

# ADAPTIVE SUPPRESSION OF WIDEBAND INTERFERENCES IN SPREAD-SPECTRUM COMMUNICATIONS USING THE WIGNER-HOUGH TRANSFORM

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## ABSTRACT

The aim of this paper is to propose an adaptive method for suppressing wideband interferences in spread spectrum (SS) communications. The proposed method is based on the time-frequency representation of the received signal, from which the parameters of an adaptive time-varying interference excision filter are estimated. The approach is based on the generalized Wigner-Hough Transform as an effective way to estimate the instantaneous frequency of parametric signals embedded in noise. The performance of the proposed approach are evaluated in the presence of chirp-like interferences plus noise.

## 1. INTRODUCTION

Direct sequence (DS) spread-spectrum (SS) signals are often used for their good behavior in applications such as: code division multiple access (CDMA), low probability of being intercepted (LPI) systems, communications over channels with multipath, resistance to intentional jamming [6]. Systems using SS signals offer a good interference rejection capability for narrowband interferences. In fact, a spread-spectrum sequence is not easily predictable because it appears to be noise-like (unless the code is known, of course). Conversely, if the interference is a predictable process, it can be predicted from the observation and then cancelled (see for example [5] and the references therein). Quite recently, Amin *et al.* proposed an interesting extension of conventional adaptive interference suppression techniques in spread-spectrum communications to the case of wideband interferers based on the time-frequency representations of the observed signal [1]. The method proposed in [1] consists in evaluating the Wigner-Ville Distribution (WVD), or a related time-frequency distribution (TFD), of the observed signal and estimating the parameters of the interfering signal from the WVD.

Once the parameters have been estimated, an adaptive time-varying filter can be set up to suppress the interference. Once again, the method exploits the fact that spread-spectrum signals are difficult to track, even in the time-frequency domain, whereas a large class of interferences can be tracked, and then cancelled, working in the time-frequency domain. One of the problems related to using Amin's method is that, if the interference is low with respect to the SS signal or to the noise, the estimation of the interference parameters can completely fail and the suppression filter could track the useful signal, instead of the interference, with obvious shortcomings. In this work we propose an extension of the method proposed in [1] using the so called Wigner-Hough Transform (WHT) [2]. The method assumes that : i) the interference has a constant amplitude; ii) the instantaneous frequency assumes a known parametric form (but the parameters are not known a-priori). The number of interfering terms does not need to be known a-priori but it can be estimated from the data. The paper is organized as follows: in Section 2 we apply the Wigner-Hough Transform to the estimate of the interference parameters; in Section 3 we propose optimal and suboptimal schemes for detecting SS signals superimposed to linear or sinusoidal FM interferences plus noise; in Section 4 we provide the performance of the proposed approach, expressed in terms of improvement of the signal-to-disturbance ratio and bit error rate.

## 2. ESTIMATION OF INTERFERENCE PARAMETERS

The observed signal is given by the sum of a SS signal plus interference plus noise:

$$x(t) = \sqrt{P_s}Ac(t) + \sum_{k=1}^K \sqrt{P_{I_k}}e^{j\phi(t, \theta_k)} + w(t), \quad (1)$$

where  $P_s$  is the signal power,  $A = \pm 1$  is the transmitted symbol (we assume BPSK modulation),  $w(t)$  is a

complex additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_w^2$ ,  $c(t)$  is a maximum-length sequence (MLS) [6] whose entries belong to the binary alphabet  $\{-1, 1\}$ . We assume perfect synchronism on the receiver and that the duration of the observation is equal to one codeword, whose length is forced to be  $L = 2^m - 1$ , where  $m$  is the length of the shift register used to generate the code. We also assume that each interference signal has a constant amplitude and an instantaneous phase expressed in a parametric form (the parameters are the entries of vector  $\theta_k$  in (1)). Constant modulus signals are commonly used whenever the interferer wants to maximize the power of the transmitted signal and then uses a signal with a constant amplitude  $\sqrt{P_{I_k}}$  equal to the peak value. In this paper, we estimate the interference parameters using the so called Wigner-Hough Transformation (WHT) [2]. The WHT-based approach was proposed in [2] for detecting linear FM signals and was then extended in [3] to the analysis of generic FM modulation laws. Given a signal  $x(t)$ , its WHT is defined as:

