A Frequency Offset Estimation Technique for Nonstationary Channels

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ABSTRACT

In this paper a new parameter estimation technique based on channel impulse response estimation is introduced which is desirable in randomly time-varying systems such as burst transmission mobile systems since it is robust in environments with considerable delay spread, thus ideal for wireless channels, and has a fast convrgence, thus desirable in burst transmission systems. The shortcomings of some of the existing algorithms are also discussed here.

1. Introduction

Parameter estimation in nonstationary systems with rapid changes is an important issue in many real time signal processing and communications systems. Real time restriction necessitates an efficient and reliable estimator for proper performance of the system. For example, estimation of frequency mismatch between the received signal carrier and the local oscillator frequencies in digital communications is essential for reliable performance of the receiver. In particular, in wireless systems a reliable link in the presence of non-negligible delay spread is not possible unless some sort of adaptive filtering is performed at the receiver. In such cases we can simplify the equalizer by estimating the amount of frequency offset and removing as much of it as possible before adaptive equalization starts.

An efficient estimator must be unbiased and consistent to provide accurate estimation of frequency offset. In addition, it must converge rapidly and have recusive structure to facilitate updating process as the system characteristics varies rapidly. As an example, in a mobile system with bursty transmission, random time variations due to the Doppler effect amounts to considerable variation of signal to noise power ratio which can be detrimental to the estimation accuracy. On the other hand, the burstiness (noncontiguity) of the transmitted signal limits the number of received symbols used for estimation. In interactive systems where real time data is being transmitted, such as voice, we have a limited amount of time to spend on receiver processing and received bursts must be processed as quickly as possible in order not to undermine the overall performance.

Many classical estimation algorithms [1], consider additive noise as the only major impairment. However, in wireless systems the channel impulse response is a random field $h(t, \tau)$ which is stationary with respect to the first parameter and white with respect to the second parameter so the estimation problem implies a recursive deconvolution in the presence of noise. Most of the optimum or near optimum estimation techniques for additive noise channel do not perform properly in time-varying random systems due to sensitivity to the time or frequency selective fading, large delay spreads, etc. due to sensitivity to these parameters.

The approach introduced in this paper is based on estimating the instantanous impulse response of the system at two or more instants not too far from each other using a training sequence. The estimation points are selected relatively close so any changes in the channel can be assumed negligible except for changes in the desired parameter which in this case is any rotation due to frequency offset. The estimation points can even be chosen as close to each other as one sample time. As discussed in more detail in the following sections, this scheme can be implemented very simply, only depends on a few samples of a training sequence, thus arrives at an estimate rather quickly, and takes into account the channel characteristics by deconvolving the channel impulse response. Also, as a by-product of this scheme we get an initial impulse response estimate of the channel which can be useful, for instance for equalizer initial estimates, and can also give estimates of delay spread during the burst, which in turn can be very helpful in the final detection scheme. Another important fact to mention is that in the scheme presented here we can design the training signal to arrive at better estimates as will be shown later, however, if the training signal is already fixed, as in the case of systems designed according to standards, we still arrive at superior estimates.

In the following section we model the system and give the optimum unbiased channel impulse response estimator in the discrete, i.e., sampled time case. Based on the system model and results we will give optimum signal design guidelines and bounds on the performance.

2. The Model and Analysis

We model the system by taking complex T/m samples, where T is the symbol period, so we have m complex samples per transmitted symbol. Let the channel be sufficiently represented by l such samples, so the vector $\vec{H} = (h(0), h(1), \ldots, h(l-1))^t$, represents the complex channel impulse response. The known transmitted symbols (i.e., the samples corresponding to the sync-word) at sample times can be similarly represented by $\vec{X} = (x(0), x(1), \ldots, x(N-1))$, where we assume that the sequence of $x(i)$, for $0 \le i < N$, is a known pattern, and the transmitted samples $x(i)$, for $i < 0$, or $i \ge N$, are unknown data samples. At the receiver we have the transmitted samples convolved with the channel impulse response plus additive noise. In order to estimate the channel impulse response we have to excite it with known samples, and since the channel impulse response is l samples long we will concentrate our observation at the receiver to the received samples corresponding to $x(l-1)$ to $x(N-1)$ of the known transmitted pattern. To arrive at our estimate we can take as many received contiguous samples beyond the one corresponding to $x(l-1)$ but not beyond the received sample corresponding to $x(N-1)$ since transmitted samples beyond that point are random and not known to us, resulting in different accuracies. Without loss of generality let us take M, $1 \le M \le N-l$, received samples corresponding to $x(k)$ where $l-1 \le k \le N-M-1$, namely $r(k)$, to $x(k+M-1)$, namely $r(k+M-1)$.

