# CHIP INTERLEAVING IN DIRECT SEQUENCE CDMA SYSTEMS

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## ABSTRACT

Direct sequence Code Division Multiple Access (CDMA) systems are considered in this paper, in a multipath environment. We address the problem of estimating each user's signature waveform without requiring training sequences. We show that the problem is considerably simplified if a novel transmission strategy is adopted, which combines spreading and interleaving. In this setup, chip sequences corresponding to successive bits are interleaved before transmission. Novel channel estimation algorithms are developed in this chip interleaving framework and their performance is analyzed. Adaptive implementations are derived and some illustrative simulations are presented.

## 1. INTRODUCTION

In CDMA communication systems, each user transmits information using a distinct signature waveform. Knowledge of those waveforms at the receiver is essential for separating the users and reducing multiuser interference. In a frequency selective multipath environment however, the received signature waveforms are distorted, resulting in degradation of system performance [1]. The estimation of the distorted user signatures is of great importance in adjusting the receiver parameters to ensure optimal operation. They are needed in the design of both single-user (e.g. RAKE) receivers [7] and multiuser ones (e.g. decorrelating receivers) [1].

If training is available, the problem is somewhat easier and can be addressed using linear MMSE estimation techniques [2]. If blind solutions are of interest, the problem appears to be more complicated. Existing solutions proposed for synchronous [3] and asynchronous systems [4] are based on subspace decompositions and have high computational complexity.

The goal of the present work is to derive simpler suboptimal solutions that can be adaptively implemented. To this end, we introduce a novel transmission strategy in which the chip sequences of consecutive bits are interleaved. It turns out that this approach considerably simplifies the channel estimation task (compared to [3]). Simple batch and adaptive algorithms are derived based on the proposed framework, and their performance is analyzed. Relevant results on single user, narrowband channels were reported in [5].

## 2. CHIP INTERLEAVING SYSTEM MODEL

In direct sequence CDMA systems, user $j$, $j = 1, \dots, J$, transmits an information symbol stream $w_j(n)$ using a spreading sequence of length $P$, $c_j(n)$, $n = 0, 1, \dots, P-1$. Then, the transmitted discrete-time signal at the chip rate is [6]

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k) c_j(n - kP) \ . \tag{1}$$

The chip sequence $s_j(n)$ is transmitted at a rate $1/T_c$ using a spectral shaping pulse modulated at the carrier frequency. The received signal $y(n)$ (after demodulation, matched filtering and sampling at the chip rate) is a superposition of signals from all users

$$y(n) = \sum_{j=0}^{J-1} y_j(n) + v(n) \ , \quad y_j(n) = \sum_{k=0}^{q} h_j(k) s_j(n-k) \tag{2}$$

where $h_j(k)$ is the impulse response of channel $j$ and $v(n)$ is additive white Gaussian noise.

In the sequel, the transmission scheme of (1) is further manipulated by the following interleaving procedure. For each block of $M$ bits $\mathbf{w}_{M,j}(n) \stackrel{\triangle}{=} [w_j(Mn) \dots w_j(Mn + M - 1)]^T$, let us consider the $M \times P$ interleaving matrix $\mathcal{I}(n)$, which is filled row-wise,

$$\mathcal{I}(n) = \begin{bmatrix} w_j(Mn)\mathbf{c}_j^T \\ \vdots \\ w_j(Mn + M - 1)\mathbf{c}_j^T \end{bmatrix} , \tag{3}$$

where

$$\mathbf{c}_j^T = [c_j(0), \dots, c_j(P-1)] \tag{4}$$

and then read column-wise. Hence, each block $\mathbf{w}_M(n)$ is transmitted $P$ times multiplied by the constants $c_j(k)$, $k = 0, \dots, P-1$, i.e.,

$$\mathbf{s}_{int,j} \stackrel{\triangle}{=} [\underbrace{c_j(0)\mathbf{w}_{M,j}^T(0), \dots, c_j(P-1)\mathbf{w}_{M,j}^T(0)}_{P \text{ times}}, \dots,$$
$$\underbrace{c_j(0)\mathbf{w}_{M,j}^T(n), \dots, c_j(P-1)\mathbf{w}_{M,j}^T(n)}_{P \text{ times}}, \dots]^T \ . \tag{5}$$

More formally, the transmitted (interleaved) signal is

$$s_{int,j}(PMl + k) =$$
$$\begin{cases} c_j(0) w_j(Ml + k) & 0 \le k < M \\ c_j(1) w_j(Ml + k - M) & M \le k < 2M \\ \vdots & \\ c_j(P-1) w_j(M(l - P + 1) + k) & (P-1)M \le k < PM \end{cases} \tag{6}$$

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A different viewpoint to this interleaving operation is to consider oversampling successive code vectors by $M$ and then multiplexing them together. Then it is clear that each bit waveform will now extend to $PM$ chips (compared to $P$ without interleaving).

