METHODS FOR FAST BLIND IDENTIFICATION AND EQUALIZATION OF COMMUNICATION CHANNELS

Chong-Meng Samson See

Defence Science Organisation
20 Science Park Drive SINGAPORE 118230
schongme@starnet.gov.sg

Colin F.N. Cowan

Department of Electrical Engineering
Queen's University of Belfast
C.F.N.Cowan@ee.qub.ac.uk

ABSTRACT

We propose new methods for fast blind identification and equalization of communication channels based on channel outputs sampled at symbol rate. These methods exploit the FA property of the transmitted sequence as well as the algebraic structures and relationships of the symbol and channel parameter matrices to achieve blind equalization. The block based methods proposed herein are fast converging and data efficient. Simulated studies have illustrated the proposed methods capable of achieving reliable blind equalization with sequence length as short as 20 samples over a wide-range of SNR.

1.. INTRODUCTION

Blind channel identification and equalization is of considerable interest and importance in the area digital communications. Blind equalizers may offer higher transmission efficiency and bandwidth conservation by shortening and eliminating the training sequence. Particularly in mobile communications where training is not efficient for rapidly time-varying channels.

One class of blind equalization methods is based on exploiting the higher order statistical information in the sampled channel outputs[1][2]. A common drawback of these methods is their slow convergence due to the large number of sampled channel outputs needed to obtain good estimates of the higher order statistics. Another class of blind equalization methods exploits the FA property and trellis relationship between the channel input and outputs. Among these methods include blind trellis search [3] and "quantized" channel approach[4]. These methods converge rapidly and can offer optimal solution after achieving global convergence. For channels with long impulse responses, the computationally complexity may become prohibitive. Recent research focus on blind equalization are based on oversampled channel outputs (see [3] and the references therein). While having the advantage of rapid convergence, these algorithms may faced singularity problem when the channel parameter matrix achieves full rank through excessive oversampling.

This paper proposes methods for fast blind equalization and identification of data communication channels. The methods proposed herein may offer a mean to alleviate the problem of slow convergence, excessive oversampling and, to some extent, computational complexity that are commonly encountered in most blind equalization methods. The key idea of the approach proposed herein is to exploit the FA property of the transmitted symbols, the algebraic structures and, the relationships between the channel output and symbol and channel parameter matrices. We formulate the blind identification problem in a least squares(LS) framework based channel outputs sampled at symbol-rate. Three block-based algorithms for jointly estimating the channel parameters and transmitted symbols are proposed. While sub-optimal, these proposed methods achieve substantial computational saving by relaxing the structural constraints on the symbol matrix.

Simulation results show the proposed methods converge rapidly and blind equalization and identification are achieved with small number of channel outputs. We use the maximum likelihood channel estimator using known sequence to examine the performance of the methods proposed herein. While locally convergent, the mean squares errors of the channel estimates achieved by the proposed algorithms are very close to the maximum likelihood estimator over a wide range of signal to noise ratio and sequence length.

2.. PROBLEM FORMULATION

The output of a FIR channel sampled at symbol rate is given by

$$x_n = \sum_{k=n}^{n-L+1} s_k h_{n-k} + v_n. \tag{1}$$

v_n is a sequence of white noise and s_k belongs to a FA set. Given an equaliser of order m and assuming the channel to be quasi-stationary for the period $P+1$, the channel output vector in matrix notation,

$$\mathbf{x}_n = \mathbf{C}(\mathbf{h})\mathbf{s}_n + \mathbf{v}_n \tag{2}$$

where

$$\begin{aligned}
\mathbf{x}_n &= [x_n, x_{n-1}, \cdots, x_{n-m+1}]^T \\
\mathbf{C}(\mathbf{h}) &= \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{L-1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{L-1} \end{bmatrix} \\
\mathbf{s}_n &= [s_n, s_{n-1}, \cdots, s_{n-L-m+2}]^T \\
\mathbf{v}_n &= [v_n, v_{n-1}, \cdots, v_{n-m+1}]^T .
\end{aligned} \tag{3}$$

