A NEW METHOD FOR ESTIMATION OF THE AMPLITUDE DISTRIBUTION OF SIGNALS[*]

Gary D. Brushe[†]   W. Paul Malcolm   Langford B. White

Signal Analysis Discipline, Communications Division, ESRL, DSTO,
Bldg. 203L, PO Box 1500, Salisbury, SA 5108, AUSTRALIA.

Cooperative Research Centre for Robust and Adaptive Systems
c/- RSISE, Australian National University, Canberra, ACT, Australia.

[†]Tel. +61 8 8259 6662, Fax +61 8 8259 6328, E-mail: Gary.Brushe@dsto.defence.gov.au

ABSTRACT

A hidden Markov model method for estimating an *a posteriori* distribution of the amplitude of communications signals is presented. As the signal to noise ratio decreases the hidden Markov model method is shown to perform significantly better than a conventional histogram method for characterising the amplitude distribution. The HMM estimation is performed within a Expectation Maximisation method in order to improve the estimates of the transition probabilities used in the HMM and the resulting estimated amplitude distribution.

1. INTRODUCTION

A new method for estimation of *a posteriori* distribution of the amplitude of communications signals is presented. The approach which is commonly used to acquire the amplitude distribution of a communications signal (in baseband quadrature form) is to simply compute the signal's amplitude at each time instant and then plot the histogram of their values. For signal's with sufficiently high Signal to Noise Ratio (SNR) the features of the signal's amplitude distribution may (from inspection) be evident from this histogram. It should be noted that this method is invariant to the effects of carrier drift However in the case of digital signals (which have a number of distinct amplitude levels, i.e a 16 QAM signal [2] has 3 distinct amplitude levels), as the signal's strength decreases the histogram method begins to coalesce features of the amplitude distribution which may be adjacent to each other, thereby making them unresolvable, i.e. for a 16 QAM signal of the three distinct amplitude levels which should be present only two are visible. Further, increasing the number of amplitude bins used in the histogram, does not improve the resolution.

Parker [1] recently developed some techniques for estimating the SNR of Quadrature Amplitude Modulated (QAM) signals [2]. These techniques are based on histograms of the baseband signals' real or complex raw amplitudes. However, if carrier drift is not compensated for in a signal at baseband, these techniques will fail as the real / complex amplitudes will be continuously varying due to the carrier drift. White and Ross [3] recently presented a method for adaptive demodulation of phase modulated signals using Hidden Markov Models (HMMs) [4]. They stated that the method can be extended to QAM signals, if estimates of the amplitudes could be initialised suitably close to the correct amplitude. If the signal strength is such that a histogram of the raw amplitudes can not extract the amplitude features correctly, and the number of amplitude levels is unknown, how can this be done?

In this paper we present a HMM method for estimating the posterior distribution of the amplitude of a signal in quadrature baseband form. This method is invariant to carrier drift and suitable for signals which have either continuously valued amplitudes (as in analogue signals) or discrete amplitude levels (as in digital signals). It is shown that the HMM method can extract the features of the amplitude distribution at SNR levels for which the histogram method fails.

2. THE ALGORITHM

The signal of interest is modelled in the quadrature baseband representation:

$$z_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = a_t \begin{bmatrix} \cos(\phi_t) \\ \sin(\phi_t) \end{bmatrix} + v_t \qquad (1)$$

where $a_t \geq 0$ denotes the amplitude, $\phi_t \in (-\pi, \pi)$ denotes the phase and v_t is a zero mean white Gaussian process with independent real and imaginary parts, v_t^r and v_t^i respectively, each of variance σ^2. The signal amplitude and phase are assumed to be either continuously valued (i.e. takes values in an interval) in the case of analogue modulated signals, or to belong to a discrete set in the case of digital signals.