In the sequel we will consider the case where the block length $M > q$. This is not restrictive in practice since $M$ is a design parameter and usually $q \ll P$. Under this assumption there is no interference between successive chips of the same bit (the channel length is less than the block length) but only from interleaved chips from other bits. This facilitates the estimation problem but introduces ISI within each block of $M$ bits, (ISI is negligible between successive blocks). Hence, each block of $M$ bits has to be jointly decoded to avoid performance penalties. This topic however, is beyond the scope of this paper.

## 3. CHANNEL ESTIMATION

It will be easier to develop the proposed method for the synchronous case first. Thus let us asume for the time being that all users are synchronized and the receiver has timing information. Then from (6) and (2) the received signal can be expressed as

$$\begin{aligned} y(PMn + lM + \tau) &= \sum_{j=0}^{J-1} c_j(l) \sum_{k=0}^{\tau} h_j(k) w_j(Mn + \tau - k) \\ &+ \sum_{j=0}^{J-1} c_j(l-1) \sum_{k=\tau+1}^{q} h_j(k) w_j(Mn + M + \tau - k) \\ &+ v(PMn + lM + \tau) \end{aligned} \quad (7)$$

for $0 < l < P$. Notice that the channel $h_j(k)$ is partially convolved with block $l$ and partially with block $l-1$. The interleaving procedure described above provides a rich structure to the transmitted signal, which can be exploited to estimate $h_j(k)$. Let us consider the signal correlation

$$r_{l,\tau} \triangleq E\{y^*(PMn) y(PMn + lM + \tau)\} \ , \quad (8)$$

for $l = 1, \ldots, P-1$, $\tau = 0, \ldots, M-1$. It follows from (7) that

$$r_{l,\tau} = \sum_{j=0}^{J-1} \gamma_j c_j(l) h_j(\tau) \quad (9)$$

where $\gamma_j = \sigma_w^2 h_j^*(0) c_j^*(0)$ and $\sigma_w^2 = E\{|w_j(n)|^2\}$. Eq. (9) can be expressed in matrix form as,

$$\mathbf{r}(\tau) = \mathbf{C}\, \tilde{\mathbf{h}}(\tau) \ , \quad (10)$$

$$\mathbf{r}(\tau) = \begin{bmatrix} r_{1,\tau} \\ \vdots \\ r_{P-1,\tau} \end{bmatrix} , \quad \tilde{\mathbf{h}}(\tau) = \begin{bmatrix} \tilde{h}_0(\tau) \\ \vdots \\ \tilde{h}_{J-1}(\tau) \end{bmatrix} \quad (11)$$

$$\mathbf{C} = \begin{bmatrix} c_0(1) & \cdots & c_{J-1}(1) \\ \vdots & & \vdots \\ c_0(P-1) & \cdots & c_{J-1}(P-1) \end{bmatrix} , \quad (12)$$

where $\tilde{h}_j(\tau) = \gamma_j h_j(\tau)$. Notice that $\tilde{h}_j(\tau)$ coincides with the impulse response $h_j(\tau)$ within a scaling ambiguity $\gamma_j$. Hence, one may recover $\tilde{h}_j(\tau)$ from the correlations $\mathbf{r}(\tau)$ by solving (11) for each $\tau$.

In practice $\mathbf{r}(\tau)$ is replaced by sample estimates,

$$\begin{aligned} \hat{\mathbf{r}}(\tau) &= \frac{1}{N} \sum_{n=0}^{N-1} y^*(PMn) \\ &\times \ [y(PMn + \tau), \ldots, y(PMn + (P-1)M + \tau)]^T \end{aligned} \quad (13)$$

and (11) is solved in the LS sense,

$$\hat{\mathbf{h}}(\tau) = \mathbf{C}^{\dagger} \hat{\mathbf{r}}(\tau) \ , \quad \mathbf{C}^{\dagger} = \left(\mathbf{C}^H \mathbf{C}\right)^{-1} \mathbf{C}^H \ . \quad (14)$$

Matrix $\mathbf{C}$ has full rank as long as the code vectors $\mathbf{c}_j$ are linearly independent and $J \leq P-1$. Notice that $\mathbf{C}$ is a priori known, and hence $\mathbf{C}^{\dagger}$ may be precomputed to save computations.