At the receiver the received vector $\vec{R}_M^k = (r(k), r(k+1), \ldots, r(k+M-1))^t$, is

$$\vec{R}_M^k = \Psi_0 X_M^k \vec{H} + \vec{W} \qquad (1)$$

where $\vec{W} = (w(0), w(1), \ldots, w(M-1))^t$, is the vector of M independent samples of an additive Gaussian noise with covariance matrix $\Gamma^{-1} = G^\dagger G$, where G is also known as the whitening filter, also

$$X_M^k = \begin{bmatrix} x(k) & x(k-1) & \dots x(k-l+1) \\ x(k+1) & x(k) & \dots x(k-l+2) \\ \cdot & \cdot & \dots \\ \cdot & \cdot & \dots \\ x(k+M-1) & x(k+M-2) & \dots x(k+M-l) \end{bmatrix}$$

in which all elements are known and complex, and

$$(\Psi_0)_{kl} = \begin{cases} \exp(j((2\pi k\Delta fT)/m)) & k = l \\ 0 & else \end{cases}$$

which accounts for the unknown frequency offset Δf introduced at the receiver. Note again that we are assuming that the channel does not change during the transmission of the $N-l$ known samples, so that $\vec{H}$ does not change, however, is unknown, and the only change in channel characteristics is due to Ψ, namely from the frequency offset. Now let us take another set of M contiguous samples at the receiver corresponding to transmitted known samples $x(k+i)$ to $x(k+i+M-1)$, where $k+i+M \le N$, resulting in the received vector $\vec{R}_M^{k+i} = (r(k+i), r(k+i+1), \dots, r(k+i+M-1))^t$, which similar to (1) is

$$\vec{R}_M^{k+i} = \Psi_i X_M^{k+i}\vec{H} + \vec{W} \qquad (2)$$

where, X_M^{k+i} is the same as X_M^k except that each element is chosen with an offset of i with respect to the elements of X_M^k. Also $\Psi_i = \exp\left(j\frac{2\pi i\Delta fT}{m}\right)\Psi_0$, simply because the sample elements in $\vec{X}_M^{k+i}$ suffer i times more angular offset rotation by the time they get to the receiver compared to the ones in $\vec{X}_M^k$.

Now, note that in (1) both $\vec{H}$, and Ψ_0 are unknown and must be estimated. This cannot be easily done unless Ψ_0 can be approximated by an $M \times M$ identity matrix, in other words, we assume that the rotation of samples due to frequency offset at the receiver is negligible over M contiguous samples, which is a reasonable assumption to make in practice, as simulation results also show, specially when M is very small. Because of this reason we will choose M to be relatively small in practice and in the case of $M = 1$, this point is moot. Under this assumption, (1) reduces to

$$\vec{R}_M^k = X_M^k\vec{H} + \vec{W} \qquad (3)$$

and (2) reduces to

$$\vec{R}_M^{k+i} = X_M^{k+i}\vec{H}e^{j\frac{2\pi i\Delta fT}{m}} + \vec{W} \qquad (4)$$

From (3) (also refer to [3]), we can get an estimate of $\vec{H}$ with offset k, namely H_k by

$$H_k = [(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}X_M^{k\dagger}\Gamma^{-1}]\vec{R}_M^k \qquad (5)$$

where $.^\dagger$ denotes the Hermitian transpose of a matrix. Note that in (5) the terms in the bracket are known a priori and can be calculated off line and stored in the receiver. It can be implemented as cascades of two FIR filters operating on the received samples. Substituting (3) in (5) we get

$$H_k = \vec{H} + [(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}X_M^{k\dagger}\Gamma^{-1}]\vec{W} \qquad (6)$$

Taking the expected value of both sides of (6) we see that $E\{H_k\} = \vec{H}$, which shows that (5) is an unbiased estimate. Also note from (6) that the AWGN has been colored by this linear process and we can suffer from noise enhancement. The amount of noise enhancement clearly depends on the term in the bracket on the right hand side of (6), and can be detrimental in cases where $X_M^{k\dagger}\Gamma^{-1}X_M^k$ is singular [4]. However, since all the elements in the bracket are known a priori we can choose the offsets k and $k+i$ in such a way that they will result in the least amount of noise enhancements among all offset choices possible.