The task is to estimate the posterior distribution of the amplitude given a block of data, $Z = \{z_1,...,z_T\}$. We assume that the amplitude statistics are stationary over this interval. We also wish to impose no prior information on the phase, so we assume the phase is an independently identically distributed (i.i.d.) sequence with uniform prior. The observed data likelihood is given by:

$$p(Z \mid A,\Phi) = (2\pi\sigma^2)^{-T/2} \times$$
$$\prod_{t=1}^{T} \exp\left\{\frac{(x_t - a_t\cos\phi_t)^2 + (y_t - a_t\sin\phi_t)^2}{-2\sigma^2}\right\} \quad (2)$$

where $A = \{a_1,..., a_T\}$ and $\Phi = \{\phi_1,...,\phi_T\}$. Taking the expected value over Φ yields:

$$E_\Phi[p(Z \mid A,\Phi)] = (2\pi)^{-3T/2}\sigma^{-T} \times$$
$$\prod_{t=1}^{T}\int_{-\pi}^{\pi}\exp\left\{\frac{1}{-2\sigma^2}\left[(x_t - a_t\cos\phi_t)^2 + (y_t - a_t\sin\phi_t)^2\right]\right\}d\phi_t$$

$$E_\Phi[p(Z \mid A,\Phi)] = \prod_{t=1}^{T}\frac{|z_t|}{\sigma^2}\exp\left\{\frac{|z_t|^2 + a_t^2}{-2\sigma^2}\right\}I_0\left(\frac{|z_t|a_t}{\sigma^2}\right) \quad (3)$$

where I_0 is the zero order modified Bessel function of the first kind. Thus the expected likelihood with iid uniform phase priors is identical to the likelihood for the amplitude of the signal (1), i.e. $E_\Phi[p(Z \mid A,\Phi)] = p(R \mid A)$, where $R = \{r_1, ..., r_T\}$ with $r_t = |z_t|$, i.e. Rician distributed [2, pp. 60].

The next step is to apply the on-line HMM smoothing methods described in [5] to obtain the set of (approximate) a posteriori probabilities $p(a_t \mid Z)$, $t = 1,...,T$. We assume a discretised set of amplitude levels (as in the histogram method), and a set of associated a priori transition probabilities. The amplitude distribution is then re-estimated using the ergodic average:

$$\hat{p}(a) = \frac{1}{T}\sum_{t=1}^{T}p(a_t \mid Z) \quad (4)$$

If one assumes the amplitude is continuous, a method for obtaining the transition probabilities is to assume they have a normalised Gaussian distribution with an estimated variance of $\hat{\sigma}_a^2$, although other distributions could also be just as valid. Use of a normalised Gaussian distribution allows the transition probabilities to be controlled via the variance which can be estimated via an Expectation Maximisation (EM) approach [4] (see Appendix).

In the case of digital signals where the amplitudes can change instantaneously between discrete levels, the algorithm is modified so that a fixed interval smoothing HMM algorithm is used and reinitialised and calculated over each of the baud periods (where the amplitude is continuous again), but the ergodic average is taken over all baud periods. The intervals for which the amplitude is continuous (i.e. the baud periods) can be determined by using baud rate estimation techniques [6] (which are also insensitive to carrier drift). The HMM equations and EM approach for the digital case are described in the appendix. For the analogue case the HMM equations need to be modified from fixed interval smoothing over each baud period to an on-line approach [5] over all the data.

3.   RESULTS AND CONCLUSION

We test the digital version of the method presented above using a square 16 QAM signal (see [2, pp. 6]) which has three distinct amplitude levels. A 16 QAM signal is generated and mixed down to baseband. First a high SNR signal (30 dB) was used and figure 1 shows the amplitude distribution as determined using a histogram and the HMM method (solid line). For the HMM method the baud periods were first determined using a baud rate estimation method and a posteriori probabilities calculated for each baud period but averaged over all baud periods. Figure 2 shows the histogram and HMM estimate (solid line) of the amplitude distribution for a signal with a SNR of 15dB. The variance $\hat{\sigma}_a^2$ of the normalised Gaussian distributed amplitude transition probabilities was estimated using the EM approach described in the appendix, however it was noted that $\hat{\sigma}_a^2$ could converge to local maxima of the likelihood which may result in an unsatisfactory estimate of the amplitude distribution (dashed line). Also the noise variance used in the estimation was significantly smaller (corresponding to a 30dB signal) than the 15 dB signal noise variance. The authors found that the HMM's amplitude distribution estimate deteriorated considerably if the correct noise variance (corresponding to 15dB) was used. One could also use an EM approach to estimate the noise variance which should be used (however this has not been done in this paper). Figure 3 is for a 20dB pulse shaped signal (standard raised cosine filter with the roll off set to 0.75) and again shows the estimate of the amplitude distribution for the histogram and HMM method (solid line).

