ON THE ROBUSTNESS OF THE FRACTIONALLY-SPACED CONSTANT MODULUS CRITERION TO CHANNEL ORDER UNDERMODELING: PART II

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ABSTRACT

This paper studies the constant modulus (CM) criterion specifically for the case where the time span of the fractionally-spaced equalizer (FSE) is less than that of the channel. Hence, there necessarily exists an error in the equalized signal. Results for the binary case (Part I) are extended to multi-level signalling. This analysis is connected with the previous work of Fijalkow et al. on misadjustment of a CM receiver to suggest a finite interval of acceptable FSE length which shows that a longer FSE may not be better than a shorter FSE–in some cases, matching the FSE length to that of the channel may not reduce the MSE to a prescribed threshold, while a shorter FSE may be successful in achieving this threshold.

1. INTRODUCTION

The Constant Modulus (CM) criterion was first proposed by Godard in [3] and developed independently by Treichler and Agee in [11]. The stochastic gradient descent implementation, or Constant Modulus Algorithm (CMA), is widely used in practice (see [10]) and shown in [8] to be globally convergent under certain assumptions. One of these assumptions is that the length of the fractionally-spaced CMA equalizer (CMA-FSE) is at least as long as that of the channel. Little work in addressing the undermodeled case exists ([12], [1] and this paper are exceptions).

In Part I of this work, [1], we studied the CM criterion with noiseless, binary signalling, specifically for the case where the fractionally-spaced equalizer (FSE) does not satisfy the length condition. We concluded that though a FSE length equal to the channel length may be needed for perfect equalization, far fewer CMA-FSE coefficients may be required to achieve a Transfer Level MSE corresponding to an error rate when CMA-FSE is typically transferred to decision directed (DD) LMS. This paper extends the analysis in [1] to real, multi-level (PAM) signalling. The change in CM cost from a perfect equalization setting is derived when the length condition is violated–either from perturbations to the channel outside the time span of the FSE, or from FSE truncation. This analysis is connected with the work in [2] on CMA-FSE misadjustment to suggest a finite interval of FSE length where the CMA-FSE is successful in achieving the Transfer Level MSE. As with classical LMS [5] if the CMA-FSE length is too short, then there remains too much intersymbol interference (ISI), while if the CMA-FSE length is too long, then excess MSE can dominate ISI reduction. An example in §3.3 shows that the "proper" FSE length is less than the actual channel length in some cases. The analyses approaches are also combined with the CM power constraint first proposed for the binary, baud-spaced case in [6] to derive upper and lower bounds on the MSE of the CM receiver.

The paper is organized as follows: §2 describes the proposed analyses approaches in addressing the undermodeling problem. §3 extends these results to the PAM CM criterion, relates the results to MSE, and presents examples. §4 derives a bound on the MSE of the CM receiver based on the CM power constraint, and §5 contains concluding remarks.

2. ANALYSES APPROACHES

2.1. Channel Perturbation

The first approach taken in addressing the robustness of a CM receiver to undermodeling is to consider those channel coefficients that are outside the time span of the FSE as channel perturbations, in order to study the CM cost incurred. This CM criterion is specifically the one minimized by CMA-FSE in a stochastic gradient descent implementation, though MSE or even BER may be the ultimate performance measure. Let

$$\mathbf{c} \;=\; [c_0\; c_1 \ldots c_{L_c-1}]^T \qquad (1)$$

be the length-L_c channel impulse response vector, which is zero outside this finite time support. Define two length-L_c vectors; vector $\mathbf{c_m}$ contains L_m ($L_m < L_c$) consecutive taps of $\mathbf{c}$ in the same positions as they occurred in $\mathbf{c}$ with zeros in the remaining $L_c - L_m$ positions, and vector $\mathbf{c_p}$ contains the $L_c - L_m$ taps of $\mathbf{c}$ that are not in $\mathbf{c_m}$ in the same positions as they occurred in $\mathbf{c}$, with zeros in the remaining L_m positions. Hence, $\mathbf{c} = \mathbf{c_m} + \mathbf{c_p}$; with a length-$L_m$ FSE, the "full length" channel is composed of a "modeled" portion which may be perfectly equalizable (baud-spaced, combined channel-FSE is a pure delay), and a "perturbation" portion which is potentially non-zero outside the time support of the FSE. For example, one such partitioning of the channel taps is