## 4. ASYNCHRONOUS CASE

The method of (13), (14) can be extended to the case of asynchronous users with some modifications, if timing information is available. Let us assume that user $j$ has an offset of $l_j M + \tau_j$ chips with respect to the user of interest and let us consider the correlation (c.f. (8))

$$r_{l,\tau,j} \triangleq E\{y_j^*(PMn + l_j M + \tau_j) y_j(PMn + (l_j + l)M + \tau_j + \tau)\} \ , \quad (15)$$

for $0 < l_j < P$, $0 \leq \tau < M$. If $l_j + l > P$, then the two data points depend on two different $M$ blocks and their correlation is zero. If $0 < l_j + l < P$, then an expression that is similar to (9) (but more complicated) can be derived (c.f. (7),(15)),

$$\begin{aligned} r_{l,\tau,j} &= c^*(l_j) c(l + l_j) \sigma_w^2 \sum_{k=0}^{\tau_j} h_j^*(k) h_j(k + \tau) \\ &+ c^*(l_j - 1) c(l + l_j) \sigma_w^2 \sum_{k=M-\tau}^{q} h_j^*(k) h_j(k + \tau - M) \\ &+ c^*(l_j - 1) c(l + l_j - 1) \sigma_w^2 \sum_{k=\tau_j+1}^{q-\tau} h_j^*(k) h_j(k + \tau) \end{aligned} \quad (16)$$

Notice that two consecutive code coefficients are now involved $[c_j(l+l_j), \ c_j(l+l_j-1)]$ in the expression (compared to one in (9)) due to the lack of synchronization. Nevertheless (16) can be expressed as a linear combination of those coefficients by grouping the first two terms together

$$r_{l,\tau,j} = c_j(l + l_j) \alpha_j(\tau) + c_j(l + l_j - 1) \beta_j(\tau) \ , \quad (17)$$

for some constants $\alpha_j(\tau)$, $\beta_j(\tau)$ (not depending on $l$). Finally, due to the independence of different users

$$\begin{aligned} r_{l,\tau} &= \gamma_0 c_0(l) h_0(\tau) \\ &+ \sum_{j=1}^{J-1} c_j(l + l_j) \alpha_j(\tau) + c_j(l + l_j - 1) \beta_j(\tau) \end{aligned} \quad (18)$$

where we assume without loss of generality that user 0 is the user of interest and is synchronized. Collecting all equations (17) in a matrix form we obtain

$$\mathbf{r}(\tau) = \mathbf{C}_{as}\, \mathbf{h}_{as}(\tau) \ , \quad (19)$$

where

$$\mathbf{C}_{as} = [\ \mathbf{C}_{0,as} \ | \ \mathbf{C}_{1,as} \ | \ \cdots \ | \ \mathbf{C}_{J-1,as} \ ] \ , \quad (20)$$

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$$\mathbf{C}_{0,as} = \begin{bmatrix} c_0(1) \\ \vdots \\ c_0(P-1) \end{bmatrix}, \ \mathbf{C}_{j,as} = \begin{bmatrix} c_j(l_j+1) & c_j(l_j) \\ \vdots & \vdots \\ c_j(P-1) & c_j(P-2) \\ 0 & c_j(P-1) \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \quad (21)$$

for $j = 1, \cdots, J-1$ and $\mathbf{h}_{as}(\tau) = [\tilde{h}_0(\tau) \mid \alpha_1(\tau), \beta_1(\tau) \mid \cdots \mid \alpha_{J-1}(\tau)\ \beta_{J-1}(\tau)]$. By solving (19) in the LS sense we can recover $\mathbf{h}_{as}(\tau)$ (c.f. (14))

$$\hat{\mathbf{h}}_{as}(\tau) = \mathbf{C}_{as}^{\dagger}\hat{\mathbf{r}}(\tau), \quad \mathbf{C}_{as}^{\dagger} = (\mathbf{C}_{as}^H \mathbf{C}_{as})^{-1}\mathbf{C}_{as}^H . \quad (22)$$

Notice however, that in the asynchronous case $\mathbf{h}_{as}(\tau)$ contains information only about the channel of the user of interest $\tilde{h}_0(\tau)$ while all the parameters $\alpha_j(\tau)$, $\beta_j(\tau)$ are nuisance parameters. Hence, the above procedure should be repeated for every user we wish to recover. However, all solutions depend on the same estimated statistics vector $\hat{\mathbf{r}}(\tau)$ and all matrices $\mathbf{C}_{as}^{\dagger}$ can be precomputed. In fact only the first row of $\mathbf{C}_{as}^{\dagger}$ needs to be stored since only the first element of $\mathbf{h}_{as}(\tau)$ is of interest.

