DETECTION OF SIGNALS WITH UNKNOWN PARAMETERS IN GBK-DISTRIBUTED INTERFERENCE

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ABSTRACT

In this paper, the design of optimal schemes for detecting deterministic narrowband signals with unknown parameters in correlated interference modelled by the recently developed GBK distribution is considered. Theoretical derivations of an optimal detector, in the Neyman-Pearson sense, are given for the case where the signal amplitude and phase are unknown. The performance of the detector is then evaluated using extensive computer simulations.

1. INTRODUCTION

The research dedicated to the design and optimisation of detectors of signals in interference is a constantly evolving field of study. In some applications the considered interference can be assumed Gaussian. Such an assumption greatly simplifies the analysis and usually leads to optimal solutions. However, it has been found that in many applications the Gaussianity assumption is not valid [7]. This has lead to the area of modelling interference by non-Gaussian probability distributions. Non-Gaussian probability models include, for example, the generalised Gaussian or generalised Laplace distribution. In the case where the data is positive (e.g. an amplitude of a narrowband process, or in the case of life distribution) the Log-Normal, the Gamma, the Weibull, and the K-distributions are frequently used models. Recently a new distribution, called the *Generalised Bessel function K* (GBK) distribution was proposed [4]. The GBK distribution includes a large number of the well known interference models, such as the ones mentioned above, while retaining mathematical tractability. It was also shown that the GBK-distribution can represent the amplitude distribution of a *spherically invariant random vector* (SIRV), which in turn enables coherent modelling of the underlying interfering phenomenon [4].

The GBK distribution encompasses a large number of known amplitude models including the Weibull and the K-distribution. Thus, it is of interest to design an optimal system for detecting signals in GBK-distributed interference. Such a system will possess an important feature of having the same structure for all spherically invariant models included in the GBK distribution. It is also expected that such a detection system will perform better than systems designed for more specific models in situations where the interference statistics changes from time to time (or from location to location).

In this paper we design an optimal detector (in the Neyman-Pearson sense) for deterministic signals with unknown amplitude and phase in GBK-distributed interference and analyse its performance using extensive computer simulation. As an example, we consider a radar scenario where the aim is to detect a target signal in clutter. We show, that the optimal schemes for detecting signals in Rayleigh, Weibull, K, and many other distributed interference processes are identical.

2. THE GBK DISTRIBUTION

The SIRV multivariate representation of the GBK distribution is given by [4]

$$\begin{aligned} f_{\boldsymbol{X}}(\boldsymbol{x}) &= \frac{4a^N c^{N-1}}{\Gamma(\alpha_1)\Gamma(\alpha_2)(2\pi)^N|\boldsymbol{M}|^{\frac{1}{2}}} \\ &\times \left(\sqrt{a}\|\boldsymbol{x}\|\right)^{\frac{c}{2}(\alpha_1+\alpha_2+N-1)-2N}\sum_{k=1}^{N}(-1)^{m+k} \\ &\times \frac{P_{(N,k)}K_{(\alpha_2-\alpha_1-k+1)}\left[2\left(\sqrt{a}\|\boldsymbol{x}\|\right)^{\frac{c}{2}}\right]}{\left(\frac{c}{2}\sqrt{a}\|\boldsymbol{x}\|^{\frac{c}{2}}\right)^{N-k}}, \end{aligned} \quad (1)$$

where $\boldsymbol{x} = [x_1, \ldots, x_{2N}]$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $c > 0$ are the distribution parameters, $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν, $\boldsymbol{M}$ is the covariance matrix (assumed to be invertible), $\|\cdot\|$ is the Euclidean norm,

$$a = \frac{\Gamma\left(\alpha_1 + \frac{2}{c}\right)\Gamma\left(\alpha_2 + \frac{2}{c}\right)}{2\Gamma(\alpha_1)\Gamma(\alpha_2)},$$

$$m = \begin{cases} 1, & N \text{ odd} \\ 0, & N \text{ even,} \end{cases}$$

and the coefficients $P_{(N,k)}$ are calculated recurrently

$$P_{(N,k)} = P_{(N-1,k)}\,C_{(N,k)} + P_{(N-1,k-1)},$$

with

$$C_{(N,k)} = \begin{cases} 0, & k > N \\ 1, & k = N \\ \frac{c}{2}\alpha_1 - (N-1) + \frac{c(k-1)}{2}, & k < N, \end{cases}$$

$P_{(0,0)} = 1$, $P_{(N,0)} = 0$, and $P_{(0,k)} = 0$.