$$WHT_x(\theta_k) = \int_{-\infty}^{\infty} W_x(t, f(t; \theta_k)) dt, \quad (2)$$

where  $W_x(t, f)$  is the Wigner-Ville Distribution of  $x(t)$ :

$$W_x(t, f) = \int_{-\infty}^{\infty} x(t + \tau/2) x^*(t - \tau/2) e^{-j2\pi f\tau} d\tau \quad (3)$$

and  $f(t; \theta_k)$  is the interference instantaneous frequency:

$$f(t; \theta_k) = \frac{1}{2\pi} \frac{d\phi(t; \theta_k)}{dt}. \quad (4)$$

For example, when dealing with linear FM signals, each signal component gives rise to energy concentrations along straight lines in the time-frequency plane of equation:  $f(t; \theta_k) = f_k + g_k t$ . The integration over all possible lines, obtainable by applying a Hough or, equivalently, a Radon Transform to the WVD, gives thus rise to peaks in the final parameter space: each peak corresponds to one linear FM signal, whose modulation parameters ( $f_k$  and  $g_k$ ) are the coordinates of the peak. As an example, Fig.1 shows the WHT of the sum of two linear FM interferences added to a SS signal plus noise. The number of samples is 63 and is equal to the number of chips in a MLS codeword; the power ratio between each interference and the SS signal is 3 dB and the signal-to-noise ratio (SNR) is also 3 dB. Due to the low interference-to-signal ratio, the WVD would not be very meaningful in such a case. Conversely, the WHT shows two evident peaks, witnessing the presence of the two interferences.

### 3. ADAPTIVE TIME-VARYING CANCELLATION FILTER

In the following we will make reference to the adaptive cancellation scheme depicted in Fig.2. We introduce

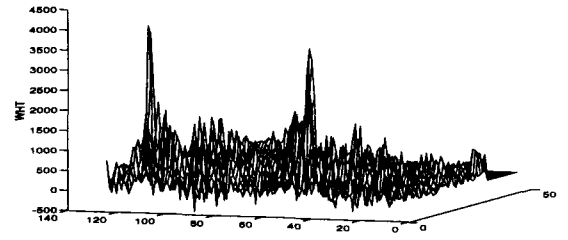


Figure 1: WHT of an SS signal plus two linear FM interferences plus noise.

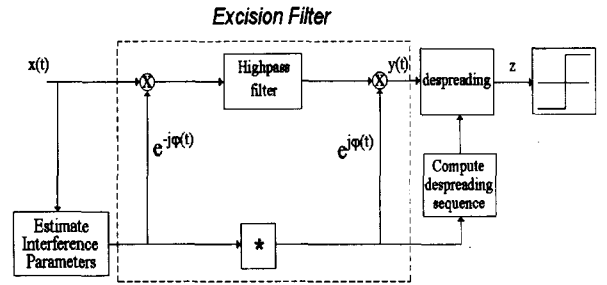


Figure 2: Adaptive time-varying excision filter.

the following vector notation to indicate the samples of the observed signal and the filter coefficients (a unitary sampling rate is assumed hereafter):

$$\begin{aligned} \mathbf{c} &:= (c(1), \dots, c(L))^T, \mathbf{w} := (w(1), \dots, w(L))^T, \\ \mathbf{d} &:= (e^{j\phi(1)}, \dots, e^{j\phi(L)})^T, \end{aligned} \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} -e^{-j\phi(1)} & e^{-j\phi(2)} & \dots & 0 \\ 0 & -e^{-j\phi(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -e^{-j\phi(L-1)} & e^{-j\phi(L)} \end{pmatrix} \quad (6)$$

$$\Phi = \text{diag}\{e^{j\phi(2)}, e^{j\phi(3)}, \dots, e^{j\phi(L)}\}. \quad (7)$$

where  $\mathbf{F} \in \mathbb{C}^{L-1 \times L}$  and  $\Phi \in \mathbb{C}^{L-1 \times L-1}$ . The noise vector  $\mathbf{w}$  is assumed to have zero mean and a diagonal covariance matrix  $\sigma_w^2 \mathbf{I}$ . The input sequence can then be written as (we will assume the presence of one interference, e.g.  $K = 1$  in (1), but the overall approach can be extended to the general case of several interferences):

$$\mathbf{x} = \sqrt{P_s} \mathbf{A} \mathbf{c} + \sqrt{P_I} \mathbf{d} + \mathbf{w}, \quad (8)$$

whereas the sequence at the output of the excision filter is:

$$\mathbf{y} = \Phi \mathbf{F} \mathbf{x}. \quad (9)$$

Indicating by  $\mathbf{h}$  the vector containing the coefficients (conjugated) of the despreading filter, the overall output is

$$z = \mathbf{h}^H \mathbf{y} = \mathbf{h}^H \mathbf{y}_s + \mathbf{h}^H \mathbf{y}_n = z_s + z_n. \quad (10)$$

