DOPPLER-BASED MOTION ESTIMATION FOR WIDE-BAND SOURCES FROM SINGLE PASSIVE SENSOR MEASUREMENTS

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ABSTRACT

We address the problem of estimating the motion of a wide-band source from single passive sensor measurements, for example, estimation of the speed and position of a car moving on a road from the recording of its acoustic signature at a microphone located next to the road. We present a new computationally efficient method based on a time-varying ARMA model for Doppler-shifted random processes. Unlike previously proposed approaches which rely on a "local" periodicity hypothesis for the signal source, or a cyclostationary assumption, our method assumes only that the source is stationary and admits a rational (ARMA) model. The method is tested on synthetic and real acoustic data.

1. INTRODUCTION

The estimation of the motion of a point source from the signal received at fixed passive sensors is a classical problem in statistical signal and array processing, with numerous applications in communication, radar, sonar, etc. If a single passive sensor is used, the only information available on the source movements will arise from wave propagation effects. Whenever a wave source moves at a speed non-negligible with respect to the wave celerity and the wave propagate under the free field hypothesis, the received signal will be subject to dilations of its time axis when compared to the emitted signal, in addition to propagation delays. For harmonic signals, these dilations take the form of the well-known Doppler frequency shift. If some additional assumptions can be made on the nature of the source and its movement, it becomes usually possible to exploit these dilations, called here Doppler effect in general, to estimate the motion (speed and position) of the source.

In a classical Doppler system, like radar or active sonar, the signal "emitted" by the source is in fact a "reflection" of a signal generated by the active sensor. Thus, it is usually known up to a set of parameters. In this paper, the sensor is *passive*, and all that is known about the signal emitted by the source is that it can be modeled by a member of a given family of stochastic processes (*viz.*, ARMA processes).

Our formulation of the Doppler-based motion estimation from single passive sensor measurments problem is motivated by the following practical application: how to estimate the (possibly time-varying) speed and position of a vehicle moving on a known path (e.g., a car on a road) from its acoustic signature at a microphone located next to the path? Previous attempts to estimate the motion of an acoustic source from Doppler effects relied on a "periodic" assumption of some sort for the source: temporal periodicity with deterministic and/or random variations of the frequency [1][2][3][4], or cyclostationarity [5]. Here, the moving source is modeled as a stationary continuous-time rational (ARMA) process. Thus, the model can be applied to wide-band signals without harmonic components.

The paper is organized as follows. In Section 2 we define a model for Doppler-shifted rational processes corresponding to a moving (wide-band) ARMA sources. An acoustical example is also given. A method for the estimation of the motion of the source from the recordings at a fixed sensor based on that model is introduced in Section 3. Section 4 presents some experimental results with synthetic and real data. Conclusions are drawn in Section 5.

2. A MODEL FOR DOPPLER-SHIFTED RATIONAL PROCESSES

2.1. Constant Doppler Shift

Let $x_c(t)$ be a continuous-time Gaussian zero-mean stationary random process modeling the signal emitted by the moving point source S and let $y_c(t)$ be the signal observed at the fixed receiver O (Fig. 1). Denote by v_0 the absolute speed of the random source, by v_r its radial speed toward O, and let $d = |OS|$. If the radial speed v_r is constant, then, in the far field [6],

$$y_c(t) \approx \sigma x_c(\alpha t - \delta), \tag{1}$$

where

$$\alpha = 1 - \frac{v_r}{c} \tag{2}$$

is the Doppler shift factor, σ is an attenuation factor inversely proportional to d, δ is a propagation delay related to d, and c is the wave propagation speed (roughly 300 m/s in the case of acoustic waves in air, for example). Let $y[n] = y_c(nT_s)$ be a sampled version of $y_c(t)$ with sampling period T_s. We will say that $x_c(t)$ is rational if

$$S_{x_c}(\omega) = \eta \left|H_{x_c}(j\omega)\right|^2 = \eta \left|\frac{P(j\omega)}{Q(j\omega)}\right|^2, \tag{3}$$