$\mathbf{C}(\cdot)$ is the channel parameter matrix parameterized only by sampled channel impulse response $\mathbf{h} = [h_0, \cdots, h_{L-1}]^T$. Augmenting the $P+1$ number of channel output vectors into $\mathbf{X}_n = [\mathbf{x}_n, \mathbf{x}_{n+1}, \cdots, \mathbf{x}_{n+P}]$, we have

$$\mathbf{X}_n = \mathbf{C}(\mathbf{h})\mathbf{S}_n + \mathbf{V}_n, \quad (4)$$

where $\mathbf{S}_n = [\mathbf{s}_n, \cdots, \mathbf{s}_{n+P}]$ and $\mathbf{V}_n = [\mathbf{v}_n, \cdots, \mathbf{v}_{n+P}]$. Note $\mathbf{S}_n$ and $\mathbf{X}_n$ are Toeplitz matrices parametrized by $\tilde{\mathbf{x}}_n = [x_{n+1-m}, \cdots, x_n, \cdots, x_{n+P}]^T$ and $\tilde{\mathbf{s}}_n = [s_{n-L-m+2}, \cdots, s_n, \cdots, s_{n+P}]^T$, respectively. The blind identification and equalization problem can be casted: *Given* $\mathbf{X}_n$, *jointly estimate* $\mathbf{h}$ *and* $\mathbf{S}_n$.

3.. LEAST SQUARES ESTIMATES

From (4), we observe the number of measurements is less than the number of unknowns where the unknown parameters are a mixture of continuous and FA parameters. If all the unknown parameters are taken to be continuous, the problem considered herein will be ill-posed. However by exploiting the FA property of the transmitted data sequence, it is still possible to identify and estimate $\{\mathbf{h}, \mathbf{S}_n\}$ up to a sign ambiguity. Identifiability conditions have been derived in [2]. From (4), the LS estimates of $\{\mathbf{h}, \mathbf{S}_n\}$ is given by

$$\begin{aligned} \left\{\widehat{\mathbf{h}}, \widehat{\tilde{\mathbf{s}}}_n\right\} &= \arg\min_{\mathbf{h}, \tilde{\mathbf{s}}_n} |\mathbf{X}_n - \mathbf{C}(\mathbf{h})\mathbf{S}_n|_F^2 \\ &= \arg\min_{\mathbf{h}, \tilde{\mathbf{s}}_n} |\chi_n - (\mathbf{S}_n^T \otimes \mathbf{I}_m)\Xi\mathbf{h}|^2 \quad (5) \end{aligned}$$

with $\chi_n = \text{vec}(\mathbf{X}_n)$ and Ξ is a full rank selection matrix such that $\Xi\mathbf{h} = \text{vec}(\mathbf{C}(\mathbf{h}))$. The vec$(\cdot)$ and $\otimes$ operators denote the stacking of a matrix's column vectors and Kronecker product, respectively. Substituting the least squares estimates of $\mathbf{h}$, (5) can be concentrated into

$$\widehat{\tilde{\mathbf{s}}}_n = \arg\min_{\tilde{\mathbf{s}}_n} J_c = \left|\mathbf{P}^{\perp}_{\mathbf{Q}(\tilde{\mathbf{s}}_n)}\chi_n\right|^2, \quad (6)$$

where $\mathbf{P}^{\perp}_{\mathbf{A}} = \mathbf{I} - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ and $\mathbf{Q}(\tilde{\mathbf{s}}_n) = (\mathbf{S}_n^T \otimes \mathbf{I}_m)\Xi$. In (6), $\tilde{\mathbf{s}}_n$ is estimated with full exploitation of the Toeplitz structure of $\mathbf{S}_n$ and $\mathbf{C}(\mathbf{h})$. While dimensionality is slightly reduced, the estimation of $\tilde{\mathbf{s}}_n$ entails optimizing a highly nonlinear cost function. Such direct search has limited applications as the number of selections of $\tilde{\mathbf{s}}_n$ grows prohibitively large even for small $\{m, P, L\}$.

