, 13, 14]. In this paper, we address the *target parameter estimation problem* through the use of STAP radar array sensor data retrieved in the presence of impulsive interference. In Section 2, we present necessary preliminaries on α -stable processes and results on the modeling of real clutter data by means of $S\alpha S$ distributions. In Section 3, we formulate the space-time adaptive processing (STAP) problem for airborne radar. In Section 4, we form the maximum likelihood function, we present the Cramér-Rao analysis, and we derive bounds on the variances of the spatial and temporal frequency estimates. In particular, we derive Cramér-Rao bounds on angle and Doppler estimator accuracy for the case of additive Cauchy noise.

The work in this paper was supported by Rome Laboratory under Contract F30602-95-1-0001 and by the Office of Naval Research under Contract N00014-92-J-1034.

2. MATHEMATICAL PRELIMINARIES

In this section, we introduce the statistical model that will be used to describe the additive noise. The model is based on the class of *isotropic S α S* distributions, and is well-suited for describing impulsive noise processes [5].

A complex random variable (r.v.) $X = X_1 + jX_2$ is isotropic *S α S* if X_1 and X_2 are jointly *S α S* and have a symmetric distribution. The characteristic function of X is given by

$$\varphi(\omega) = \mathcal{E}\{\exp(j\Re[\omega X^*])\} = \exp(-\gamma|\omega|^\alpha), \quad (1)$$

where $\omega = \omega_1 + j\omega_2$. The *characteristic exponent* α is restricted to the values $0 < \alpha \leq 2$ and it determines the shape of the distribution. The smaller the characteristic exponent α , the heavier the tails of the density. The *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Stable processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem. They are described by their characteristic exponent α , taking values $0 < \alpha \leq 2$. Gaussian processes are stable processes with $\alpha = 2$. Stable distributions have heavier tails than the normal distribution, possess finite p th order moments only for $p < \alpha$, and are appropriate for modeling noise with outliers.

Figure 1 shows results on the modeling of the statistics of real radar clutter from the Mountaintop Database by means of four distributions. The three, namely Rayleigh, K, and Weibull have been used extensively in the past by radar engineers to model clutter [15, 16, 17, 18]. The impulsive nature of the clutter data is obvious in Figure 1 which demonstrates that the *S α S* distribution, whose tails decay algebraically, is superior to the Rayleigh, K, and Weibull distributions for modeling the particular radar clutter data under study. The reason why *S α S* distributions are such flexible modeling tools is that *the characteristic exponent α allows us to represent signals and/or noise with a continuous range of impulsiveness*.

3. STAP PROBLEM FORMULATION

Space-time adaptive processing (STAP) refers to multidimensional adaptive algorithms that simultaneously combine the signals from the elements of an array antenna and the multiple pulses of a coherent radar waveform to suppress interference and provide target detection [19, 6].

Consider a uniformly spaced linear radar array antenna consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) $f_r = 1/T_r$ and over a certain range of directions of interest. The array receives signals generated by q narrow-band moving targets which are located at azimuth angles $\{\phi_k; k = 1, \dots, q\}$ and have relative velocities with respect to the radar $\{\nu_k; k = 1, \dots, q\}$ corresponding to Doppler frequencies $\{f_k; k = 1, \dots, q\}$. Since the signals