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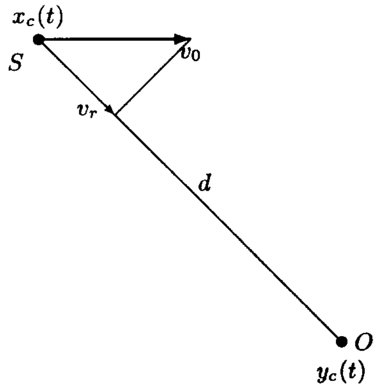

Figure 1: Radial speed of the source (S) with respect to the receiver (O).

where $P(s)$ and $Q(s)$ are polynomials in s and ω is the radian frequency. If $x_c(t)$ is rational and $x_c(t)$ is essentially band-limited so that $1/T_s$ exceeds the Nyquist rate, it was shown in [7] that $y[n]$ is also approximately rational and that its PSD is related to that of $x_c(t)$ by

$$\begin{aligned} S_y(\Omega) &\approx \frac{1}{\alpha T_s} S_{x_c}(\frac{\Omega}{\alpha}) \\ &\approx \eta \sigma^2 T_s \left| H_\alpha(e^{j\Omega}) \right|^2, \qquad \Omega \in [0, 2\pi], \end{aligned} \tag{4}$$

with

$$H_\alpha(z) = \sum_{k=1}^{p} \frac{A_k}{1 - e^{\alpha s_k T_s} z^{-1}}. \tag{5}$$

The constants A_k and s_k are defined by the partial fraction expansion

$$H_{x_c}(s) = \sum_{k=1}^{p} \frac{A_k}{s - s_k}, \tag{6}$$

where it as been further assumed that $Q(s)$ has order greater than $P(s)$ and only simple roots.

2.2. Time-Varying Doppler Shift

If the motion of the source O is such that the radial speed $v_r(t)$ and the distance $d(t)$ vary slowly enough compared to the bandwidth of the process $x_c(t)$, the received signal $y[n]$ can be assumed locally stationary with local spectrum given by (4)-(5) with time-varying Doppler parameters $\alpha(t)$ and $\sigma(t)$ function of $v_r(t)$ and $d(t)$. Thus, $y[n]$ can be represented as a time-varying ARMA model [8] of the form

$$y[n] = -\sum_{k=1}^{p} a_k[n] y[n-k] + \sum_{k=0}^{q} b_k[n] \epsilon[n-k] \tag{7}$$

where $\epsilon[n]$ is a white innovation sequence. The time-varying coefficients in (7) are related to the rational model for the source (3) and the time-varying Doppler parameters $\alpha[n] = \alpha(nT_s)$ and $\sigma[n] = \sigma(nT_s)$ by

$$\frac{\sum_{k=0}^{q} b_k[n] z^{-1}}{1 + \sum_{k=1}^{p} a_k[n] z^{-1}} = \sqrt{\eta T_s} \sigma[n] H_{\alpha[n]}(z). \tag{8}$$

For future reference, denote by $z_k[n]$, $k = 1, \ldots, p$ the time-varying poles of (7), and observe that

$$z_k[n] = e^{\alpha[n] s_k T_s}. \tag{9}$$

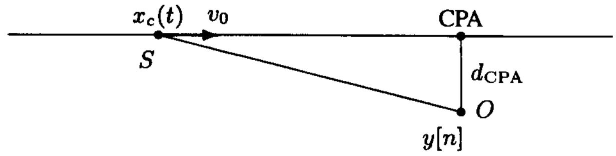

Figure 2: Source moving at a constant speed on a straight path (e.g., a car on a road).

Also, observe from (4) that the "instantaneous" power of $y[n]$, $s^2[n] = \text{var}(y[n])$ is approximately equal to the Doppler gain $\sigma^2[n]$ times a scaling factor κ independent of n,

$$s^2[n] = \sigma^2[n] \kappa. \tag{10}$$

2.3. Simulation of Doppler-shifted Processes: An Acoustical Example

Given the spectrum of the source $x_c(t)$ and a description of its motion, i.e., of $d(t)$ and $v_r(t)$, the model defined by (5)–(8) can be used to simulate a Doppler-shifted discrete time process $y[n]$ directly.