The output SNR is then:

$$SNR_{out} = \frac{|z_s|^2}{E\{|z_n|^2\}} = \frac{|\mathbf{h}^H \mathbf{y}_s|^2}{|\mathbf{h}^H \mathbf{R} \mathbf{h}|^2}, \quad (11)$$

where  $\mathbf{R}$  is the covariance matrix of the noise at the output of the excision filter in Fig.2 and is equal to [4]:

$$\mathbf{R} = E\{\Phi \mathbf{F} \mathbf{x} \mathbf{x}^H \mathbf{F}^H \Phi^H\} = \sigma_w^2 \Phi \mathbf{F} \mathbf{F}^H \Phi^H. \quad (12)$$

The vector  $\mathbf{h}$  can be simply put equal to the codeword  $\mathbf{c}$  or it can be optimized. We consider two possible choices for  $\mathbf{h}$ : 1)  $\mathbf{h}_{sub} = \mathbf{c}$  or 2)  $\mathbf{h}_{opt} = \mathbf{R}^{-1} \mathbf{y}_s$ . The first choice  $\mathbf{h} = \mathbf{c}$  is commonly adopted for its simplicity, but it is a *suboptimal* solution because it does not take into account nor the noise correlation neither the modifications on the useful signal introduced by the excision filter. Conversely, the second choice takes into account the modification of the useful signal and the noise correlation due to the excision filter and employs the vector  $\mathbf{h}$  which maximizes the improvement factor (IF), defined as the ratio between the output and the input SNRs:

$$IF = \frac{SNR_{out}}{SNR_{in}}. \quad (13)$$

The IF corresponding to the optimal despreading filter is [4]:

$$IF_{max} = \mathbf{c}^H \mathbf{F}^H (\mathbf{F} \mathbf{F}^H)^{-1} \mathbf{F} \mathbf{c} \quad (14)$$

## 4. PERFORMANCE

The performance is expressed in terms of improvement factor and bit error rate. Two interference classes will be considered: linear and sinusoidal FM signals.

### 4.1. Improvement Factor

The IF depends on the interference parameters. To extract a single performance parameter and analyze the effect of the codeword length  $L$ , we have computed the average loss between the suboptimal scheme and the optimal scheme proposed above, averaged over all values of the interference parameters. The results are reported in Table 1, for linear and sinusoidal FM interferences. For each kind of modulation, Table 1 reports the loss between the suboptimal and the optimal despreading filters (left column) and the loss between the optimal scheme and the theoretical maximum gain, i.e.  $IF_{max} = L$  (right column). As we can see, the optimal despreading filter tends to have a gain of about 3

$L$	Linear FM		Sin. FM	
7	-2.83	-0.67	-2.44	-0.32
15	-2.93	-0.3	-2.89	-0.12
31	-2.99	-0.14	-2.97	-0.04
63	-2.99	-0.07	-2.99	-0.02

Table 1: Average losses between suboptimal and optimal schemes (left column) and between optimal scheme and theoretical limits (dB) (right column).

dB with respect to the suboptimal despreading scheme and the loss with respect to the theoretical limit tends to zero as the codeword length increases. The values for linear or sinusoidal FM interferences are similar (the sinusoidal FM case shows a slightly smaller loss with respect to the linear FM case).