From (6) we see that H is a Gaussian random process, since it is a linear combination of a fixed sample of a random process $\vec{H}$, and a linear operation on a zero mean Guassian random process $\vec{W}$. So, H is a Guassian random process with mean $\vec{H}$ and covariance matrix

$$E\{(H-\vec{H})^2\} = E\Big\{([(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}X_M^{k\dagger}\Gamma^{-1}]\vec{W}) \cdot ([(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}X_M^{k\dagger}\Gamma^{-1}]\vec{W})^\dagger\Big\} = N_0(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}$$

The signal to noise ratio at the output of the estimator, SNR_e, is

$$SNR_e = \frac{\|\vec{H}\|}{N_0\|(X_M^{k\dagger}\Gamma^{-1}X_M^k)^{-1}\|} \qquad (7)$$

where $\|.\|$ denotes the l_2 norm of a vector in the numerator and of a matrix in the denominator.

In section 3 we show that, except in some peculiar circumstances, often the higher M is the higher SNR_e will be. But recall from previous arguments that the higher M is the less realistic our approximation of Ψ_0 to an $M \times M$ identity matrix will be. So, there is an optimum M for a given circumstance.

Similar to the estimate in (5) derived from (3), we can get an estimate of $\vec{H}e^{j\frac{2\pi i\Delta fT}{m}}$, or equivalently H_{k+i}, from (4) to be

$$H_{k+i} = H_k e^{j\frac{2\pi i\Delta fT}{m}} = [(X_M^{(k+i)\dagger}\Gamma^{-1}X_M^{k+i})^{-1}X_M^{(k+i)\dagger}\Gamma^{-1}]\vec{R}_M^{k+i} \qquad (8)$$

This is the exact same set up as in (5) and can be realized similarly.

Now (5) and (8) give two estimates of the channel impulse response i samples away from each other, thus besides the effect of noise, the two estimates differ only in phase which directly corresponds to the amount of frequency offset, as is obvious from (5) and (8). So, it can be easily seen from (5) and (8) that each element of the channel vector is going to give an estimate of the frequency offset. These estimates, there are l of them, can be weighted by the magnitude of their corresponding element and then averaged to give a final estimate. The estimate of the frequency offset then, is

$$\Delta f = \frac{m}{2\pi i T} \frac{\sum_{j=0}^{l-1} [|\hat{h}_k(j)|^2 * \angle(\hat{h}_{k+i}(j) - \hat{h}_k(j))]}{\sum_{j=0}^{l-1} |\hat{h}_k(j)|^2} \qquad (9)$$

in which $\angle.$ denotes the angle or phase of the operand, and $\hat{h}_k(j)$ is the j^{th} element of the vector $\hat{H}_k$.

Note that if we have the choice of the transmitted symbols we can choose a sync-word smart enough to give us less noise enhancement so the penalty we pay due to noise enhancement is minimized. However, in cases where we have no such control, such as cases in which sync-words are dictated by standards, our task mainly gets concentrated on which offsets k and $k+i$ to choose and what M should be. The choice of l is usually dictated to us by the resolution of the estimate we like and the characteristics of the channel we transmit over.

3. Noise Enhancement and Optimum Signal Design

Two important issues should be addressed here. First, the penalty of deviating from matched filter structure and second is the optimum signal design for estimation. The penalty for extra linear processing after matched filtering is noise enhacement. The signal to noise ratio at the input of the matched filter, in AWGN case, is

$$\left(\frac{S}{N}\right)_i = \frac{\|XH\|}{\sigma^2} \qquad (10)$$

As mentioned earlier the first filter is matched to the transmitted signal not the received one. Note that for simplicity we have dropped the subscripts and superscripts from the variables. This, hopefully, will not cause any confusions in following the concepts in this section. The signal to noise ratio at the output of the matched filter is

$$\left(\frac{S}{N}\right)_M = \frac{\|X^*XH\|}{\sigma^2 tr(X^*X)} \qquad (11)$$

where tr denotes the trace of the matrix. It can be shown that

$$tr(X^*X) = \sum_{i=1}^{N+l-1} \sum_{j=1}^{l} |x_{ij}|^2 = \|X\|_F^2$$

where $\| \ \|_F$ denotes the Frobenius norm of the matrix.