Consider the following example. Suppose that an acoustic source S, e.g., a vehicle on a road, is moving on a straight line at a constant speed v_0. The signal emitted by the source $x_c(t)$ is assumed to be stationary with the AR(4) spectrum of Fig. 3. The signal $y_c(t)$ is recorded at a microphone O located at a distance d_{CPA} of the closest point of approach of the vehicle (Fig. 2) and is sampled to yield $y[n]$. From simple geometric consideration, it can be shown that the Doppler parameters $\alpha(t)$ and $\sigma(t)$ with respect to O are related to v_0 and the time of closest approach of the source t_{CPA} by

$$\alpha(t) = 1 + \frac{v_0^2}{c} \frac{t_{CPA} - t}{\sqrt{v_0^2 (t_{CPA} - t)^2 + d_{CPA}^2}}, \tag{11}$$

$$\sigma(t) = \frac{1}{\sqrt{v_0^2 (t_{CPA} - t)^2 + d_{CPA}^2}}. \tag{12}$$

Figure 4 (a) gives the evolution of $\alpha(t)$ and $\sigma(t)$ in the case $v_0 =$ 10 km/h, $d_{CPA} = 10$ m, and $t_{CPA} = 0$ s. The wave velocity c is the speed of sound, which is taken to be equal to 330 m/s. Using (5)–(8) and taking a sampling frequency $1/T_s$ equal to 20 kHz, $y[n]$ can be modeled by a time-varying ARMA(4,3) process whose time-varying spectrum is represented in Fig. 4 (b).

This model can be used to simulate $y[n]$ by passing a white noise $\epsilon[n]$ with unit variance through the time-varying recursive filter (7) whose coefficients are defined by (8). Efficient implementation of the time-varying filter is easy via its parallel form obtained from (5). Figure 4 (c) shows the spectrogram obtained from a realization of $y[n]$ with a length 512 sliding Hanning window and 1/2 window overlap factor. Visual comparison of this spectrogram with spectrograms obtained from real Doppler-shifted data and informal listening tests validates the approach. The sample realization of $y[n]$ corresponding to the spectrogram is provided on the proceedings' CD-ROM in WAV format.

3. ESTIMATION OF THE MOTION PARAMETERS

Let us assume that the motion of the source is known up to a set of parameters θ. That is, $v_r(t) = v_r(t; \theta)$ and $d(t) = d(t; \theta)$. Thus,

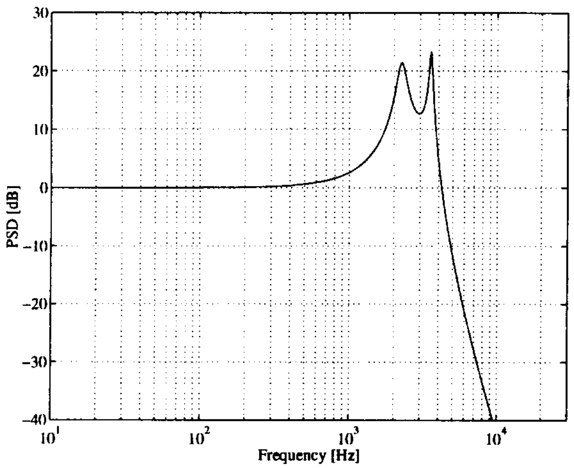

Figure 3: AR(4) spectrum of the moving noise source $x_c(t)$.

we have

$$\alpha(t) = \alpha(t;\theta) \quad \text{and} \quad \sigma(t) = \sigma(t;\theta). \tag{13}$$

In the "car pass-by" example of Section 2.3, the unknown parameters are the speed v_0 of the vehicle and the time of passage, $\theta = \{v_0, t_{\text{CPA}}\}$. The parametric model for the Doppler parameter $\alpha(t;\theta)$ and $\sigma(t;\theta)$ are given by (11) and (12). Let $\mathbf{y} = (y[1], \ldots, y[N])$ denote a sample of the signal recorded at the sensor O. The motion parameter estimation problem consist in finding θ given $\mathbf{y}$. The spectra of the moving source $S_{x_c}(\omega)$ is *a priori* unknown. Using the results of Section 2 together with (13), a computationally efficient solution can be obtained as follows.

