CONSTRAINED BEAMFORMING FOR CYCLOSTATIONARY SIGNALS

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ABSTRACT

The classical Linearly Constrained Minimum Variance (LCMV) Beamformer corresponds, in the general case, to the *Linear*, *Time Invariant* (TI) and *Spatio-Temporal* (ST) complex filter which output power is minimized under some linear constraints. Optimal for stationary signals, this Beamformer becomes sub-optimal for (quasi)-cyclostationary observations for which the optimal complex filters are *(poly)-periodic* (PP) and, under some conditions of non circularity, *Widely Linear* (WL). Using these results and the fact that PP filtering is equivalent to FREquency SHifted (FRESH) filtering, the purpose of this paper is to present a first extension of the classical LCMV Beamformer, taking into account the potential (quasi)-cyclostationarity and non circularity properties of the observations. This new Cyclic LCMV Beamformer is shown to have an equivalent Cyclic Generalized Sidelobe Canceller (GSLC) structure. The performance computation of this new Cyclic Beamformer shows the interest of the latter in cyclostationary contexts and opens a reflexion about the optimal constraint choice.

1. INTRODUCTION

The classical LCMV Beamformer [1] corresponds, in the general case, to the *Linear*, TI and ST complex filter $\boldsymbol{h}$ which minimizes, under some linear constraints, the power of the output, $y(t) = \boldsymbol{h}^{\dagger}\boldsymbol{X}(t)$, where $\boldsymbol{X}(t)$ is the vector of the complex envelopes of the ST observations at the output of the sensors. Limiting the analysis to the exploitation of the second order statistics of the data, it has been shown recently in [2] that this classical approach of array filtering is optimal only for stationary signals and becomes sub-optimal for non stationary and in particular for (quasi)-cyclostationary observations, omnipresent in radiocommunications contexts, which statistics are (quasi) or (poly)-periodic and which complex envelope may be second order non circular [3]. More precisely, for (quasi)-cyclostationary observations, the optimal complex filters, in a mean square sense, become PP [4-5] and, under some non circularity conditions, WL [6], i.e. of the form $y(t) = \boldsymbol{h}_1(t)^{\dagger}\boldsymbol{X}(t) + \boldsymbol{h}_2(t)^{\dagger}\boldsymbol{X}(t)^*$, where * means complex conjugate and where $\boldsymbol{h}_1(t)$ and $\boldsymbol{h}_2(t)$ are TV and PP complex filters. Using the previous results and the fact that PP filtering is equivalent to FRESH filtering [4-5], the purpose of this paper is to present a first extension of the classical LCMV Beamformer, taking into account the potential (quasi)-cyclostationarity and non circularity properties of the received signals. This new Cyclic LCMV Beamformer is shown to have an equivalent Cyclic GSLC [7] structure. The performance computation of this new Cyclic Beamformer shows the interest of the latter in (quasi)-cyclostationary contexts and opens a reflexion about the optimal constraint choice.

This work has been supported by D.R.E.T (French Administration)

2. PROBLEM FORMULATION

Consider an array of N Narrow-Band (NB) sensors and let us call $\boldsymbol{x}(t)$ the vector of the complex envelopes of the signals present at time t at the output of the sensors. Each sensor is assumed to receive the contribution of a useful cyclostationary signal, P cyclostationary jammers and a background noise. Under these assumptions, the observation vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{x}(t) = s(t)\, e^{j(\Delta\omega_0 t+\phi_0)}\boldsymbol{s} + \sum_{i=1}^{P} m_i(t)\, e^{j(\Delta\omega_i t+\phi_i)}\boldsymbol{J}_i + \boldsymbol{b}(t) \tag{2.1}$$

where $\boldsymbol{b}(t)$ is the noise vector, assumed spatially white and stationary, $s(t)$, $\Delta\omega_0$, ϕ_0 and $\boldsymbol{s}$ are the complex envelope, assumed zero-mean and cyclostationary, the carrier residue, the phase and the steering vector, assumed known or estimated, of the useful signal respectively, whereas $m_i(t)$, $\Delta\omega_i$, ϕ_i and $\boldsymbol{J}_i$ are the complex envelope, assumed zero-mean and cyclostationary, the carrier residue, the phase and the steering vector of the jammer i respectively.