$$\begin{aligned}\mathbf{c_m} &:= [c_0\; c_1 \ldots c_{L_m-1}\; \underbrace{0\;0 \ldots 0}_{L_c-L_m\; zeros}\;]^T \\ \mathbf{c_p} &:= [\underbrace{0\;0 \ldots 0}_{L_m\; zeros}\; c_{L_m}\; c_{L_m+1} \ldots c_{L_c-1}]^T \qquad (2)\end{aligned}$$

which considers the perturbation as appended channel taps with largest delay from the current symbol.

Further, let $\mathbf{C_m}$, $\mathbf{C_p}$ and $\mathbf{C}$ be the convolution matrices associated with $\mathbf{c_m}$, $\mathbf{c_p}$ and $\mathbf{c}$, respectively, and let $\mathbf{f_m}$ be the equalizer coefficient vector corresponding to a global

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minimum of the CM cost associated with channel $\mathbf{c_m}$. The combined channel-equalizer response before decimation is

$$\begin{aligned} \mathbf{h} &= \mathbf{C f_m} && (3) \\ &= \mathbf{C_m f_m} + \mathbf{C_p f_m} && (4) \end{aligned}$$

and the decimated (baud-spaced) version can be written as

$$\begin{aligned} \mathbf{h}_{\downarrow} &= \mathbf{C_{m\downarrow} f_m} + \mathbf{C_{p\downarrow} f_m} && (5) \\ &= \mathbf{h_m} + \mathbf{h_p} && (6) \end{aligned}$$

where $\mathbf{C_{m\downarrow}}$ and $\mathbf{C_{p\downarrow}}$ are appropriately row-decimated versions of $\mathbf{C_m}$ and $\mathbf{C_p}$, respectively. Observe that since $\mathbf{f_m}$ is a global minimum of the CM criterion with respect to channel $\mathbf{c_m}$, there is no error in the equalized signal due to the first term in (6) provided the source kurtosis is less than 3 (see [7]). The second term is the effect of channel perturbations outside the time span of the FSE.

2.2. Truncated Equalizer

A related approach to that above is to consider the effect on the CM cost due to equalizer taps lost in truncation from the FSE which achieves perfect equalization. Let

$$\mathbf{f}^* = [f_0^* \, f_1^* \cdots f_{L_c-1}^*]^T \quad (7)$$

be a CM global minimum for channel $\mathbf{c}$. Define two length-L_c vectors $\mathbf{f_t}$ and $\tilde{\mathbf{f}}$ such that $\mathbf{f_t} = \mathbf{f}^* + \tilde{\mathbf{f}}$. Vector $\mathbf{f_t}$ contains L_m consecutive taps of $\mathbf{f}^*$ in the same positions as they occurred in $\mathbf{f}^*$ with zeros in the remaining $L_c - L_m$ positions, and vector $-\tilde{\mathbf{f}}$ contains the $L_c - L_m$ taps of $\mathbf{f}^*$ that are not in $\mathbf{f_t}$ in the same positions as they occurred in $\mathbf{f}^*$, with zeros in the remaining L_m positions. For example, one such partitioning is

$$\begin{aligned} \mathbf{f_t} &:= [f_0^* \, f_1^* \cdots f_{L_m-1}^* \; \underbrace{0\,0 \ldots 0}_{L_c - L_m \; zeros} \;]^T \\ \tilde{\mathbf{f}} &:= [\underbrace{0\,0 \ldots 0}_{L_m \; zeros} \; -f_{L_m}^* \; -f_{L_m+1}^* \cdots - f_{L_c-1}^*]^T \quad (8) \end{aligned}$$