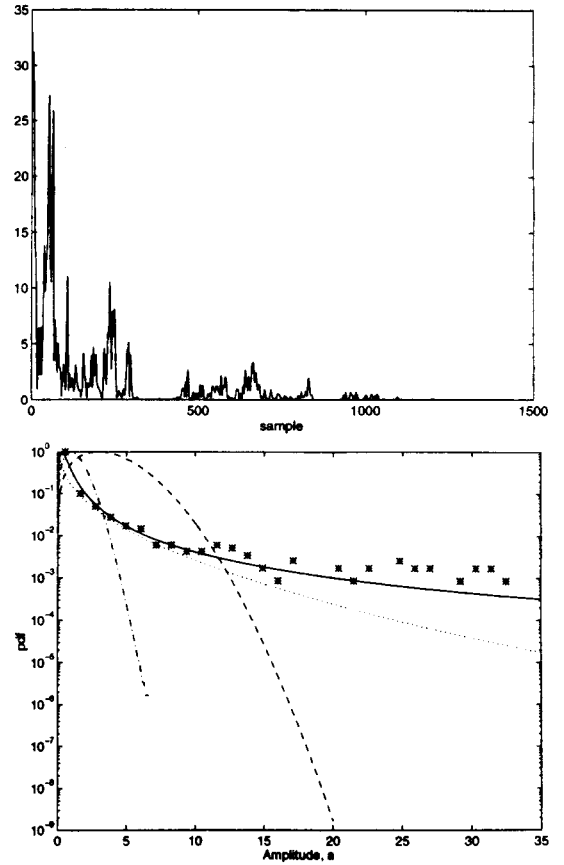


Figure 1: The magnitude of the Mountaintop clutter data corresponding to $\theta = 315^\circ$ in azimuth and $[r_{min} \ r_{max}] = [3.5 \ 80.0]$ nmi in range (top). Probability density function comparison (bottom). Stars, *, denote the empirical curves. Dashed: Rayleigh with $\sigma = 2.9743$, solid: *S α S* with $\alpha = 1.0304$, $\gamma = 0.2068$, dashdot: K-distribution with $\nu = 15.8210$, $a = 0.1738$, and dotted: Weibull with $\eta = 0.57$, $\sigma = 0.7765$.

are narrow-band, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as [19]:

$$\mathbf{x}(t) = \mathbf{V}(\psi, \omega)\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where

- $\mathbf{x}(t) = [x_1(t), \dots, x_{MN}(t)]^T$ is the array output vector (N : number of array elements, M : number of pulses, t may refer to the number of the coherent processing intervals (CPI's) available at the receiver);
- $\mathbf{s}(t) = [\beta_1(t), \dots, \beta_q(t)]^T$ is the signal vector emitted by the sources as received at the reference sensor 1 of the array;
- $\mathbf{V}(\psi, \omega) = [\mathbf{v}(\psi_1, \omega_1), \dots, \mathbf{v}(\psi_q, \omega_q)]$ is the *space-time steering matrix* where $\psi_k = 2\pi \frac{d}{\lambda_0} \sin(\phi_k)$ is the normalized angle and $\omega_k = 2\pi \frac{f_k}{f_r}$ is the normalized Doppler;

- $\mathbf{v}(\psi_k, \omega_k) = \mathbf{b}(\omega_k) \otimes \mathbf{a}(\psi_k)$ is the *space-time steering vector*, where $\otimes$ denotes the Kronecker matrix product, and
 - $\mathbf{a}(\psi_k) = [1, e^{-j\psi_k}, \dots, e^{-j(N-1)\psi_k}]^T$ is the *spatial steering vector*;
 - $\mathbf{b}(\omega_k) = [1, e^{-j\omega_k}, \dots, e^{-j(M-1)\omega_k}]^T$ is the *temporal steering vector*.

- $\mathbf{n}(t) = [n_1(t), \dots, n_{MN}(t)]^T$ is the noise vector whose components are modeled as Cauchy-distributed random variables with pdf given by

$$\chi_\gamma(r) = \frac{\gamma}{2\pi(r^2 + \gamma^2)^{3/2}}, \quad (3)$$

where the *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Assuming the availability of S coherent processing intervals (CPI's) $t_1, \dots, t_S$, the data can be expressed as

$$\mathbf{X} = \mathbf{V}(\psi, \omega)\mathbf{S} + \mathbf{N}, \quad (4)$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $MN \times S$ matrices

$$\mathbf{X} = [\mathbf{x}(t_1), \dots, \mathbf{x}(t_S)], \quad (5)$$

$$\mathbf{N} = [\mathbf{n}(t_1), \dots, \mathbf{n}(t_S)], \quad (6)$$

and $\mathbf{S}$ is the $q \times S$ matrix

$$\mathbf{S} = [\mathbf{s}(t_1), \dots, \mathbf{s}(t_S)]. \quad (7)$$

Our objective is to jointly estimate the directions-of-arrival $\{\phi_k; k = 1, \dots, q\}$ and the Doppler frequencies $\{f_k; k = 1, \dots, q\}$ of the source targets.