Under the previous assumptions, for a given ST observation $(NL \times 1)$ vector $\boldsymbol{X}(t) \triangleq (\boldsymbol{x}(t)^{T}, \boldsymbol{x}(t-\tau_1)^{T}, \ldots, \boldsymbol{x}(t-\tau_{L-1})^{T})^{T}$, the classical LCMV Beamformer corresponds to the Linear, TI and ST $(NL \times 1)$ complex filter $\boldsymbol{h} \triangleq (\boldsymbol{h}_0^{T}, \boldsymbol{h}_1^{T}, \ldots, \boldsymbol{h}_{L-1}^{T})^{T}$, which minimizes, under some linear constraints of no useful signal distortion in its classical form, the temporal mean, $<E[|y(t)|^2]>$, over the observation window [0, T], of the instantaneous power, $E[|y(t)|^2]$, at the output $y(t)$ of the filter, defined by

$$y(t) = h^{\dagger} X(t) = \sum_{i=0}^{L-1} h_i^{\dagger} x(t - \tau_i) \quad (2.2)$$

where $\tau_0 = 0$. Note that although ST structures may be interesting for NB signals, they are mainly used for Wide Band (WB) signals, whereas for NB signals instantaneously mixed, only Spatial filters are generally considered, which means that $L = 1$ and $X(t) = x(t)$. In this case, the linear constraint of no useful signal distortion can be written as $h^{\dagger} s = 1$ and the classical LCMV Beamformer for NB signals is a particular case of the well known Spatial Matched Filter (SMF).

In the presence of P cyclostationary signals, the observation vector $X(t)$ is, in the general case, quasi-cyclostationary and the Linear and TI structure defined by (2.2) becomes sub-optimal [2]. In these conditions, the optimal complex filters are PP [4-5] and, under some conditions of non circularity, WL [2] [6], which means that, for the given ST observation vector $X(t)$, their output can be written as

$$y(t) = h_1(t)^{\dagger} X(t) + h_2(t)^{\dagger} X(t)^* = \sum_{i=0}^{L-1} h_{1i}(t)^{\dagger} x(t - \tau_i) + \sum_{i=0}^{L-1} h_{2i}(t)^{\dagger} x(t - \tau_i)^* \quad (2.3)$$

where the complex filters $h_1(t)$ and $h_2(t)$, and thus the complex filters $h_{1i}(t)$ and $h_{2i}(t)$, $(0 \le i \le L - 1)$, are PP filters. As a consequence, the filters $h_{1i}(t)$, $h_{2i}(t)$ and their complex conjugate have a Fourier serial expansion and we can write, for $1 \le m \le 2$ and $0 \le i \le L - 1$,

$$h_{mi}(t)^* = \sum_k h_{mik}^* e^{j2\pi\alpha_{mik}t} \quad (2.4)$$

where the vectors h_{mik} correspond to (N x 1) TI complex filters and where the cyclic frequencies α_{mik} are related to the cyclic frequencies of the observations. Using (2.4) into (2.3), the optimal filters output, for the given ST and quasi-cyclostationary observation vector $X(t)$, can be written as

$$y(t) = \sum_k \sum_{i=0}^{L-1} h_{1ik}^{\dagger} x(t - \tau_i) e^{j2\pi\alpha_{1ik}t} + h_{2ik}^{\dagger} x(t - \tau_i)^* e^{j2\pi\alpha_{2ik}t} \quad (2.5)$$

which is the sum of FRESH TI filters outputs. However, in practical situations, only a finite number M of cyclic frequency and of TI filters can be used and we only consider, in the following, PP filters with M input vectors, called Mth-order PP filters, which output can be written as

$$y(t) = h_1^{\dagger} x(t) + \sum_{l=2}^{M} h_l^{\dagger} x(t - \Delta_l)^{\zeta_l} e^{j2\pi\alpha_l t} \quad (2.6)$$

where, for $1 \le l \le M$, h_l is a (N x 1) TI complex filter, $\Delta_l \in \{\tau_0, \tau_1, \ldots, \tau_{L-1}\}$, $\zeta_l = \pm 1$ with $x^1 \triangleq x$ and $x^{-1} \triangleq x^*$ and α_l is a cyclic frequency of the observation. Note that the 1st-order PP filter, defined by (2.6) with $M = 1$, is the classical Linear, TI and Spatial filter, whereas for $M > 1$, the Mth-order PP filter defined by (2.6) is Linear if all the ζ_l are equal to 1 and WL in the other cases, TI if all the α_l are zero and TV otherwise, Spatial if all the Δ_l are zero and ST in the other cases.