It was established that the theory of SIRV can be applied when the interference amplitude is modelled by the GBK distribution, having pdf given by

$$f_R(r) = \frac{2c\left(\frac{r}{\beta}\right)^{\frac{c}{2}(\alpha_1+\alpha_2)-1}}{\beta\Gamma(\alpha_1)\Gamma(\alpha_2)} K_{(\alpha_2-\alpha_1)}\left[2\left(\frac{r}{\beta}\right)^{\frac{c}{2}}\right], \quad (2)$$

with the shape parameter α_1 and the power parameter c, such that

$$C_{(N,k)} \leq 0, \qquad k = 1, \ldots, N-1. \quad (3)$$

3. DESIGN OF A DETECTOR

Consider a radar scenario, as an example. Let

$$\tilde{s} = s_I + js_Q = Ae^{j\theta}\tilde{v}, \quad (4)$$

be the complex envelope of a narrowband target signal, where s_I and s_Q represent the inphase and quadrature components of the target signal complex envelope, $\tilde{v}$ is a complex N-dimensional vector of the transmitted signal, A accounts for the channel attenuation and the target cross-section, and θ accounts for the initial phase of the received coherent pulse train [3]. The detection problem can be expressed in the following framework

$$\begin{aligned} \mathsf{H} &: \boldsymbol{z} = \boldsymbol{c} \\ \mathsf{K} &: \boldsymbol{z} = \boldsymbol{s} + \boldsymbol{c}, \end{aligned} \quad (5)$$

where H denotes the null hypothesis, K the alternative hypothesis, and where $\boldsymbol{z} = [\boldsymbol{z}_I, \boldsymbol{z}_Q]$, $\boldsymbol{c} = [\boldsymbol{c}_I, \boldsymbol{c}_Q]$, and $\boldsymbol{s} = [\boldsymbol{s}_I, \boldsymbol{s}_Q]$, are real vectors with $2N$ entries representing the observations of the received signal, clutter signal, and target signal, respectively.

Since the theory of SIRVs can be applied when the clutter amplitude is modelled by the GBK distribution with certain values of the parameters α_1 and c, one can use a whitening transformation without penalty. This is due to the fact that a SIRV is closed under a linear transformation [5]. Whitening the received signal leads to the following hypothesis alternative framework

$$\begin{aligned} \mathsf{H} &: \boldsymbol{x} = \boldsymbol{n} \\ \mathsf{K} &: \boldsymbol{x} = \boldsymbol{u} + \boldsymbol{n}, \end{aligned} \quad (6)$$

where $\boldsymbol{x}$ is the whitened version of the received signal vector $\boldsymbol{z}$, and $\boldsymbol{n}$ and $\boldsymbol{u}$ represent the whitened versions of the clutter signal vector $\boldsymbol{c}$ and target signal vector $\boldsymbol{s}$, respectively. Note that the whitening transformation does not change the statistical properties of a parametric detector as long as the clutter process is spherically invariant.

Since the clutter process can be assumed to be a zero-mean process, the whitened version of the target signal can be expressed as $\boldsymbol{u} = Ae^{j\theta}\boldsymbol{p}$, where $\boldsymbol{p}$, referred to as the signal pattern, is the whitened version of $\boldsymbol{v}$.

Since the GBK-distribution fulfils the requirements of a SIRV, it can be also represented as a result of compounding the Gaussian process with some other process. In other words we can represent a GBK-distributed variate as conditionally Gaussian with some modulating variate $s > 0$. The so called *characteristic probability density function*, $f_S(s)$, can be found by solving the following integral equation [1, 6]

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = (2\pi)^{-N}|\boldsymbol{M}|^{-\frac{1}{2}} \int_0^\infty \frac{1}{s^{2N}} \exp\left(-\frac{\|\boldsymbol{x}\|^2}{2s^2}\right) f_S(s)ds, \quad (7)$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is given in (1) and $\alpha_1 c \leq 2$ (for $N = 2$). However, no closed form expression for $f_S(s)$ has been found.