In this paper we alleviate this problem by relaxing the structural constraint on $\mathbf{S}_n$ and derive three iterative block algorithms for jointly estimating $\{\mathbf{S}_n, \mathbf{h}\}$. The general approach taken herein for optimizing the mixed continuous-FA parameters cost function is based on the alternating minimization procedure. Specifically, the continuous-FA parameters optimization is decoupled into two steps, wherein in the first step, the continuous variables are optimized while the FA parameters are held fixed and vice-versa in the next step. This decoupled optimization is performed iteratively until convergence is achieved. The three blind channel identification and equalization algorithms are outlined as follow.

Algorithm 1: Approximative LS I (ALS I)
This algorithm fully relaxes the constraint on the Toeplitz structure of $\mathbf{S}_n$. Given initial estimate $\mathbf{h}^{(0)}$ and set $k = 0$, we have

1. With the Toeplitz structure constraint on $\mathbf{S}_n$ fully relax, set $k = k + 1$ and estimate the "unstructured" symbol matrix from

$$\begin{aligned} \widehat{\mathbf{S}}_n^{(k)} &= \arg\min_{\mathbf{s}_n} |\mathbf{x}_n - \mathbf{C}(\widehat{\mathbf{h}}^{(k-1)})\mathbf{s}_n|^2 + \cdots \\ &\quad + \arg\min_{\mathbf{s}_{n+P}} |\mathbf{x}_{n+P} - \mathbf{C}(\widehat{\mathbf{h}}^{(k-1)})\mathbf{s}_{n+P}|^2. \quad (7) \end{aligned}$$

2. Compute $\mathbf{Q} = (\widehat{\mathbf{S}}_n^{(k)} \otimes \mathbf{I}_m)\Xi$ and estimate the channel impulse response by

$$\mathbf{h}^{(k)} = (\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{Q}\chi. \quad (8)$$

3. Repeat (1)-(2) until $|\mathbf{h}^{(k)} - \mathbf{h}^{(k-1)}| < \eta$ where η is a predefined threshold.

Algorithm 2: Approximative LS II (ALS II)
The Toeplitz structure of $\mathbf{S}_n$ relates $\mathbf{s}_n$ to $\mathbf{s}_{n+1}$ by a linear shift. Based on this property, we derive a decision feedback approach which implicitly imposes a *partial* Toeplitz structure onto estimated symbol matrix $\widehat{\mathbf{S}}_n$. Introducing the extraction operator

$$f(\mathbf{s}_n, p, q) = [s_{n-p}, s_{n-p-1}, \cdots, s_{n-q}]^T, \quad (9)$$

we can write $\mathbf{s}_n = f(\mathbf{s}_n, 0, n_{ff} + n_{fb} - 1)$ and decompose $\mathbf{s}_n$ into

$$\begin{aligned} \mathbf{s}_n^T &= [\mathbf{s}_{ff}(n)^T, \mathbf{s}_{fb}(n)^T]^T \\ \mathbf{s}_{ff}(n) &= f(\mathbf{s}_n, 0, n_{ff} - 1) \\ \mathbf{s}_{fb}(n) &= f(\mathbf{s}_n, n_{ff}, n_{ff} + n_{fb} - 1). \quad (10) \end{aligned}$$

where n_{ff} and n_{fb} are the feedforward and feedback length selected such that $n_{fb} + n_{ff} = L + m - 1$.

The proposed method is outlined as follows: Given the initial estimates of $\mathbf{h}^{(0)}$, $\mathbf{s}_{fb}^{(0)}(0)$ and $k = 0$

1. Set $k = k + 1$.
2. $\mathbf{S}_n^{(k)}$ is estimated column by column as follows. Decompose

$$\mathbf{C}(\widehat{\mathbf{h}}^{(k-1)}) = [\mathbf{C}_{ff}(\widehat{\mathbf{h}}^{(k-1)}), \mathbf{C}_{fb}(\widehat{\mathbf{h}}^{(k-1)})]$$

of appropriate dimensions.