Under all the previous assumptions, the problem we address in this paper is to find the M TI complex filters h_l, $(1 \le l \le M)$, minimizing the temporal mean, $<E[|y(t)|^2]>$, of the instantaneaous power, $E[|y(t)|^2]$, at the output, $y(t)$, of the filter (2.6), under a constraint of no useful signal distortion, i.e under the following linear constraints

$$h_1^{\dagger} s = 1 \text{ and } h_l^{\dagger} s^{\zeta_l} = 0, \ (2 \le l \le M) \quad (2.7)$$

The Cyclic LCMV Beamformer solution to the previous problem is, for NB and quasi-cyclostationary observations, a first Mth-order PP extension of the classical Linear, TI and Spatial LCMV Beamformer which corresponds to a particular version of the SMF. Note that the optimal choice of the parameters $(\Delta_l, \zeta_l, \alpha_l)$, $2 \le l \le M$, will be guided by the results of the Cyclic LCMV Beamformer performance analysis, presented in section 5.

3. OPTIMAL CYCLIC LCMV BEAMFORMER

Defining the M (MN x 1) vectors $S_l \triangleq [0^T, \ldots, 0^T, s^{\varepsilon_l\,T}, 0^T, \ldots, 0^T]^T$, $(1 \le l \le M)$, which non zero components are comprised between the indices $(l - 1)N + 1$ and lN, the (MN x M) matrix $S \triangleq [S_1, S_2, \ldots, S_M]$, the ($M$ x 1) vector $f \triangleq [1, 0, \ldots, 0]^T$, the ($MN$ x 1) vectors $H \triangleq [h_1^T, h_2^T, \ldots, h_M^T]^T$ and $X(t) \triangleq [x(t)^T, \exp(j2\pi\alpha_2 t)\, x(t - \Delta_2)^{\zeta_2\,T}, \ldots, \exp(j2\pi\alpha_M t)\, x(t - \Delta_M)^{\zeta_M\,T}]^T$ and noting $R_X(t) \triangleq E[X(t)X(t)^{\dagger}]$ the correlation matrix of $X(t)$, the filter H_o solution of the previous problem is obviously [1] given by

$$H_o = <R_X(t)>^{-1} S \,(S^{\dagger} <R_X(t)>^{-1} S)^{-1} f \quad (3.1)$$

Note that for $M = 1$, the expression (3.1) gives the classical Linear, TI and Spatial LCMV Beamformer which corresponds to the well-known SMF and which is defined by

$$h_{1o} = (s^{\dagger} <R_x(t)>^{-1} s)^{-1} <R_x(t)>^{-1} s \quad (3.2)$$

where $R_x(t) \triangleq E[x(t)\, x(t)^{\dagger}]$ is the correlation matrix of the observation vector $x(t)$.

4. EQUIVALENT CYCLIC GSLC STRUCTURE

To show that the Cyclic LCMV Beamformer (3.1) has an equivalent GSLC structure, let us consider a full rank $((N-1) \times N)$ complex matrix A such that $A\,s = \mathbf{0}$. Then, the space orthogonal to s corresponds to the one spanned by the vectors $A^{\dagger}w$ when w spans C^{N-1}. As a consequence, for each $(N \times 1)$ complex vector h_i, $1 \leq i \leq M$, it exists a $((N-1) \times 1)$ vector w_i, such that

$$h_i = (s^{\dagger}s)^{-1}(s^{\dagger}h_i)\,s - A^{\dagger}w_i \tag{4.1}$$

Using (4.1) and (2.7) into (2.6), we obtain another expression of $y(t)$ under the constraints (2.7) given by

$$y(t) = y_c(t) - y_a(t) \tag{4.2}$$

where

$$y_c(t) \triangleq (s^{\dagger}s)^{-1}\, s^{\dagger}x(t) \tag{4.3}$$

$$y_a(t) \triangleq w_1{}^{\dagger}z(t) + \sum_{l=2}^{M} w_l{}^{\dagger}z(t-\Delta_l)^{\zeta_l}\, e^{j2\pi\alpha_l t} \tag{4.4}$$