Nevertheless, the knowledge of $f_S(s)$ is not necessary when designing optimal detection structures for the case when the target signal is unknown or partially known. Following [2], the generalised log-likelihood ratio test (GLLRT) for detecting signals with unknown amplitude and phase in GBK-distributed clutter is given by

$$\begin{aligned} \Lambda(\boldsymbol{x}) &= \max_{(\theta,A)} \log\left[\frac{f_{\boldsymbol{X}}(\boldsymbol{x} - Ae^{j\theta}\boldsymbol{p})}{f_{\boldsymbol{X}}(\boldsymbol{x})}\right] \\ &= \log\left[\frac{\max_{(\theta,A)} \int_0^\infty \frac{f_S(s)ds}{s^{2N}} \exp\left(-\frac{\|\boldsymbol{x} - Ae^{j\theta}\boldsymbol{p}\|^2}{2s^2}\right)}{\int_0^\infty \frac{f_S(s)ds}{s^{2N}} \exp\left(-\frac{\|\boldsymbol{x}\|^2}{2s^2}\right)}\right] \\ &= g\left(\sqrt{\|\boldsymbol{x}\|^2 - \frac{|\langle\boldsymbol{x},\boldsymbol{p}\rangle|^2}{\|\boldsymbol{p}\|^2}}\right) - g(\|\boldsymbol{x}\|) \underset{H_0}{\overset{H_1}{\gtrless}} T \end{aligned} \quad (8)$$

respectively, where $\langle\cdot,\cdot\rangle$ denotes the dot product,

$$\begin{aligned} g(\|\boldsymbol{x}\|) &= \log\Bigg[\|\boldsymbol{x}\|^{\frac{c}{2}(\alpha_1+\alpha_2+N-1)-2N} \sum_{k=1}^{N}(-1)^{m+k} \\ &\times \frac{P_{(N,k)}K_{(\alpha_2-\alpha_1-k+1)}\left[2\left(\sqrt{a}\|\boldsymbol{x}\|\right)^{\frac{c}{2}}\right]}{\left(\frac{c}{2}(\sqrt{a}\|\boldsymbol{x}\|)^{\frac{c}{2}}\right)^{N-k}}\Bigg], \end{aligned} \quad (9)$$

and T is a suitable threshold that controls the level of false alarm. The block diagram of the optimal GLLRT detector implementing the test given in (8) is shown in Figure 1.

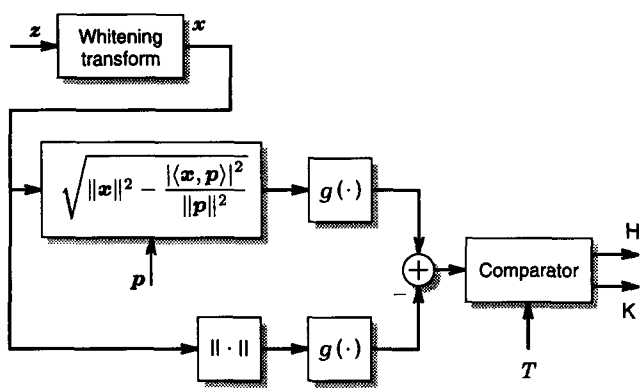

Figure 1. Optimum GLLRT detector for a target signal with unknown phase and unknown amplitude in GBK-distributed clutter.

The detection scheme based on the GLLRT given in (8) has the important feature of reducing to the optimal scheme for detecting signals in Gaussian clutter when the parameters of the GBK distribution are set to $\alpha_1 = \frac{1}{2}$, $\alpha_2 = 1$, and $c = 4$. In this particular case, the function $g(x)$ given in (9) reduces to

$$g(\|\boldsymbol{x}\|) = -\frac{\|\boldsymbol{x}\|}{2}.$$

Also, it is straightforward to show that for $\alpha_1 = 1$ and $c = 2$ the function in (9) reduces to the one given in [2, Eq. (18)] leading to the scheme for detecting signals in K-distributed clutter. Note that in this case the coefficients $P_{(N,k)}$ reduce to zero for $k = 1, \ldots, N-1$ and $P_{(N,N)} = 1$. Similarly, letting $\alpha_1 = 0.5$ and $\alpha_2 = 1$ one can obtain a detection scheme for signals in Weibull distributed clutter (provided that the shape parameter of the Weibull distribution is less than or equal to 2).