The baud-spaced, combined channel-equalizer response for the truncated equalizer $\mathbf{f_t}$ can be written as

$$\begin{aligned} \mathbf{h}_{\downarrow} &= \mathbf{C}_{\downarrow} \mathbf{f_t} && (9) \\ &= \mathbf{C}_{\downarrow} \mathbf{f}^* + \mathbf{C}_{\downarrow} \tilde{\mathbf{f}} && (10) \end{aligned}$$

where $\mathbf{C}_{\downarrow}$ is the appropriately row-decimated version of $\mathbf{C}$. Observe that (10) is the same form as (6), where the first term is analogous to the "modeled" contribution, and second term is analogous to the "perturbation" contribution. Thus, the first term satisfies the length condition and will achieve perfect equalization since $\mathbf{f}^*$ is a global minimum of the CM criterion, provided the source kurtosis is less than 3 (see [7]). Our goal in addressing CMA's robustness properties is to study the effect of the second terms of (6) and (10) on the real, multi-level CM cost function.

3. CM CRITERION

3.1. Change in Cost

It is shown in [1] that using the two analysis approaches above, the CM cost incurred for the binary-signalling case upon violation of the length condition is

$$\begin{aligned} \Delta J_{CM|BPSK} &= \left[4\|\mathbf{h_p}\|_2^2\right] && (11) \\ &+ p_\delta \left[4p_\delta^2 + 12 \sum_{i\neq\delta, i=0}^{P} p_i^2\right] \\ &+ \left[\sum_{i=0}^{P} p_i^4 + 3 \sum_{i=0}^{P} \sum_{j=0, j\neq i}^{P} p_i^2 p_j^2\right] \end{aligned}$$

where each element of the combined channel-equalizer vector is comprised of two terms, $h_i = m_i + p_i$, such that $m_i \in \mathbf{C_{m\downarrow} f_m}$ or $m_i \in \mathbf{C_{\downarrow} f^*}$ and $p_i \in \mathbf{C_{p\downarrow} f_m}$ or $p_i \in \mathbf{C}_{\downarrow}\tilde{\mathbf{f}}$. Note also that since the "modelled" portion achieves perfect equalization, $m_\delta = 1$ and $m_i = 0 \; \forall i \neq \delta$. (The case where $m_\delta = -1$ can be shown to be equivalent). We next extend this result to the PAM signalling case.

The CM cost for a Non-Constant-Modulus (NCM), white, symmetric source can be expressed as

$$\begin{aligned} J_{CM|NCM} &= \gamma^2 - 2\sigma_s^2 \gamma \sum_{i=0}^{P} h_i^2 + \nu \sum_{i=0}^{P} h_i^4 && (12) \\ &+ 3(\sigma_s^2)^2 \left(\sum_{i=0}^{P} \sum_{j=0, j\neq i}^{P} h_i^2 h_j^2 \right) \end{aligned}$$

where $\sigma_s^2 = E\{s^2\}$, $\nu = E\{s^4\}$ and $\gamma = \nu/\sigma_s^2$ is the CM dispersion constant. Hence, the CM cost at a global minimum which achieves perfect equalization is

$$J_{CM|global\ min.} = \gamma^2 - \nu \quad (13)$$

When the length condition is violated, as from channel perturbations outside the FSE time span or from FSE taps lost in truncation, it can be shown in a similar fashion as was done in deriving (11) (see [1]) that the CM cost changes from a perfect equalization setting to

$$\begin{aligned} J_{CM|length} &= \left[\gamma^2 - \nu\right] && (14) \\ &+ \left[4\nu p_\delta^2 + (6(\sigma_s^2)^2 - 2\nu) \sum_{i\neq\delta, i=0}^{P} p_i^2\right] \\ &+ p_\delta \left[4\nu p_\delta^2 + 12(\sigma_s^2)^2 \sum_{i\neq\delta, i=0}^{P} p_i^2\right] \\ &+ \left[\nu \sum_{i=0}^{P} p_i^4 + 3(\sigma_s^2)^2 \sum_{i=0}^{P} \sum_{j=0, j\neq i}^{P} p_i^2 p_j^2\right] \end{aligned}$$