4. THE CRAMÉR-RAO BOUND FOR CAUCHY NOISE

The Cramér-Rao bound for the error variance of an unbiased estimator $\hat{\Theta}$ satisfies

$$\mathbf{C}_{\hat{\Theta}} - \mathbf{J}^{-1}(\Theta) \geq 0 \quad (8)$$

where $\mathbf{C}_{\hat{\Theta}}$ is the covariance matrix of $\hat{\Theta}$, $\mathbf{J}(\Theta)$ is the Fisher information matrix, and $\mathbf{T} \geq 0$ is interpreted as meaning that the matrix $\mathbf{T}$ is semidefinite positive. Then, the following theorem holds for the case of complex isotropic Cauchy noise.

Theorem 1 *The CRB for ψ , ω and γ is given by*

$$CRB(\psi, \omega) = \left[\sum_{t=1}^S (\Sigma - \Upsilon^T(t)\Xi\Upsilon(t)) \right]^{-1}, \quad (9)$$

and

$$CRB(\gamma) = \frac{5}{4} \frac{\gamma^2}{MNS} \quad (10)$$

where

$$\Sigma = \frac{3}{5\gamma^2} \cdot \begin{bmatrix} \Re \left\{ \sum_{t=1}^S \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{b \otimes a} \beta(t) \right\} \\ \Re \left\{ \sum_{t=1}^S \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \\ \Re \left\{ \sum_{t=1}^S \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \\ \Re \left\{ \sum_{t=1}^S \beta^H(t) \mathbf{D}_{d_b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \end{bmatrix}$$

$$\Upsilon(t) = \frac{3}{5\gamma^2} \cdot \begin{bmatrix} \Re \left\{ \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{b \otimes a} \beta(t) \right\} \\ \Re \left\{ \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \\ \Im \left\{ \beta^H(t) \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \\ \Im \left\{ \beta^H(t) \mathbf{D}_{d_b \otimes a}^H \mathbf{D}_{d_b \otimes a} \beta(t) \right\} \end{bmatrix} \quad t = 1, \dots, S$$

$$\Xi = \frac{5\gamma^2}{3} \cdot \begin{bmatrix} \left[\Re \left\{ \mathbf{D}_{b \otimes a}^H \mathbf{D}_{b \otimes a} \right\} \right]^{-1} \\ - \left[\Re \left\{ \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \right\} \right]^{-1} \\ \left[\Im \left\{ \mathbf{D}_{b \otimes a}^H \mathbf{D}_{d_b \otimes a} \right\} \right]^{-1} \\ \left[\Re \left\{ \mathbf{D}_{d_b \otimes a}^H \mathbf{D}_{d_b \otimes a} \right\} \right]^{-1} \end{bmatrix}$$

and

$$\mathbf{D}_{b \otimes a} = [\mathbf{b}(\omega_1) \otimes \mathbf{a}(\psi_1), \dots, \mathbf{b}(\omega_q) \otimes \mathbf{a}(\psi_q)],$$

$$\mathbf{D}_{d_b \otimes a} = [\mathbf{d}_b(\omega_1) \otimes \mathbf{a}(\psi_1), \dots, \mathbf{d}_b(\omega_q) \otimes \mathbf{a}(\psi_q)],$$

$$\mathbf{D}_{b \otimes d_a} = [\mathbf{b}(\omega_1) \otimes \mathbf{d}_a(\psi_1), \dots, \mathbf{b}(\omega_q) \otimes \mathbf{d}_a(\psi_q)],$$

$$\text{where } \mathbf{d}_a(\psi_j) = [d_1^a(\psi_j) \dots d_N^a(\psi_j)] = \left[\frac{\partial a_1(\psi_j)}{\partial \psi_j} \dots \frac{\partial a_N(\psi_j)}{\partial \psi_j} \right],$$