4. PERFORMANCE ANALYSIS

There exist no close form expression for the pdfs of the statistic $\lambda(\boldsymbol{x})$ neither under the null hypothesis nor under the alternative. Thus, we resort to computer simulations to study the performance of the detector.

It is assumed that the parameters of the GBK-distribution, namely α_1, α_2, and c, are known. This corresponds to the case where the clutter statistics are obtainable before the detection process is performed.

The performance analysis of the detectors of signals in GBK-distributed clutter has been evaluated using 10000 realisations of the clutter process in each case.

4.1. The Known Signal Case

Consider first the case where the amplitude and the phase of the target signal are known. The detection problem in this case can be resolved by using the log-likelihood ratio test (LLRT) given by

$$\Lambda(\boldsymbol{x}) = g(\|\boldsymbol{x} - \boldsymbol{u}\|) - g(\|\boldsymbol{x}\|) \underset{H_0}{\overset{H_1}{\gtrless}} T, \qquad (10)$$

In this case, the LLRT statistic, $\Lambda(\boldsymbol{x})$, does not depend on the target signal parameters but depends on the target signal-to-clutter SCR ratio,

$$\text{SCR} = \frac{\|\boldsymbol{u}\|^2}{\mathsf{E}[\|\boldsymbol{N}\|^2]},$$

the number of integrated pulses, N, and the parameters of the GBK-distributed clutter. Specifically, the LLRT statistic depends only on the shape parameters α_1 and α_2, and the power parameter c. Since the shape parameters α_1 and α_2 are interchangeable, it is sufficient to evaluate the performance analysis of the detector only as a function of α_1. It should be noted that the performance analysis of the LLRT based detector is of the greatest interest since it gives a benchmark for the GLLRT based detectors. In other words, the detector of known signals has the best possible performance.

In Figure 2, the probability of detection is shown for the LLRT based detector as a function of SCR for four different values of the power parameter c of the GBK distribution, namely for $c = 0.5$, $c = 1$, $c = 2$, and $c = 3$. The detection performance was evaluated for two integrated pulses. The remaining parameters of the GBK-distributed clutter were set to $\alpha_1 = 0.5$ and $\alpha_2 = 2$. Since all detection schemes are independent of the scale parameter β, it was fixed to 1 in all simulations.

Similarly, in Figure 3, the probability of detection is shown for the LLRT based detector as a function of SCR for four different values of the shape parameter α_1 of the GBK distribution, namely for $\alpha_1 = 0.25$, $\alpha_1 = 0.5$, $\alpha_1 = 0.75$, and $\alpha_1 = 1$. The remaining parameters of the GBK-distributed clutter were set to $\alpha_2 = 2$, $\beta = 1$, and $c = 2$.

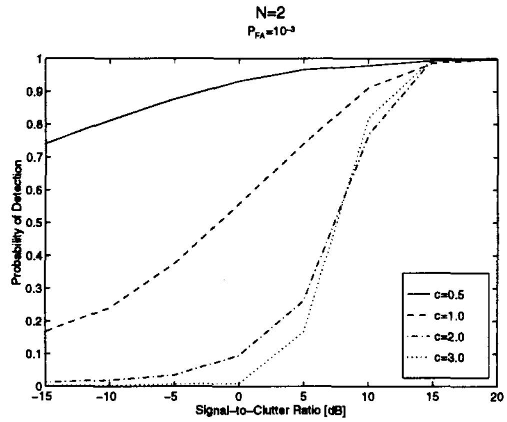

Figure 2. Probability of detection for the LLRT based detector of known signals as a function of SCR and the power parameter c of the GBK distribution for $N = 2$. The shape parameters α_1 and α_2 are set to 0.5 and 2, respectively.

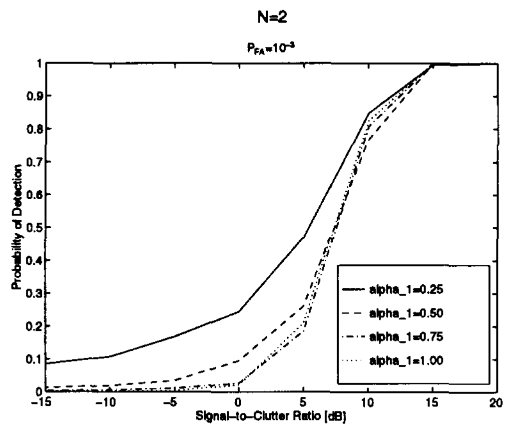

Figure 3. Probability of detection for the LLRT based detector of known signals as a function of SCR and the shape parameter α_1 of the GBK distribution for $N = 2$. The shape parameter α_2 and the power parameter c are both set to 2.