Algorithm:

1. Fit an adaptive or a windowed batch ARMA spectral estimator to $\mathbf{y}$ using any of the standard techniques [9]. Denote by $\hat{a}_k[n]$, $k = 1, \ldots, p$, $n = 1, \ldots, N$, the coefficients of this adaptive ARMA spectral estimator and by $\hat{s}^2[n]$ its variance.
2. Compute the roots $\hat{z}_k[n]$, $k = 1, \ldots, p$, $n = 1, \ldots, N$, of the time-varying AR polynomials $1 + \hat{a}_1[n]z^{-1} + \cdots + \hat{a}_p[n]z^{-p}$.
3. Let ξ denote the extended set of parameters $\theta \cup \{\kappa\} \cup \{s_1, \ldots, s_p\}$. Compute $\hat{\xi}$ as the solution to the non-linear least-squares problem

$$\begin{aligned}\hat{\xi} \;=\; & \arg\min_{\xi} \sum_{n=1}^{N} \Big\{ \rho_1 \left|\hat{s}^2[n] - \sigma^2(nTs;\theta)\kappa\right|^2 \\ & + \rho_2 \sum_{k=1}^{p} \left|\hat{z}_k[n] - \exp[\alpha(nT_s;\theta)s_k T_s]\right|^2 \Big\}\end{aligned}$$

   for some weighting constants ρ_1 and ρ_2.
4. Take the estimator of the motion parameter $\hat{\theta}$ to be the adequate subset of $\hat{\xi}$.

Note that only the AR part of the time-varying ARMA model computed at step 1 of the algorithm is necessary later. An estimator of the AR the AR part of an ARMA model, like a windowed