The elements of matrix X^*X are:

$$[X^*X]_{ij} = \begin{cases} \sum_{n=0}^{N-1} x(n)x^*(n+i-j) & i \geq j \\ [X^*X]_{ij}^* & i < j \end{cases}$$

The signal to noise ratio at the output of the estimator is

$$\left(\frac{S}{N}\right)_o = \frac{\|H\|}{\sigma^2 tr(X^*X)^{-1}} \qquad (12)$$

Notice that the denominators in the above two equations are in fact Frobenius norms of the positive definite matrix X^*X. On the other hand, The trace of a matrix equals the sum of eigenvalues and $eig(X^*X) = \frac{1}{eig(X^*X)^{-1}}$. We can show that the signal to noise ratio at the the output of the estimator is less than S/N at the output of the filter matched to the transmitted signal $\left(\frac{S}{N}\right)_o < \left(\frac{S}{N}\right)_M$.

The second issue is optimum choice of the signal. By using Cauchy-Schwarz inequality we can easily conclude that

$$var(\hat{h}_i) \geq \frac{\sigma^2}{(X^*X)_{ii}} \qquad (13)$$

and the lower bound will be acheived if the martrix (X^*X) is diagonal. On the other hand (X^*X) is a toeplitz matrix whose elements are $(X^*X)_{ij} = \sum_{n=0}^{N-1-|i-j|} x(n)x(n+|i-j|))$.

So in order to have an orthogonal matrix the sync-word should be a pseudorandom noise and then the optimum estimate is

$$\hat{h}_i = \frac{r_{rx}(i)}{r_{xx}(0)}.$$

It should be mentioned that minimum variance estimate results in the optimum estimate for any sync-word, i.e., meets Crame-Rao lower bound. The above argument proves that the lower bound will be minimized by using a proper estimator signal.

4. Simulation Results

In Figure 1 we show a situation in which we try to estimate a fixed frequency offset introduced equal to -1/72 of the symbol rate, or equivalently, every symbol time. This is a roughly 1.4% of symbol rate frequency offset which is considerable. Also a delay spread of one symbol time with an image power equal to that of the main signal has been introduced. This is a rather harsh delay spread and most other algorithms either fall apart or suffer severely under such circumstances. Note from this figure that the performance is not considerably different from the case with no delay spread present.

We like to point out that there are two ways we can use the general estimation scheme presented in this paper. One is for packet switched calls, such as in control channels or point of sale applications, wherein one or very few bursts are transmitted per call and there is a relatively big gap between transmissions, also consecutively received bursts do not necessarily come from the same source. The other is for circuit switched calls, such as in regular voice or other continuous bit stream calls, wherein once the call is established, numerous bursts will be transmitted regularly over the channel from the source to the receiver. In the former case, we use the algorithm presented here once at a time and get an initial coarse estimate (coarseness depends on the SNR) of the frequency offset and remove it. The rest of the frequency offset will be removed by, say, an equalizer which could much more easily handle it once the bulk of the frequency offset is removed. In the circuit switched scenario, however, since we have very many coming bursts from the same source, we can average the results of this algorithm across several bursts to overcome the variance inherent in the algorithm output when the SNR is very low. We will show the estimate outputs once they are averaged over several bursts.

In the case of packet switched calls, we have to rely on single estimates. Even though on the average the estimate is very close to the actual frequency offset injected but in low SNR's there is a variance. So, the estimate is correct only within a range of the actual frequency offset. For example, for the case of Figure 1, the single estimate is within 10 percent of the actual frequency offset only 26 percent of the time at an SNR = 10 dB. However, at an SNR = 30 dB, the estimate is always within 10 percent of actual. Figure 2 shows the fraction of time the estimate is within 10, 20 and 30 percent of the actual in the aforementioned scenario for several SNR's.