## 5. ADAPTIVE IMPLEMENTATION

It follows from (13) that $\hat{\mathbf{r}}(k)$ can be recursively computed by

$$\hat{\mathbf{r}}^{(n)}(\tau) = \frac{n-1}{n}\hat{\mathbf{r}}^{(n-1)}(\tau) + \frac{1}{n}y^*(PMn)\mathbf{y}_\tau(n) \quad (23)$$

where $\mathbf{y}_\tau(n) = [y(PMn+\tau), \cdots, y(PMn+(P-1)M+\tau)]^T$.

If a constant step algorithm is desired, (23) can be modified to

$$\hat{\mathbf{r}}^{(n)}(\tau) = \lambda\hat{\mathbf{r}}^{(n-1)}(\tau) + (1-\lambda)y^*(PMn)\mathbf{y}_\tau(n) \quad (24)$$

for some $0 < \lambda < 1$. Equations (24) and (14) provide an adaptive implementations of the proposed methods

$$\hat{\mathbf{h}}^{(n)}(\tau) = \lambda\hat{\mathbf{h}}^{(n-1)}(\tau) + (1-\lambda)y^*(PMn)\mathbf{C}^{\dagger}\mathbf{y}_\tau(n) . \quad (25)$$

## 6. PERFORMANCE ANALYSIS

It would be interesting to study the estimation accuracy of the proposed algorithm, i.e., to evaluate

$$\mathbf{R}_{\hat{\mathbf{h}}(\tau)} = E\{[\hat{\mathbf{h}}(\tau) - \mathbf{h}(\tau)][\hat{\mathbf{h}}(\tau) - \mathbf{h}(\tau)]^H\} . \quad (26)$$

Let us focus on the synchronous case in the sequel, where the expressions are simpler. Similar arguments however are valid for the asynchronous case too.

From (14) it follows that

$$\mathbf{R}_{\hat{\mathbf{h}}(\tau)} = \mathbf{C}^{\dagger}\mathbf{R}_{\hat{\mathbf{r}}(\tau)}[\mathbf{C}^{\dagger}]^H \quad (27)$$

where $\mathbf{R}_{\hat{r}(\tau)} = E\{[\hat{\mathbf{r}}(\tau) - \mathbf{r}(\tau)][\hat{\mathbf{r}}(\tau) - \mathbf{r}(\tau)]^H\}$. It is clear from (14) that $\hat{\mathbf{r}}(\tau)$ is unbiased, i.e., $E\{\hat{\mathbf{r}}(\tau)\} = \mathbf{r}(\tau)$, while

$$\mathbf{R}_{\hat{\mathbf{r}}(\tau)} = E\{\hat{\mathbf{r}}(\tau)\hat{\mathbf{r}}^H(\tau)\} - \mathbf{r}(\tau)\mathbf{r}^H(\tau) . \quad (28)$$

Substituting $\hat{\mathbf{r}}(\tau)$ from (13) we obtain

$$\begin{aligned} \mathbf{R}_{\hat{\mathbf{r}}(\tau)} &= \sum_{n_1,n_2}^{N-1} \frac{1}{N^2} E\{y^*(PMn_1)y(PMn_2)\mathbf{y}_\tau(n_1)\mathbf{y}_\tau^H(n_2)\} \\ &- \mathbf{r}(\tau)\mathbf{r}^H(\tau) \quad (29) \\ &= \frac{1}{N}E\{\mid y(PMn)\mid^2 \mathbf{y}_\tau(n)\mathbf{y}_\tau^H(n)\} - \frac{1}{N}\mathbf{r}(\tau)\mathbf{r}^H(\tau) . \end{aligned}$$

The expectation term in (29) can be expressed in terms of the system parameters and $\mathbf{R}_{\hat{r}(\tau)}$ can be evaluated. Notice that the estimation variance is reduced at a rate $1/N$ as expected.