$$z(t) \triangleq A\ x(t) \tag{4.5}$$

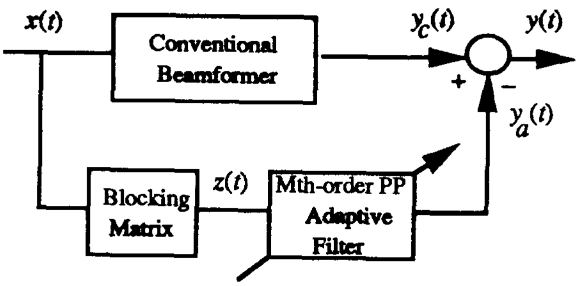

Figure 1. *Equivalent GSLC structure of the Cyclic LCMV Beamformer*

The previous expressions show that the Cyclic LCMV Beamformer defined by (3.1) has an equivalent GSLC structure depicted at figure 1, where $y_c(t)$, defined by (4.3), is the output of a Conventional Beamformer in the direction of the useful signal whereas $y_a(t)$, defined by (4.4), is the output of a Mth-order PP filter which input, $z(t)$, defined by (4.5), is an observation vector containing no useful signal. Thus, implementing the Cyclic LCMV beamformer (3.1) is equivalent to implement the Cyclic GSLC structure of figure 1 where the vectors w_i, $(1 \leq i \leq M)$, minimize the MSE between $y_c(t)$ and $y_a(t)$.

5. CYCLIC LCMV BEAMFORMER PERFORMANCE

Using (3.1), the Signal to Interference plus Noise Ratio (SINR) at the output of the Mth-order Cyclic LCMV Beamformer, noted $\mathrm{SINR_o}[M]$, can be computed. Note that the output SINR is defined, in the paper, by the ratio between the temporal mean of the output instantaneous useful signal power and interference plus noise power respectively. After some elementary algebraic manipulations, we obtain

$$\mathrm{SINR_o}[M] = \frac{\pi_s}{f^{\dagger}(S^{\dagger}\langle R_B(t)\rangle^{-1}S)^{-1}f} \tag{5.1}$$

which gives, for $M = 1$, the expression of the SINR at the output of the SMF, given by

$$\mathrm{SINR_o}[1] = \pi_s\, s^{\dagger}\langle R(t)\rangle^{-1}s \tag{5.2}$$

where $\pi_s \triangleq \langle E[|s(t)|^2]\rangle$, $\langle R_B(t)\rangle$ corresponds to $\langle R_x(t)\rangle$ in the absence of useful signal and $\langle R(t)\rangle$ is the temporal mean of the input interference plus noise correlation matrix.

5.1 One jammer case ($P = 1$) with $M = 2$

Assuming $P = 1$ in (2.1) and $M = 2$ in (2.6), the expressions (5.1) and (5.2) can be developed and the gain in SINR, $G[2]$, obtained in using (3.1) with $M = 2$ instead of the SMF and defined by the ratio between $\mathrm{SINR_o}[2]$ and $\mathrm{SINR_o}[1]$, can be computed and written, after tedious computations and for $T = +\infty$, as

$$G[2] = 1 + \frac{\varepsilon^2\gamma^2|\alpha_{1s}|^2(1-|\alpha_{1s}|^2)}{[1+\varepsilon(1-|\alpha_{1s}|^2)][1+\varepsilon+\varepsilon(1-|\alpha_{1s}|^2)(1+\varepsilon(1-\gamma^2))]} \tag{5.3}$$

where α_{1s} is the spatial correlation coefficient between the jammer and the signal, defined by the normalized inner product of J_1 and s, $\varepsilon \triangleq J_1{}^{\dagger}J_1\langle\pi_1(t)\rangle/\langle\eta_2(t)\rangle$, $\pi_1(t) \triangleq E[|m_1(t)|^2]$, $\langle\eta_2(t)\rangle$ is the background noise power per sensor and γ, $(0 \leq \gamma \leq 1)$, is a jammer's normalized cyclic correlation coefficient defined by

$$\gamma \triangleq |\langle E[m_1(t)m_1(t-\Delta_2)^{\zeta_2 *}]e^{j[(1-\zeta_2)\Delta\omega_1 - 2\pi\alpha_2]t}\rangle| \,/\langle\pi_1(t)\rangle \tag{5.4}$$