3. For $t = n$ to $n + P$, compute

$$\mathbf{x}_t^{\dagger} = \mathbf{x}_t - \mathbf{C}_{fb}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{fb}^{(k)}(t)$$

and estimate $\widehat{\mathbf{s}}_{ff}^{(k)}(t)$ from

$$\widehat{\mathbf{s}}_{ff}^{(k)}(t) = \arg\min_{\mathbf{s}_{ff}(t)} |\mathbf{x}_t^{\dagger} - \mathbf{C}_{ff}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{ff}(t)|^2.$$

4. Update symbol vector $\widehat{\mathbf{s}}_t^{(k)} = [\widehat{\mathbf{s}}_{ff}^{(k)}(t)^T \mathbf{s}_{fb}^{(k)}(t)^T]^T$.

5. Extract the next feedback vector to be used at $t+1$:

$$\widehat{s}_{fb}^{(k)}(t+1) = f(s_t^{(k)}, n_{ff}-1, n_{ff}+n_{fb}-2).$$

6. Compute $\mathbf{Q} = (\widehat{\mathbf{S}}_n^{(k)} \otimes \mathbf{I}_m)\Xi$, then estimate

$$\mathbf{h}^{(k)} = (\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{Q}\chi.$$

7. Repeat (1)-(6) until $|\mathbf{h}^{(k)} - \mathbf{h}^{(k-1)}| < \eta$ where η is a predefined threshold.

Algorithm 3: Approximative LS III (ALS III)

We note that **ALS II** requires *a priori* knowledge of the initial feedback vector $s_{fb}(0)$. As we shall show in the simulation results, the impact of errors in $s_{fb}(0)$ can be quite severe resulting in error propagation. To deal with this problem, we first introduce the notion of time-reversal. Specifically, we can write

$$\mathbf{X}_{nr} = \mathbf{C}(\mathbf{h}_r)\mathbf{S}_{nr} + \mathbf{V}_{nr} \tag{11}$$

where $\mathbf{h}_r = [h_{L-1}, h_{L-2}, \cdots, h_1, h_0]^T$ and $\mathbf{S}_{nr}$ and $\mathbf{X}_{nr}$ are Toeplitz
matrices parametrized by $\tilde{\mathbf{x}}_{nr} = [x_{n+P}, \cdots, x_{n+1}, \cdots, x_n]^T$ and $\tilde{\mathbf{s}}_n = [s_{n+P}, \cdots, s_{n-L-m+3}, s_{n-L-m+2}]^T$, respectively. Based on the observation the error propagation persist only over a finite period, we present next an improved version of **ALS II** by incoporating time-reversal. The proposed method is outlined as follows. Given an initial estimates of $\mathbf{h}^{(0)}$, an *arbitrary* $s_{fb}(0)$ and $k=0$

1. Set $k = k+1$.
2. Execute Step (2)-(6) of **ALS II** using $\mathbf{h}^{(k)}$, $s_{fb}^{(k)}(0)$ and $\mathbf{X}_n$.
3. Perform time-reversal to obtain $\mathbf{h}_r^{(k)}$ and $\mathbf{S}_{nr}^{(k)}$. Initialize

$$s_{fb}^{(k)}(0) = f({s_{r0}}^{(k)}, n_{ff}-1, n_{ff}+n_{fb}-2) \tag{12}$$

where ${s_{r0}}^{(k)}$ is the first column of the estimated symbol matrix $\mathbf{S}_{nr}^{(k)}$.