The change in CM cost due to undermodeling is therefore

$$\begin{aligned} \Delta J_{CM} &= J_{CM|length} - J_{CM|global\ min.} && (15) \\ &= \left[4\nu p_\delta^2 + (6(\sigma_s^2)^2 - 2\nu) \sum_{i\neq\delta, i=0}^{P} p_i^2\right] && (16) \\ &+ p_\delta \left[4\nu p_\delta^2 + 12(\sigma_s^2)^2 \sum_{i\neq\delta, i=0}^{P} p_i^2\right] \\ &+ \left[\nu \sum_{i=0}^{P} p_i^4 + 3(\sigma_s^2)^2 \sum_{i=0}^{P} \sum_{j=0, j\neq i}^{P} p_i^2 p_j^2\right] \end{aligned}$$

Note that (16) is grouped according to powers of the p_i. For the quadratic contribution, the NCM source weights the p_δ^2 element more heavily than the other p_i^2 elements, since $\nu \geq (\sigma_s^2)^2$. Note that the quadratic terms were equally weighted for the binary case in (11). For example, with a 4-PAM unit-variance constellation, $4\nu = 6.56$ while $(6(\sigma^2)^2 - 2\nu) = 2.72$. These factors should be related to the MSE cost incurred using the same analysis methods.

3.2. Relation to MSE

The MSE criterion is

$$J_{MSE} = E\{(y - s_\delta)^2\} \tag{17}$$

$$= \sigma_s^2 \left(\sum_{i=0}^{P} h_i^2 + 1 - 2h_\delta \right) \tag{18}$$

where y is the FSE output, s_δ is a source symbol, and σ_s^2 is the source power. The change in MSE from a perfect equalization setting due to undermodeling is then

$$\Delta J_{MSE} = \sum_{i=0}^{P} (m_i + p_i)^2 + 1 - 2(m_\delta + p_\delta) \tag{19}$$

$$= \|\mathbf{h_P}\|_2^2, \ (\sigma_s^2 = 1) \tag{20}$$

Hence, when the perturbation terms are "small", the change in CM cost in the vicinity of a global minimum is approximately a scaled version of the change in MSE cost

$$\Delta J_{CM} \approx (6 - 2\nu)\Delta J_{MSE} \tag{21}$$

with source power normalized to unity. This result generalizes that for the binary case in [1], $\Delta J_{cm} \approx 4(\Delta J_{MSE})$.

3.3. Examples

Undermodeling:

Based on the binary analysis in [1], it is proposed that the FSE length be chosen to span the "significant" channel taps–those whose magnitudes are approximately 20% or greater of the largest tap. We wish to compare this rule with that which may be suggested by the preceding PAM analysis. Hence, we evaluate (16) with a 16-PAM source for the two approaches of channel perturbations and equalizer truncation as described in §2. The $T/2$-spaced channel models are derived from empirical measurements of digital microwave radio signals in the San Francisco Bay Area which are described in [4] and now available over the *www* from the database at *http://spib.rice.edu/spib/microwave.html*. The channel taps for the approach of §2.1 are partitioned according to (2) and the partitioning of FSE coefficients for the approach of §2.2 is according to (8). The results are scaled (by a factor of $\frac{1}{(6-2\nu)}$) to approximate the MSE cost. The results for Channel 2 of the database are shown below; the results for other channels of the database have similar behavior and can be found over the *www* at *www.ee.cornell.edu/~johnson/BERG*.

Figure 1 contains two plots. The top plot contains the magnitude of the $T/2$ channel impulse response coefficients. The bottom plot contains the graphs of three functions: i) (solid) the approximation of MSE from a scaled version of (16) due to channel perturbations outside the FSE time span (§2.1), ii) (dashed) the approximation of MSE from a scaled version of (16) due to FSE truncation (§2.2), and iii) (dotted) the true MSE described by (20) according to the approach in §2.1–note that, unlike the BPSK case, this quantity is *not* necessarily a scaled version of the quadratic contribution of (16). The graphs may be referenced to the dashed line of constant MSE which corresponds to a Transfer Level for which CMA is typically transferred to decision directed (DD) LMS when further error-rate reduction or tracking is required.