Discussion. From Figure 2, it is readily seen that the power parameter c of the GBK-distributed clutter process greatly affects the performance of the LLRT based detector.

Specifically, for small values of the power parameter c, say $c \leq 1$, the probability of detection increases for small SCR while it decreases for large SCR. Conversely, for a larger c the probability of detection increases for large SCR while it decreases for small SCR.

Similarly, from Figure 3, one concludes that the shape parameter α_1 of the GBK-distributed clutter also affects the performance of the LLRT based detector. Specifically, for small values of α_1 the probability of detection decreases for large SCR and increases for low SCR. Conversely, for larger values of α_1 the probability of detection increases for large SCR and decreases for low SCR. Thus, the shape parameter α_1 has a similar affect on the probability of detection as the power parameter c.

4.2. The Unknown Signal Case

Following the same reasoning as for the LLRT based detector, we conclude that the GLLRT based detector is independent of the phase of the signal but depends on the signal amplitude and the mean clutter power.

In Figure 4 and Figure 5 we present two typical examples of the receiver operating characteristics (ROCs), for three distinct cases where: (i) the target signal is known, (ii) the target signal phase is unknown, and (iii) the target signal amplitude and phase are unknown. The SNR was 10 dB.

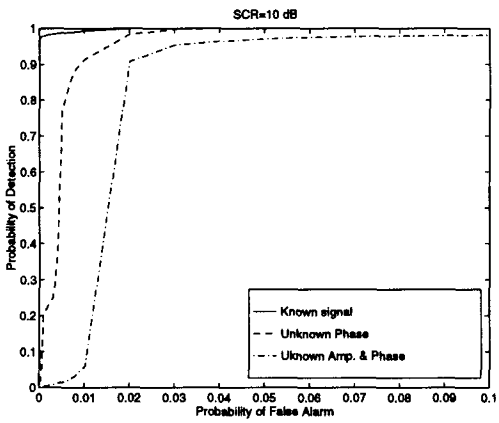

Figure 4. Receiver operating characteristics for the LLRT and GLLRTs based detectors of target signals in GBK-distributed clutter for $N = 2$. The shape parameters α_1 and α_2, and c are set to 0.5, 2, and 0.5, respectively.

Discussion. From the simulation results, it has been observed that the power parameter c affects the ROC curves in a much grater rate than the shape parameter α_1. It should be noted that the parameters of the GBK-distributed clutter have similar affect on the GLLRT based detector as they have on the LLRT based detector. Also, the performance of the GLLRT based detector quickly deteriorates at low SCR.

5. CONCLUSIONS

In this paper we have designed an optimal scheme for detecting signals with unknown parameters, such as the amplitude or phase, in interference modelled by the recently

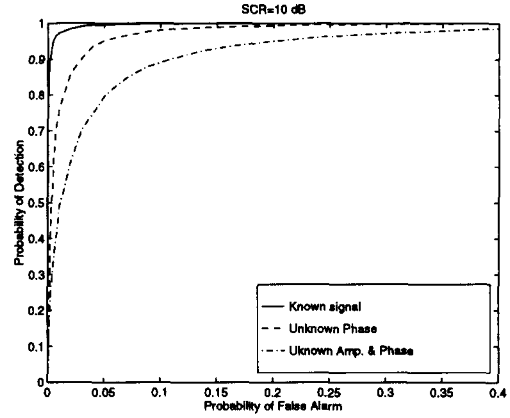

Figure 5. Receiver operating characteristics for the LLRT and GLLRTs based detectors of target signals in GBK-distributed clutter for $N = 2$. The shape parameters α_1, α_2, and c are set to 0.25, 2, and 2, respectively.

developed GBK-distribution. We have shown that the optimal schemes for detecting signals in Rayleigh, Weibull, K, Gamma, Generalised Half Gaussian, Generalised Half Laplace, and many other interference distribution are identical in their structure. Performance analysis of the proposed detector has been investigated using extensive computer simulations. The GBK distribution is attractive in the sense that the classification of the interference model can be avoided.

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