4. Execute Step (2)-(6) of **ALS II** using $\mathbf{h}_r^{(k)}$, $s_{fb}^{(k)}(0)$ and $\mathbf{X}_{nr}$.
5. Perform time-reversal to obtain $\mathbf{h}^{(k)}$ and $\mathbf{S}_n^{(k)}$. Initialize

$$s_{fb}^{(k)}(0) = f(s_0^{(k)}, n_{ff}-1, n_{ff}+n_{fb}-2) \tag{13}$$

where $s_0^{(k)}$ is the first column of the estimated symbol matrix $\mathbf{S}_n^{(k)}$.

6. Set $k = k+1$
7. Repeat (1)-(5) until $|\mathbf{h}^{(k)} - \mathbf{h}^{(k-1)}| < \eta$ where η is a predefined threshold.

It can be easily seen by incorporating the time-reversal mechanism, **ALS III** is not constrained by erroneous choice of $s_{ff}(0)$. The improved channel estimates and estimated $s_{ff}(0)$ will reinforce each other towards better estimates in the next iteration.

Remarks

While **ASL I** appears to be more computationally expensive, it can easily lend itself for efficient parallel implementation. **ASL II & III** decision feedback structures can only offer limited parallelizm. Also $\mathbf{h}$ can be efficiently computed by exploiting the sparse nature of $\mathbf{Q}$. The proposed approach and methods can be readily extended for oversampled channel outputs to achieve robustness to sampling phase.

Initial estimates of $\mathbf{h}$ play a critical role on the global convergence of the methods proposed herein. From our extensive simulations, we found by choosing $\mathbf{h}^{(k)} = [0, \cdots, 0, 1, 0, \cdots, 0]^T$, where the channel impulse response peaks at the position of 1, the proposed methods can achieve global convergence over a wide range of SNR and for a variety of channels with high probability. To be truely blind, $\widehat{\mathbf{h}}$ and $\widehat{\mathbf{S}}_n$ that attain the lowest $|\mathbf{X}_n - \mathbf{C}(\widehat{\mathbf{h}})\widehat{\mathbf{S}}_n|_F$ are chosen over L candidates of $\mathbf{h}^{(0)}$. This is somewhat brute force and more efficient approaches are currently being investigated [7].

Alternatively, relatively good estimates of the 'shape' of the channel can be obtained from using very short training sequence, which can be used as $\mathbf{h}^{(0)}$ for the methods proposed in this paper. Performance analysis of this approach is currently carried out [7].

4.. SIMULATION RESULTS

We consider a dispersive channel modelled by a 3-tap FIR filter with $\mathbf{h} = [0.4079, 0.8168, 0.4079]^T$. The transmitted symbols are an i.i.d. sequence of $\{-1, 1\}$. All the results are averaged from 500 independent trials. Fig 1 compares the performance of **ALS I, II&III** over a range of SNR. $s_{fb}^{(k)}(0)$ is assumed to be known exactly. It suffices to note that **ALS III** performs better than the other two algorithms. The relative computational load of direct search using (6), **ALS I,II& III** in this simulation example is 1612 : 16 : 1. Fig. 2 depicts the performance of the proposed methods for various symbol sequence length. Fig. 3 compare the performance of **ALS II & III** where $s_{fb}^{(k)}(0)$ is not known exactly. The equalizer order is fixed at $m = 4$. Clearly **ALS III** achieve good performance with unknown $s_{fb}^{(k)}(0)$. Note that most of the performance curves follow the maximum likelihood estimates of $\mathbf{h}$ using known data sequence very closely and convergence is achieved typically in < 4 iterations.

5.. CONCLUDING REMARKS

In this paper, we propose three original fast blind channel identification and equalization methods that only use channel outputs sampled at symbol rate. While only real-valued parameters are considered here, extensions to complex-valued channel impulse response and higher order modulation schemes are straight forward. Simulation results have demonstrated them to achieve reliable blind channel identification with short sequences over a wide range of SNR. This demonstrates the proposed fast converging and data efficient blind channel identification and equalization methods to have potential for applications in mobile communications.