Let us turn our attention now at the analysis of the adaptive algorithm. We will try to evaluate its convergence properties in terms of bias and mean square error. From (24) we conclude that

$$E\{\hat{\mathbf{r}}^{(n)}(\tau) - \mathbf{r}(\tau)\} = \lambda E\{\hat{\mathbf{r}}^{(n-1)}(\tau) - \mathbf{r}(\tau)\} \quad (30)$$

and therefore

$$E\{\hat{\mathbf{r}}_e^{(n)}(\tau)\} = \lambda^n \mathbf{r}_e^{(0)}(\tau), \quad \mathbf{r}_e^{(n)}(\tau) = \hat{\mathbf{r}}^{(n)}(\tau) - \mathbf{r}(\tau) . \quad (31)$$

Hence, the algoritms bias reduces exponentially

$$E\{\hat{\mathbf{h}}^{(n)}(\tau) - \mathbf{h}(\tau)\} = \lambda^n \mathbf{C}^{\dagger}\mathbf{r}_e^{(0)}(\tau) . \quad (32)$$

In order to study the behavior of the MSE, we obtain the following recursion from (24)

$$\hat{\mathbf{r}}_e^{(n)}(\tau) = \lambda\hat{\mathbf{r}}_e^{(n-1)}(\tau) + (1-\lambda)e_\tau(n) \quad (33)$$

where $e_\tau(n) = y^*(PMn)\mathbf{y}_\tau(n) - \mathbf{r}(\tau)$.

Then it follows from (33) using the independence assumption that

$$\mathbf{R}_{\hat{\mathbf{h}}}^{(n)} = \lambda^2 \mathbf{R}_{\hat{\mathbf{h}}}^{(n-1)} + (1-\lambda)^2 \mathbf{C}^{\dagger}\mathbf{R}_e[\mathbf{C}^{\dagger}]^H \quad (34)$$

where $\mathbf{R}_{\hat{\mathbf{h}}}^{(n)} = E\{[\hat{\mathbf{h}}^{(n)}(\tau) - \mathbf{h}(\tau)][\hat{\mathbf{h}}^{(n)}(\tau) - \mathbf{h}(\tau)]^H\}$, $\mathbf{R}_e = E\{e_\tau(n)e_\tau^H(n)\}$.

Notice that for the synchronous case the independent assumption is justified and does not imply any approximation. Finally, from (34) we can get an expression for the excess estimation error

$$\mathbf{R}_{\hat{\mathbf{h}}}^{(n)} \longrightarrow \frac{(1-\lambda)^2}{(1-\lambda^2)}\mathbf{C}^{\dagger}\mathbf{R}_e[\mathbf{C}^{\dagger}]^H \quad \text{as } n \to \infty . \quad (35)$$

The effect of the choice of $\lambda$ on the excess error is evident from (35).

## 7. SIMULATIONS

Simulation results presented in this section confirm the applicability and performance of the proposed method. The method was tested on both synchronous and asynchronous CDMA systems. In all the simulations, Gold sequences of length 31 were used as spreading codes and blocks of $M = 5$ bits were chosen for interleaving. The algorithm was tested at noise level SNR=20 dB.

**Synchronous case:** Different channels of order $q = 3$ were used for the 10 users (see Fig. 1), scaled so that all the users have the same power. In Fig. 1, results from 500 Monte Carlo realizations of the proposed method are shown ($N = 2000$). The true impulse responses (solid line), mean estimates (dashed line) and mean $\pm$ standard deviation (dashdot lines) are shown.

**Asynchronous case:** The method was tested on a asynchronous CDMA system with 5 users. An arbitrary delay was assigned to each of the interfering users assuming that time delay of the user of interest is known. The time delay of the interfering users also needs to be known within an $M$-chip block. Fig. 2 shows the result of the method for 500 Monte Carlo runs ($N = 2000$). Only the user of interest is obtained in the asynchronus case as shown in Fig. 2.

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**Performance:** In the chip interleaving framework, we also investigated possible performance loss due to interchip interference in each $M$-bit block. Fig. 3 shows the performance of a single user system with multipath and no interleaving (probability of word error $M = 5$ vs SNR). The solid line shows the performance when the receiver filter is matched to the distorted pulse while the dashed-dotted when it is matched to the spreading code ignoring the effects of the channel.