### 4.2. Bit error rate

Assuming that the estimation of the interference parameters is correct, the output of the excision filter contains only signal and noise. Transmitting a BPSK signal and considering a Gaussian noise, we can express the error probability (bit error rate) in a closed form. The overall system shown in Fig.2 is linear and can be described by the equivalent weighting vector  $\mathbf{q}$ . We have considered three possible choices for vector  $\mathbf{q}$  [4]: 1)  $\mathbf{q}_{opt} = (\Phi \mathbf{F})^H \mathbf{h}_{opt}$ ; 2)  $\mathbf{q}_{sub} = (\Phi \mathbf{F})^H \mathbf{c}$ ; 3)  $\mathbf{q} = \mathbf{c}$ . The first choice corresponds to a scheme with excision filter followed by the optimal despreading filter; with the second choice the excision filter is followed by the suboptimal despreading system; the third choice refers to a conventional receiver which does not use any excision filter and demands all rejection capabilities to the despreading operations. In the first two cases, assuming perfect estimation of the interference parameters (the effect of the estimation error is considered in [4]), the interference is completely cancelled. The decision is taken comparing the real part of the filter output with a zero threshold<sup>1</sup>. In the BPSK case, the error probability is [4]:

$$P_e = Q\left(\sqrt{SNR_{in}} \frac{\Re(\mathbf{q}^H \mathbf{c})}{\sqrt{\mathbf{q}^H \mathbf{q}}}\right); \quad (15)$$

where  $SNR_{in} = P_s/\sigma_w^2$  and  $Q(x) := \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-u^2/2} du$ . In the third case, in the presence of the interference,

<sup>1</sup>Indeed the output signal is complex. We decided to take the real part because the imaginary part of the useful signal component, due to the mismatching introduced by the excision filter, is much smaller than the real part; in this way we maintain the linearity of the filter and avoid any unnecessary addition of noise.

defining the interference to signal ratio as  $ISR = P_I/P_s$ , the bit error rate is [4]:

$$P_e = \frac{1}{2}Q\left(\sqrt{SNR_{in}L}(1 + \sqrt{ISR}\frac{\Re(c^T d)}{L})\right) + \frac{1}{2}Q\left(\sqrt{SNR_{in}L}(1 - \sqrt{ISR}\frac{\Re(c^T d)}{L})\right) \quad (16)$$

The BER has been computed for linear FM interferences and is shown in Fig.3. The solid line reports the

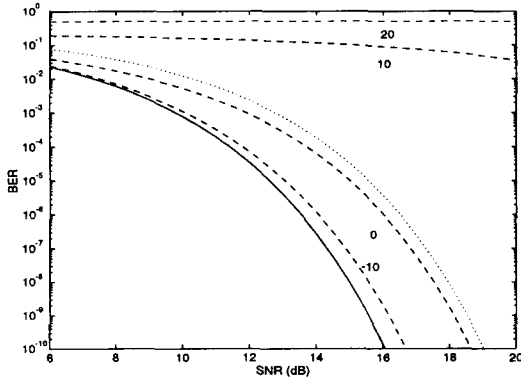


Figure 3: BER for linear FM interference vs.  $SNR_{in}L$ .

BER obtained using the excision filter and the optimal despreading filter; the dotted line shows the BER obtained using the excision filter and the suboptimal despreading filter; the dashed lines report the BER obtainable with the simple scheme that does not use any excision filter (the labels on each curve indicates the interference-to-signal ratio (ISR) in dB). Although we have assumed a perfect interference cancellation, the error probability depends on the interference parameters because the excision filter coefficients depend on the interference parameters and then SS signal and noise are consequently modified. To make a conservative estimate, we selected the interference parameters which produce the maximum error probability (worst case analysis). We can observe that the proposed scheme provides a consistent gain with respect to the simpler schemes, especially at high ISRs. The gain is moderate only at low ISR, which suggests that, in that case, we could decide do not use the excision filter, thus avoiding any unnecessary complication in the receiver.

## 5. CONCLUSION

In this paper we have proposed a novel interference rejection scheme for wideband interferences using the generalized Wigner-Hough Transform for estimating the

interference parameters and an adaptive time-varying filter for the interference excision. The despreading filter coefficients are optimized in order to maximize the improvement factor, defined as the gain between input and output SNRs. The proposed method provides the following advantages with respect to similar techniques, e.g. [1]: 1) it is able to reliably estimate the interference parameters at lower SNR, exploiting the signal model; 2) the despreading filter is optimal and takes into account the presence of the excision filter. The disadvantage of the proposed method, besides the higher computational cost, is that it is not robust against mismatching between the observed data and the assumed model. Investigations are in progress to establish a rigorous criterion for thresholding the Wigner-Hough Transform, to avoid the excision filter at low interference-to-signals ratios, where the interference rejection provided by simple despreading is sufficient to recover the useful data.

## 6. REFERENCES

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