These figures confirm the conclusions drawn from the binary analysis and examples in [1]. Far fewer CMA-FSE taps are needed to reach the Transfer Level than for perfect equalization. The "significant" portion of the channel appears to be those coefficients greater than approximately 20% of the magnitude of the largest channel tap. Little improvement in the MSE is observed by increasing the FSE length to span channel coefficients less than this threshold. Also, in this region, the true MSE as described in iii) above is essentially the same as the approximated MSE as described in i) above, i.e., the CM cost remains essentially a scaled version of the MSE cost. Moreover, the change in costs remains small, suggesting that the CM minimum stays in a tight neighborhood of the MSE minimum. These figures also show that the two different but related approaches described in §2 are not order-able (one is not always greater than the other), suggesting the validity of both.

Misadjustment:

The perfect equalization result requiring the length condition and also our analysis thus far both study the CM error surface. The adaptive implementation to descend this error surface (CMA-FSE), however, typically uses a gradient descent approach with non-vanishing, but small, stepsize μ. A NCM source causes a misadjustment term in the equalizer update equation, since the instantaneous CMA error is generally nonzero, effectively causing a "rattling around" in both the CM and MSE minima. This behavior usually forces the transfer from CMA-FSE to DD-LMS for further MSE reduction. Note that LMS suffers misadjustment due to noise, but not due to a NCM source. The misadjustment in MSE terms of a CM receiver updated with CMA-FSE is approximated when the length condition is satisfied and in the absence of noise in [2]:

$$J_{MSE|mis} \approx \mu L_m \frac{\frac{E\{s^6\}}{(\sigma_s^2)^3} - \kappa}{(3-\kappa)} \sigma_s^2 \sigma_r^2 \tag{22}$$

where $\kappa = \nu/(\sigma_s^2)^2$ is the source kurtosis and σ_r^2 is the received signal power. The result mimics that for LMS in its dependence on the FSE length (see [5]), which suggests a classical compromise: the FSE length must be chosen long enough to cover the "significant" portion of the channel so that the undermodeling does not dominate the MSE of the CM receiver, but not too long so that the misadjustment dominates the MSE of the CM receiver. We approximate the MSE due to both violation of the length condition and misadjustment in the receiver implementing CMA-FSE from (16) and (22) as

$$J_{MSE} \approx \frac{1}{6-2\nu}\Delta J_{CM} + J_{MSE|mis} \tag{23}$$

Figure 2 shows this approximation for Channel A with a 4-PAM source and $\mu = 1.5 \times 10^{-3}$. This result suggests a FSE length less than that needed for perfect equalization! In fact, this example shows that choosing the FSE length equal to the length of the channel is precisely the wrong thing to do in attempting to reach the MSE Transfer Level.

4. POWER CONSTRAINT

The analyses approaches of §2 can be applied to the CM receiver power constraint first proposed for the baud-spaced binary case in [6] and generalized in [9]. A bound on the ℓ_2 norm of the portion of the combined channel-equalizer arising from violation of the length condition follows easily from this power constraint.

Define $\mathbf{Q_m} := \mathbf{C_{m\downarrow}}^T \mathbf{C_{m\downarrow}}$. The CM receiver satisfies

$$k \le \mathbf{f_m}^T \mathbf{Q_m} \mathbf{f_m} \le 1 \tag{24}$$

where k depends on the source kurtosis (see [9]). An alternative expression for (24) can be written in terms of the minimum and maximum eigenvalues of $\mathbf{Q_m}$.

$$\lambda_{min}(\mathbf{Q_m})\mathbf{f_m}^T\mathbf{f_m} \le \mathbf{f_m}^T\mathbf{Q_m}\mathbf{f_m} \le \lambda_{max}(\mathbf{Q_m})\mathbf{f_m}^T\mathbf{f_m} \tag{25}$$

Together, (24) and (25) bound the ℓ_2 norm of the CM equalizer coefficients.

$$\frac{k}{\lambda_{max}(\mathbf{Q_m})} \leq \mathbf{f_m}^T\mathbf{f_m} \leq \frac{1}{\lambda_{min}(\mathbf{Q_m})} \qquad (26)$$

Define $\mathbf{Q_p} = \mathbf{C_{p\downarrow}}^T\mathbf{C_{p\downarrow}}$ so that a similar expression can be written for the portion of the combined channel-equalizer resulting from violation of the length condition.