$$\mathbf{d}_b(\omega_j) = [d_1^b(\omega_j) \dots d_M^b(\omega_j)] = \left[\frac{\partial b_1(\omega_j)}{\partial \omega_j} \dots \frac{\partial b_M(\omega_j)}{\partial \omega_j} \right] \text{ and } \beta(t) \text{ is given by}$$

$$\begin{aligned} \beta(t) &= \text{diag}[\beta_1(t) \dots \beta_q(t)] \\ &= \text{diag}[\beta_{\Re,1}(t) \dots \beta_{\Re,q}(t)] + \\ &\quad j \cdot \text{diag}[\beta_{\Im,1}(t) \dots \beta_{\Im,q}(t)] \\ &= \beta_{\Re} + j \cdot \beta_{\Im} \quad t = 1, \dots, S \end{aligned}$$

where $\beta_{\Re,i}(t) = \Re\{\beta_i(t)\}$, $\beta_{\Im,i}(t) = \Im\{\beta_i(t)\}$, and $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary part operators, respectively.

Proof Given in [20].

We should note that the above bound can be achieved only when there exist unbiased estimators for all the models parameters γ , $\beta(t)$, ψ and ω . A useful insight on the CRB can be gained if we consider the case of a single target ($q = 1$) located in $\psi = \frac{2\pi d}{\lambda_0} \sin \phi$, with normalized Doppler shift $\omega = 2\pi f T_r$ and constant target amplitude β . In this case, we have

$$CRB(\phi) = \frac{\gamma^2}{S|\beta|^2} \frac{\lambda_0^2}{(2\pi d)^2} \cdot \frac{5N(\|\mathbf{d}_b\|^2 - \delta_b^2/M)}{3\xi} \cdot \frac{1}{\cos^2(\phi)} \quad (11)$$

$$CRB(f) = \frac{\gamma^2}{S|\beta|^2} \frac{1}{(2\pi T_r)^2} \cdot \frac{5M(\|\mathbf{d}_a\|^2 - \delta_a^2/N)}{3\xi}, \quad (12)$$

where

$$\delta_a = \sum_{i=1}^N |d_i^a|, \quad \delta_b = \sum_{i=1}^M |d_i^b|, \quad \rho = \sum_{i=1}^{MN} |d_{g(i)}^a| |d_{f(i)}^b|,$$

and

$$\xi = (M \| \mathbf{d}_a \|^2 - \frac{M}{N} \delta_a^2)(N \| \mathbf{d}_b \|^2 - \frac{N}{M} \delta_b^2) - (\delta_a \delta_b - \rho)^2.$$

Finally, if we consider an airborne radar system that utilizes a uniform linear array antenna and a waveform with a uniform pulse repetition interval, the bounds (11) and (12) are given by:

$$CRB(\phi) = \frac{\gamma^2}{S|\beta|^2} \cdot \frac{\lambda_0^2}{(2\pi d)^2} \cdot \frac{20}{M^2 N^2 (N^2 - 1)} \cdot \frac{1}{\cos^2(\phi)} \quad (13)$$

and

$$CRB(f) = \frac{\gamma^2}{S|\beta|^2} \cdot \frac{1}{(2\pi T_r)^2} \cdot \frac{20}{N^2 M^2 (M^2 - 1)}. \quad (14)$$

As can be seen in (11) and (12), target angle accuracy is a function of Doppler frequency and vice-versa. In addition, the bounds are functions of the generalized SNR function given by $\frac{S|\beta|^2}{\gamma^2}$, similarly to the Gaussian case where the bounds are functions of the SNR. The larger the dispersion γ of the noise, the higher the CRB.

5. CONCLUSIONS

In this paper, we addressed the modeling of the amplitude statistics of radar clutter by means of $S\alpha S$ distributions and the estimation of the parameters of the stable distributions from real clutter of the Mountain Top Database. In addition, we considered the problem of target-angle and Doppler estimation with an airborne radar employing space-time adaptive processing. In particular, we derived Cramér-Rao bounds on angle and Doppler estimator accuracy for the case of additive Cauchy interference.

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