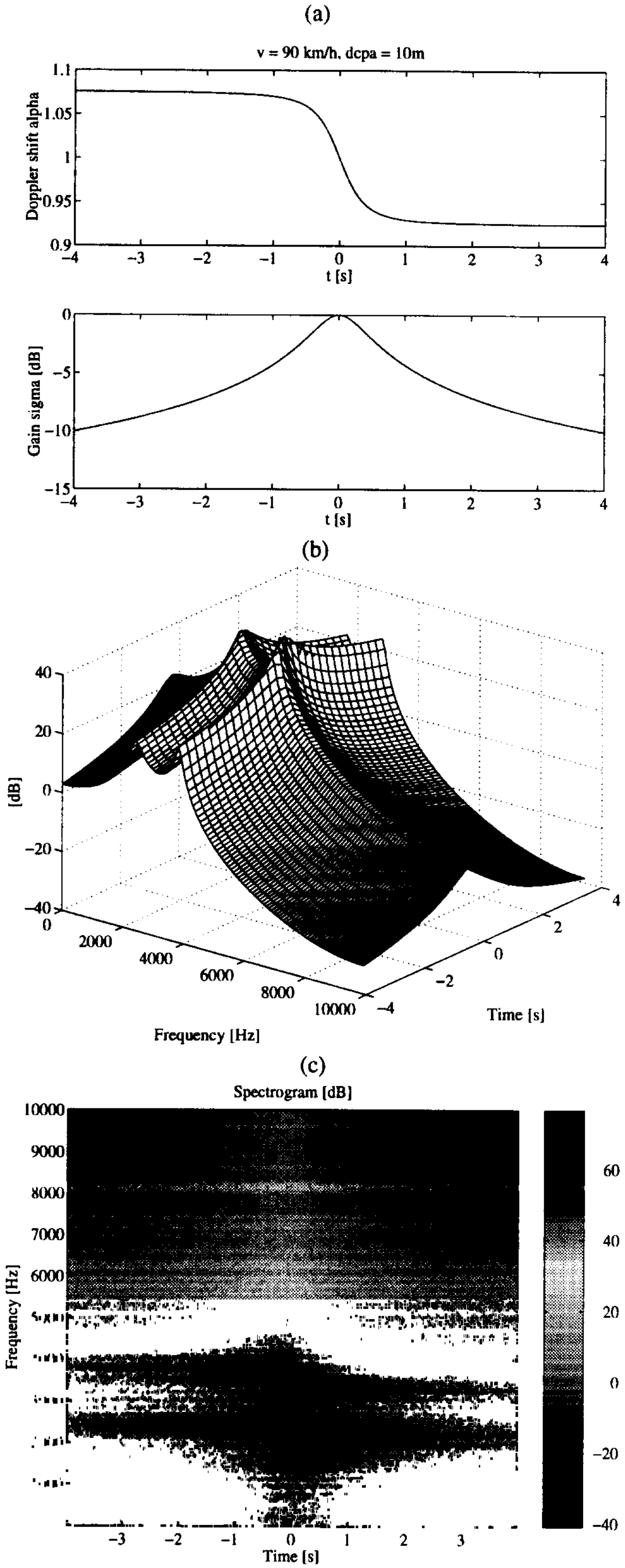

Figure 4: Doppler parameters (a), time-varying ARMA spectrum (b), and spectrogram of a realization (c) for the simulation of a vehicle "pass-by" at constant speed.

Table 1: Performance of $\hat{v}_0$ on 100 Monte-Carlo runs.

v_0 [km/h]	d_{CPA} [m]	mean $\hat{v}_0$	std dev. $\hat{v}_0$
90	10	89.67	1.86
50	10	49.92	0.86
20	10	19.99	0.36
50	20	49.99	0.73
30	50	29.98	1.61

modified Yule-Walker estimator [9], can be used instead of a "full" ARMA estimator.

The computation of the time-varying roots $\hat{z}_k[n]$ can be performed efficiently by a Gauss-Newton type root-finding algorithm [10][11]. Indeed, under the local stationarity hypothesis, $\hat{z}_k[n]$ should change slowly. It is thus possible to initialize the roots-finding algorithm at time n by the values found at time $n-1$; convergence should be fast. The root-finding step can be combined with the AR update of step 1 to provide an efficient pole tracking algorithm.

The non-linear maximization of step 3 can be performed by any numerical optimization method. For non-linear least-squares, Levenberg-Marquardt is usually a good choice. The constant ρ_1 and ρ_2 are chosen experimentally to ensure proper weighting of the fitting errors between the "power" part and the "poles" part. It has also been found experimentally that using a logarithmic (dB) scale for the "power" term improved the performance of the estimator.

4. EXPERIMENTAL RESULTS

A series of experiments have been conducted to test the proposed algorithm. First, Monte-Carlo simulations have been performed in Matlab using the model described in Section 2.3 with various combinations of speed v_0 and microphone position d_{CPA}. The AR part of the ARMA(4,4) model used for $y[n]$ was estimated using a modified Yule-Walker algorithm on length 4096 data frames with an overlap factor of 1/2. Table 1 presents some typical results for the estimation of the speed v_0 from 8 s samples of $y(t)$. The best performance were obtained by using a set of weights $\{\rho_1, \rho_2\}$ strongly in favor of the "pole" fit to the detriment of the "power" fit.

Next, real cars and trucks were recorded using a microphone located next to a straight road. The microphone was placed at a distance $d_{CPA} = 8$ m from the center of road. The speeds of the vehicles were measured using a calibrated speedometer. The recordings were made originally on a DAT and later downsampled to 20 kHz and transfered on a workstation for further processing. The same speed estimation algorithm as above was then applied to the samples. Various order of ARMA models have been tried ranging from AR(2) to ARMA(8,8) processes. The best have been obtained by AR(6) processes, with a weighting $\{\rho_1, \rho_2\}$ strongly in favor of the "power" fit. Typical speed estimates presented an undershoot of 5–10 km/h.

5. CONCLUDING REMARKS

The new Doppler-based passive method for motion estimation introduced in this paper is simple to implement and is quite general in its application, in the sense that, unlike previously proposed methods, it does not require any "periodicity" property of the sound source. It did not perform on the real data as well as could have been expected from the simulations. In addition to the fact that ARMA processes are perhaps not well suited to model vehicle noise, we suggest several reasons that can be put forward to explain this poor performance. First, there was some background noise and wind noise during the recording of the sound events. Next, a car or a truck is only approximately modeled by a stationary point source. Finally, the Doppler model (11)–(12) does not account properly for all the sound wave propagation effects. None of these difficulties is unsurmountable; we believe that the proposed method has shown some promises and should be improved to account for these phenomena.

6. ACKNOWLEDGMENTS

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