The chip interleaving strategy was applied to that system ($M = 5$) and the performance was computed (dashed line) through the union bound (assuming joint decoding of the $M$ bits). No performance loss was observed as there is no significant ISI between successive blocks of $M$ bits. We should stress here once more that Fig. 3 involves a single user system and adresses performance losses due to ISI only (not MUI).

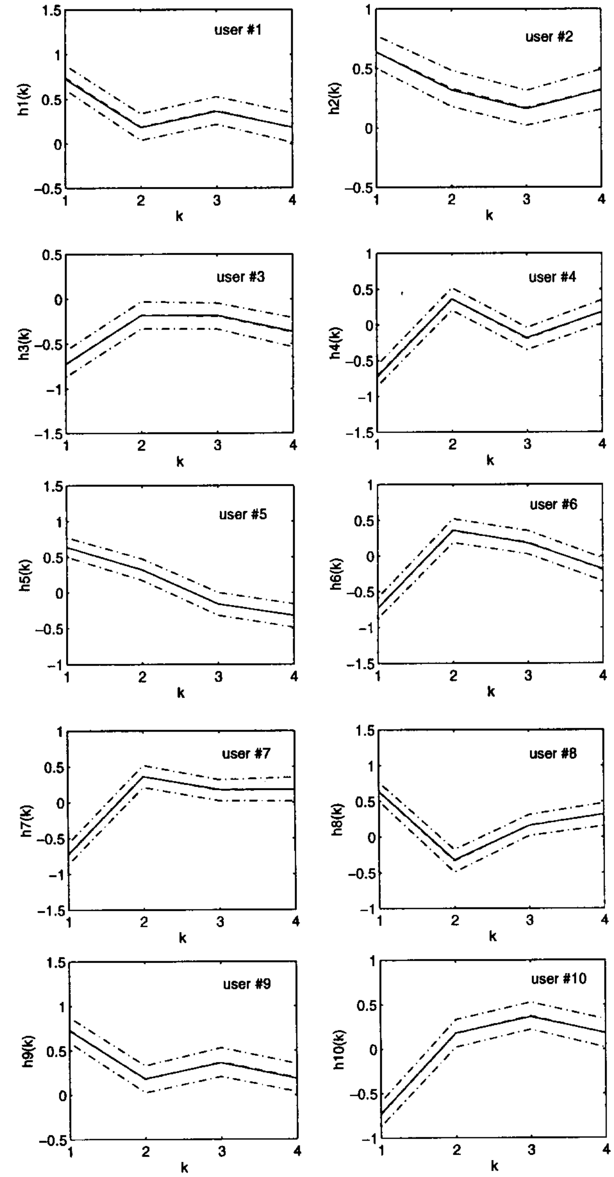


Figure 1. True and estimated channel tap coefficients

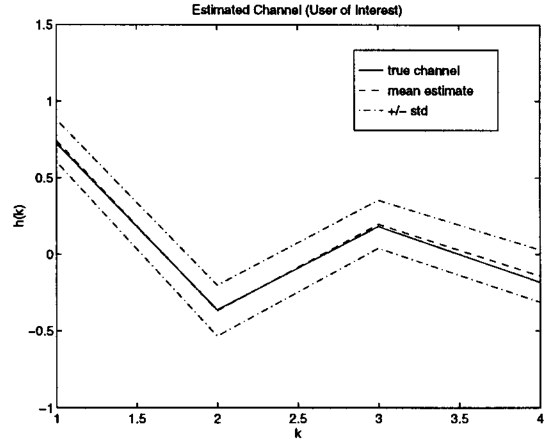


Figure 2. True and estimated channel for user of interest

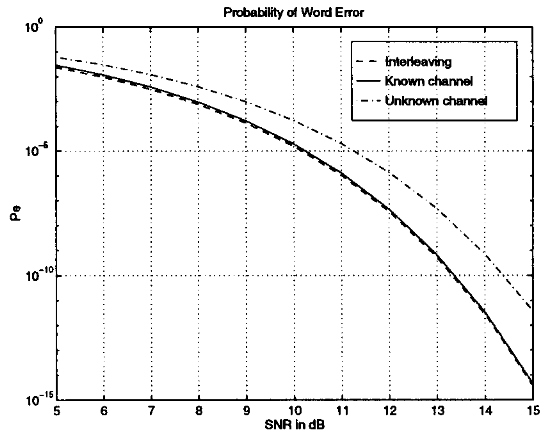


Figure 3. Receiver Performance

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