$$\lambda_{min}(\mathbf{Q_p})\mathbf{f_m}^T\mathbf{f_m} \leq \mathbf{f_m}^T\mathbf{Q_p}\mathbf{f_m} \leq \lambda_{max}(\mathbf{Q_p})\mathbf{f_m}^T\mathbf{f_m} \qquad (27)$$

Substituting the bounds of the equalizer norm from (26) into (27) implies that

$$k\frac{\lambda_{min}(\mathbf{Q_p})}{\lambda_{max}(\mathbf{Q_m})} \leq ||\mathbf{C_{p\downarrow}f_m}||_2^2 \leq \frac{\lambda_{max}(\mathbf{Q_p})}{\lambda_{min}(\mathbf{Q_m})} \qquad (28)$$

These bounds offer some insight into the manifestation of error due to common subchannel roots. As the "modeled" channel loses disparity, $\lambda_{min}(\mathbf{Q_m})$ approaches zero and the upper bound approaches infinity. As the FSE length is increased to match the channel length, $\lambda_{min}(\mathbf{Q_p})$ goes to zero, which forces the lower bound to zero.

Similarly, when FSE taps are truncated as in §2.2,

$$||\mathbf{C_\downarrow\tilde{f}}||_2^2 \quad \leq \quad \frac{\lambda_{max}(\mathbf{Q})}{\lambda_{min}(\mathbf{Q})} \qquad (29)$$

where $\mathbf{Q} := \mathbf{C_\downarrow}^T\mathbf{C_\downarrow}$. Note that the bound is precisely the condition number of the autocorrelation matrix of the baud-spaced received signal.

Though the bounds in (28) and (29) offer insight into error due to loss of channel disparity, and therefore may (or may not) merit further study, the bounds do not become tight when applied to the microwave channels from [4].

5. CONCLUSION

This paper has studied the robustness of the CM criterion for PAM signalling when the length condition is violated, a nearly unavoidable condition in practice for high data rate communications. Presumably, the results for the complex (QAM) case are a straightforward extension of the above analysis. It is shown that the effect on the CM cost and the MSE cost is small when perturbations outside the time span of the FSE are small, so that the CM cost is approximately a scaled version of the MSE cost. Hence, the CM minimum stays in a tight neighborhood around the MSE minimum. This analysis is connected with the previous work in [2] on CMA-FSE misadjustment which illuminates conflicting contributors to FSE length selection; the FSE length should be chosen long enough to span the significant channel coefficients, but not so long that the misadjustment dominates the MSE. An example shows that this finite interval of FSE length need not include that needed for perfect equalization.

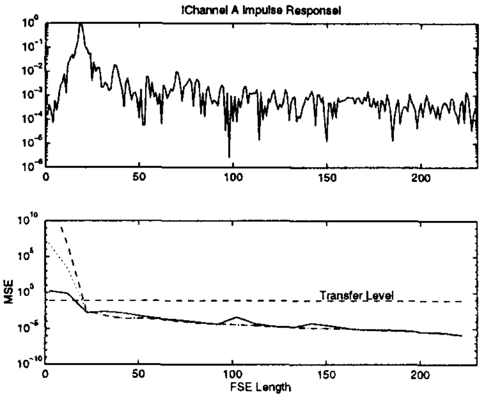

Figure 1. AppSigTec Channel A, 16-PAM

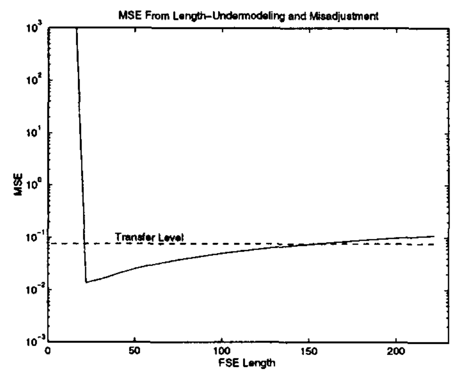

Figure 2. Undermodeling and Misadjustment MSE

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