As can be seen in Figures 1 - 3, the HMM method (with parameters tuned, i.e. the value of $\hat{\sigma}_a^2$) performs significantly better than the histogram method when determining the amplitude distribution of baseband signals.
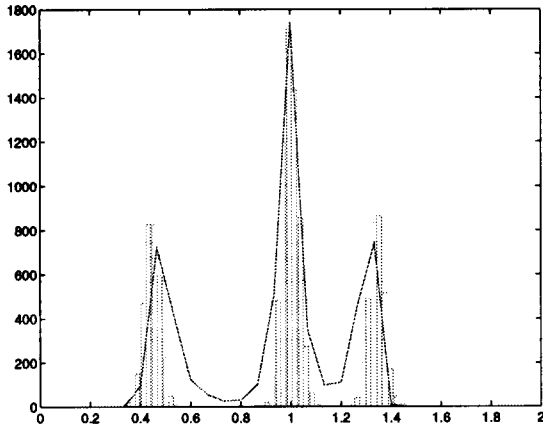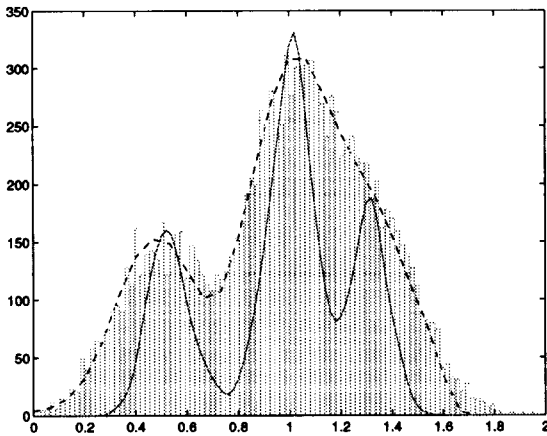
Figure 1: Amplitude distribution (30dB Signal).

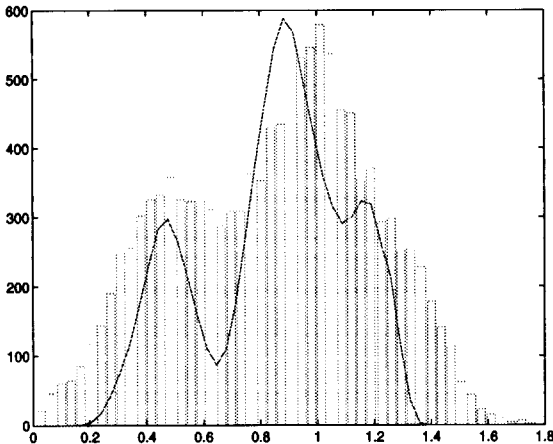Figure 2: Amplitude distribution (15dB Signal).

Figure 3: Amplitude distribution (20dB pulse shaped Signal).

REFERENCES

[1] G. J. Parker, "Techniques for the blind estimation of signal to noise ratio for quadrature modulated signals", *Proc. ISSPA '96*, Gold Coast, Australia, 1996, pp. 238 - 241

[2] W. T. Webb and L. Hanzo, *Modern Quadrature Amplitude Modulation*, (IEEE Press USA and Pentech Press London, 1994).

[3] L. B. White and J. D. Ross, "Adaptive demodulation of phase modulated signals using hidden Markov models", *Proc. ICASSP '93*, Minneapolis, USA, 1993.

[4] L. R. Rabiner, "A tutorial on hidden Markov models and selected applications in speech recognition", *Proc. IEEE*, vol. 77, no. 2, pp. 257 - 286, Feb. 1989.

[5] V. Krishnamurthy and J. B. Moore, "On-Line estimation of hidden Markov model parameters based on the Kullback - Leibler information measure", *IEEE Transactions on Signal Processing*, vol. 41, no. 8, pp. 2557 - 2573, 1993.

[6] A. W. Wegener, "Practical Techniques for Baud Rate Estimation", *Proc. ICASSP '92*, vol. IV, pp. 681 - 684, 1992.

APPENDIX

The reader is referred to [4] for an explanation on hidden Markov models and the Expectation Maximisation (EM) method. The following description only explains the equations used in the HMM & EM estimation methods.