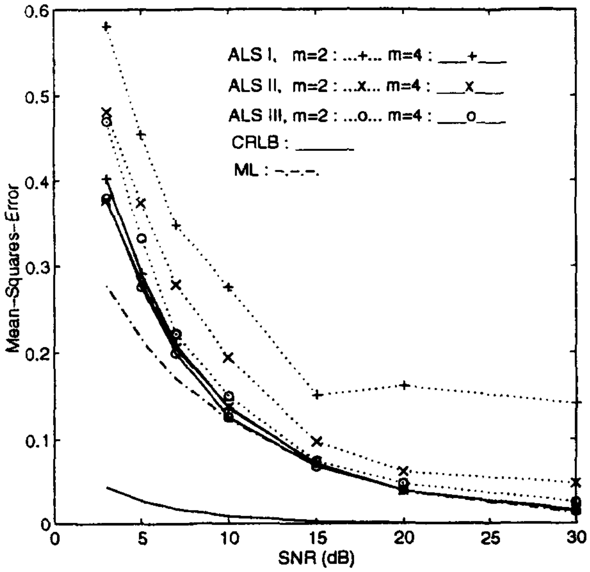

Figure 1. Mean-Square-Error of Channel Estimation versus SNR (dB), $P = 20$

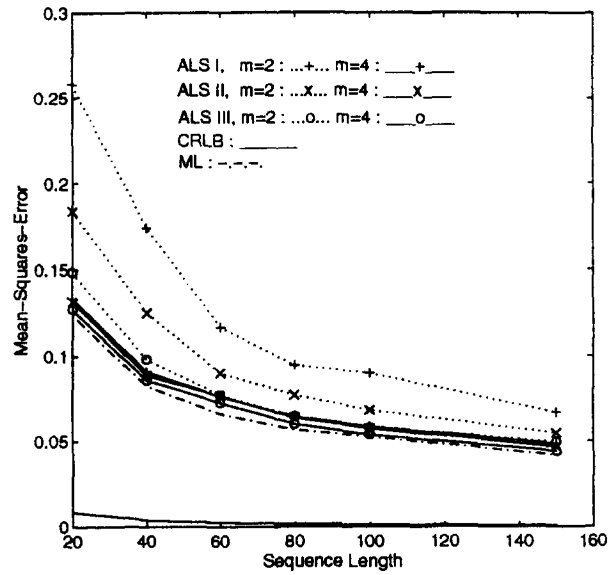

Figure 2. Mean-Square-Error of Channel Estimation versus Sequence Length, SNR: 10dB

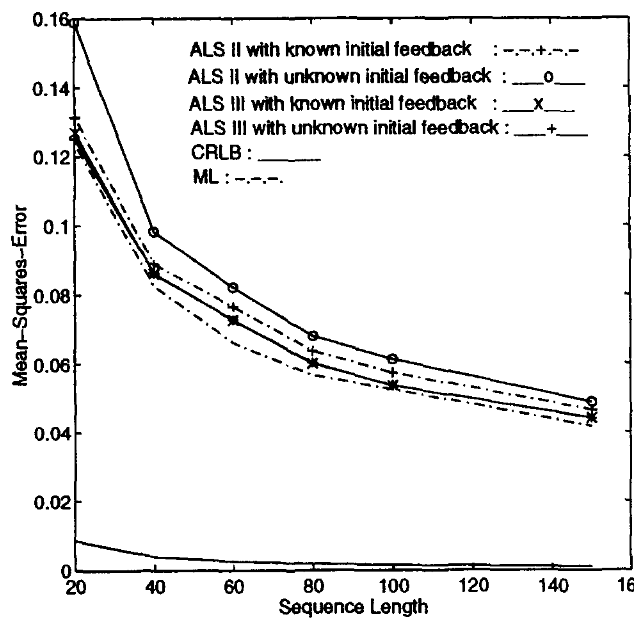

Figure 3. Mean-Square-Error of Channel Estimation versus Sequence Length, SNR: 10dB

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