The expression (5.3) shows that the Cyclic LCMV Beamformer does not improve the output performance with respect to that of the SMF ($G[2] = 1$) in the absence of jammer ($\varepsilon = 0$), when the latter is orthogonal ($|\alpha_{1s}| = 0$) or colinear ($|\alpha_{1s}| = 1$) to the useful signal or when the normalized cyclic correlation coefficient of the jammer is zero ($\gamma = 0$). In the other cases, $G[2]$ is an increasing function of γ which does not depend on the signal's and jammer's phase. As a consequence, to maximize the gain in SINR at the output, the parameters $(\Delta_2, \zeta_2, \alpha_2)$ must be chosen so as to maximize γ, which can be done after an a priori estimation of the cyclic correlation function of the observations. For example, for a BPSK jammer we have to choose $\Delta_2 = 0$, $\zeta_2 = -1$ and $2\pi\alpha_2 = 2\Delta\omega_1$.

Nevertheless, the expression (5.3) shows that the gain $G[2]$ is always upper-bounded by 2, which is relatively weak compared to that obtained in some cases in radiocommunications with particular 2th-order PP filters when a reference signal correlated with the useful signal is available [2] [8]. In fact, this weak value of the gain in SINR obtained in using a 2th-order Cyclic GSLC structure in (quasi)-cyclostationary context instead of the SMF has already been found in [9] in using 3th-order or 5th-order Volterra GSLC structures in non Gaussian contexts. These results are directly related to the constraints of no useful signal distortion, chosen in this paper (2.7) and in [9] to obtain a first Cyclic or Volterra extension of the classical LCMV Beamformer respectively, which are obviously too strong to obtain higher gain in performance. This remark shows off the problem of the optimal constraints choice, which may be solved for example by a maximum likelihood approach and which will be presented in another paper. These results are illustrated in Figure 2 which shows the variations of $G[2]$ as a function of the DOA of the jammer for several values of γ and for a ULA of 4 sensors.

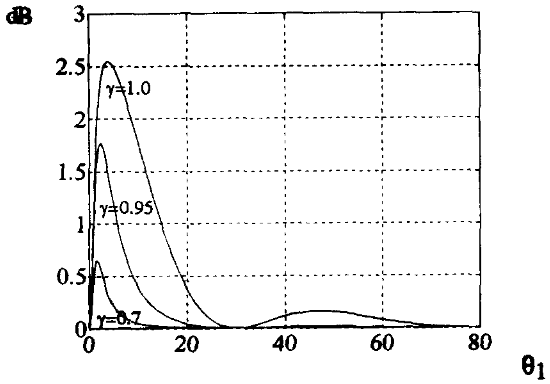

Fig. 2 - *G[2] as a function of θ_1*
$\theta_s = 0°$, ULA, N = 4, INR1 $\triangleq \langle\pi_1(t)\rangle / \langle\eta_2(t)\rangle$ = 16 dB

5.2 Two jammers case (P = 2) with M = 2

In the case of two jammers, the gain $G[2]$ is, in particular, a function of the phase difference ψ between the two jammers. Although this gain remains relatively weak in most cases, there are some situations for which this gain can be higher as it is shown on figure 3 which shows the variations of $G[2]$ as a function of ψ for several values of γ_2, the γ coefficient of the jammer 2, and for an ULA of 3 sensors.

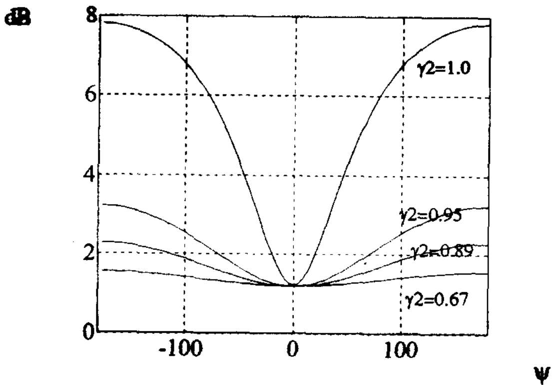

Fig. 3 - *G[2] as a function of ψ, ULA, N = 3, γ_1 = 1*
$\theta_s = 0°$, $\theta_1 = 12°$, $\theta_2 = 45°$, INR1 = 16 dB, INR2 = 20 dB

6. CONCLUSION

A first Cyclic LCMV Beamformer has been presented to improve the performance of the SMF in (quasi)-cyclostationary contexts. This new beamformer has an equivalent Cylic GSLC structure which associated constraints on the useful signal prevent from obtaining high gain in performance in most situations. This opens a reflexion about the optimal constraint choice which will be presented in an other paper.

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