Expectation Step (E-Step):

Using the current (or initial) value of $\hat{\sigma}_a^2$, determine the normalised Gaussian distributed transition probabilities for the N amplitude states of the HMM, i.e.

$$a_{ij} \triangleq \Pr\left\{a_t = q_j \big| a_{t-1} = q_i\right\}$$

$$a_{ij} = \frac{\exp\left(\dfrac{(i-j)^2}{2N^2\hat{\sigma}_a^2}\right)}{\displaystyle\sum_{k=0}^{N-1}\exp\left(\dfrac{(i-k)^2}{2N^2\hat{\sigma}_a^2}\right)}$$

The HMM forward probabilities are recursively calculated as follows:

$$\alpha_{t|t}^{(n)}(i) \triangleq \Pr\{r_{nT}, \ldots, r_{nT+t}, a_{nT+t} = q_i\}$$

$$\alpha_{t|t}^{(n)}(i) = b_i(r_{nT+t}) \sum_{j=0}^{N-1} a_{ji} \alpha_{t-1|t-1}^{(n)}(j)$$

for $t = 0, \ldots, T-1$ and $i = 0, \ldots, N-1$. T is the baud period of the signal, n corresponds to the nth baud period and q_i is the ith amplitude state of the N uniformly discretised amplitude levels. Also:

$$b_i(r_{nT+t}) = \frac{r_{nT+t}}{\sigma^2} \exp\left\{\frac{r_{nT+t}^2 + q_i^2}{-2\sigma^2}\right\} I_0\left(\frac{r_{nT+t}q_i}{\sigma^2}\right)$$

is the probability that the observation r_{nT+t} was produced by amplitude state q_i.

$$\alpha_{0|0}^{(n)}(i) = \pi(i) b_i(r_{nT})$$

$\pi(i)$ is set uniformly.

The HMM backward probabilities are recursively calculated as follows:

$$\beta_{t|t+1}^{(n)}(i) \triangleq \Pr\{r_{nT+t+1}, \ldots, r_{(n+1)T-1} | a_{nT+t} = q_i\}$$

$$\beta_{t|t+1}^{(n)}(i) = \sum_{j=0}^{N} a_{ij} \beta_{t+1|t+1}^{(n)}(j)$$

where $\beta_{t+1|t+1}^{(n)}(j) = \beta_{t+1|t+2}^{(n)}(j) b_j(r_{nT+t+1})$, $t = T-1, \ldots, 0$ and $i = 0, \ldots, N-1$. Initialisation is by $\beta_{nT|nT+1}^{(n)} = \frac{1}{N}$ (note that this initialisation prevents data from the next baud period from being used in the current baud period estimates, i.e. each baud period is calculated independently).

Now the probability that the amplitude at time t is amplitude state q_i given all the observations in a baud period is:

$$\gamma_{t|T}^{(n)}(i) \triangleq \Pr\{a_{nT+t} = q_i | r_{nT}, \ldots, r_{(n+1)T-1}\}$$

$$\gamma_{t|T}^{(n)}(i) = \frac{\alpha_{t|t}^{(n)}(i) \beta_{t|t+1}^{(n)}(i)}{\sum_{j=0}^{N-1} \alpha_{t|t}^{(n)}(j) \beta_{t|t+1}^{(n)}(j)}$$

for $i = 0, \ldots, N-1$.

Now we also calculate:

$$\xi_{t|T}^{(n)}(i,j) \triangleq \Pr\{a_{nT+t} = q_i, a_{nT+t+1} = q_j | r_{nT}, \ldots, r_{(n+1)T-1}\}$$

$$\xi_{t|T}^{(n)}(i,j) = \frac{\alpha_{t|t}^{(n)}(i) \beta_{t+1|t+1}^{(n)}(j) a_{ij}}{\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} \alpha_{t|t}^{(n)}(m) \beta_{t+1|t+1}^{(n)}(k) a_{mk}}$$

$$\bar{\xi}(i,j) = \sum_n \sum_t \xi_{t|T}^{(n)}(i,j)$$

Maximisation Step (M-Step):

Update $\hat{\sigma}_a^2$ by:

$$\hat{\sigma}_a^2 = \arg\max_{\sigma_a^2}\left[J(\sigma_a^2)\right]$$

where

$$J(\sigma_a^2) = \sum_{i,j} \bar{\xi}(i,j) \log(a_{ij})$$

remembering that a_{ij} is a function of $\hat{\sigma}_a^2$.

The process is then repeated and EM theory guarantees convergence of $\hat{\sigma}_a^2$ to a local maximum of the likelihood.