Another important performance criterion to be addressed is how much of a frequency offset can this algorithm track? Simulation results for SNR = 40, 30, 20, and 10 dB show that the proposed scheme tracks the frequency offset rather well for all SNR's up to a difference equivalent to over 4% of the symbol rate, a frequency offset which corresponds to over 15 degrees of phase offset per symbol time. This is very robust. Now the question is how does this proposed algorithm compare with the classic ones. This algorithm with $M = 1$ and $l = 1$ degenerates to a classic scheme. Figure 3 shows the 100 burst averaged estimate result of this case compared with the proposed algorithm herein in the case with no delay spread for various values of SNR. Note from this figure that our proposed algorithm arrives at closer estimates for all values of SNR with smaller variances. However, the proposed algorithm is robust in the presence of severe delay spread and only a minor penalty in performance is paid in that case, whereas simulation shows that the classic scheme breaks down under considerable delay spread situations and arrives at nonsense results.

5. Maximum likelihood estimation:

Another parameter estimation method is directly based on maximum likelihood estimation. For a system with a carrier frequency offset, the frequency offset is one such parameter. In addition, in a system with delay spread due to multipath propagation, the channel impulse response may also be an unknown parameter. We can show that the likelihood fuction to be maximized is

$$\Lambda_l(\Delta f) = R*\Psi_0 X\tilde{H} + \tilde{H}^* X*\Psi_0*R = 2\mathrm{Re}[\tilde{H}^* X*\Psi_0*R]$$

Since analytic solution of gradient equations are difficult we suggest to implement in real time, we propose an alternative method based upon the equations for $\tilde{H}$ and the equation for $\Lambda_l(\Delta f)$. This involves the use of a bank of matched filters, and does not suffer from the limitations of a simple maximum of a matched filter bank. This scheme will be presented in future.

6. Conclusions

In this document we have proposed a new algorithm for frequency offset estimation for burst transmissions over wireless channels. The proposed algorithm is very robust against severe delay spread as opposed to the existing schemes. The proposed algorithm depends on the transmission of a known symbol pattern and is very simple to implement, especially more so when the known pattern meets certain requirements described above. These requirements are easily met in practical applications, e.g., in IS-136 standard, etc.

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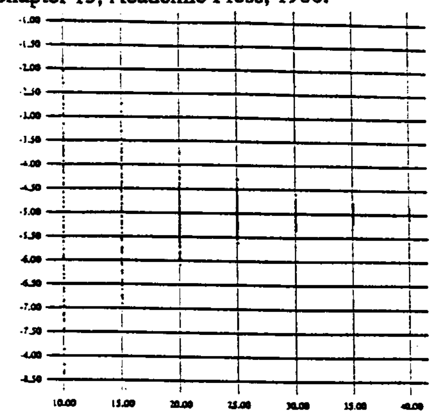

Figure 1: Freq. offset estimated vs. SNR.

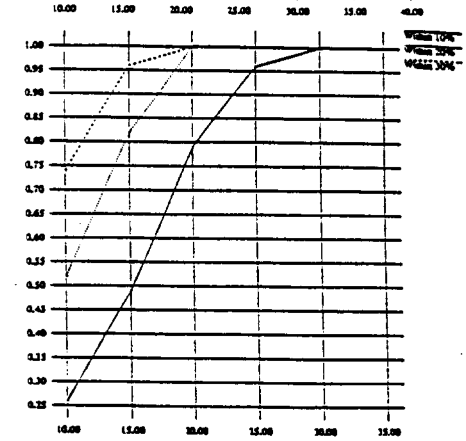

Figure 2: Fraction of time estimate is within 10, 20, or 30% of the actual vs. SNR.

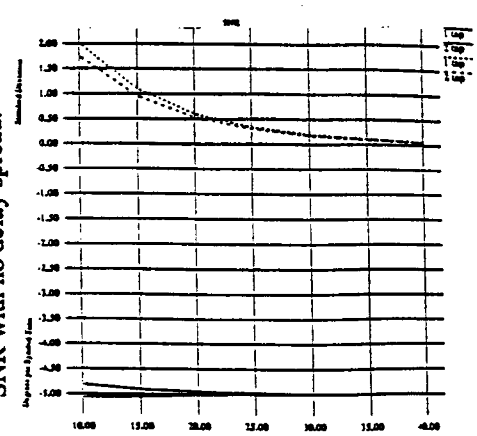

Figure 3: Comparison of single tap and the proposed approach vs. SNR with no delay spread.