NOVEL APPROACH TO PARAMETER ESTIMATION OF A CLASS OF NONLINEAR SYSTEMS

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ABSTRACT

A new identification algorithm based on over-sampling scheme is proposed for a Hammerstein model which consists of a nonlinear element followed by a linear dynamic model. The unknown linear transfer function model can be identified by making use of the information obtained from an over-sampled output, and the intermediate input to the linear part can also be estimated as well as an arbitrary continuous or discontinuous function type of nonlinear element by a deconvolution approach. The *prior* information of the nonlinear element is not needed in the new algorithm.

1. INTRODUCTION

Generally nonlinear systems can only be characterized by a nonlinear model adequately, so nonlinear system identification becomes important in theory and practice. Since the Hammerstein model, which consists of a nonlinear memoryless element followed by a linear dynamic system, is particularly simple but can describe a class of nonlinear system efficiently, it has attracted much attention for a long time. However, in almost all previous algorithms [1]-[3], the nonlinear element is limited to the systems with parametric nonlinearity, *i.e.*, it can be characterized or approximated by a finite sum of polynomials or other known basic functions, which needs the *prior* information. Therefore the consistency of the parameters estimate of the linear part may not be ensured due to the affection of approximation errors of nonlinearity. In [4], an algorithm without requirement of nonlinearity information was presented, but it used a specific input test signal.

Estimation of the Hammerstein model can also be considered as a blind identification problem, however, the second-order cyclostationary statistics based approaches [5] have restricted availability to an FIR model. High order cumulants can also be used for a blind identification problem, but it requires a large number of data and its convergency is very slow.

The main purpose of this paper is to clarify how to estimate the parameters in the linear part when the *prior* information about the nonlinearity can not be available. In this algorithm, we employ an over-sampling scheme for observing the system output to estimate the linear transfer function model, then estimate the unknown input to the linear part to abtain a nonparametric model of the nonlinear element. One of the distinctive features is that the nonlinear function may be an arbitrary nonlinear function which is not limited to a polynomial series expansion. The estimation of the linear part is not affected by the expression of the nonlinear part, and its consistency can be assured.

2. SYSTEM DESCRIPTION AND OVER-SAMPLING SCHEME

We deal with a Hammerstein model, which is frequently utilized to characterize a class of nonlinear system effectively. For the simplicity, we assume the system is an SISO system, $r(t)$, $y(t)$ are input, output respectively, $u(t)$ is its intermidiate input to the linear part as well as the output of nonlinearity. Further we assume that the nonlinear element is described by $u = f(r)$, which is an unknown nonlinear function, and the linear part is described by a transfer function with the order n.

$r(t)$ is a discrete-time white random signal for the identification whose duration time is T. Now the problem is how to identify the nonlinear function and the linear dynamic model $G(z)$ by using only the accessible output signal $y(t)$.

Now we introduce an over-sampling scheme for observing the output signal $y(t)$. The sampling interval Δ is chosen such that $\Delta = T/p$, where $p \geq n+1$. In the following discussion, we select $p = n+1$, then the linear dynamics can be expressed by a linear discrete-time system with the sampling interval Δ as

$$y(k) + \sum_{i=1}^{n} a_i y(k-i) = \sum_{i=1}^{n} b_i u(k-i) + v(k) \quad (1)$$

where $u(k)$, $y(k)$ and $v(k)$ are the system input, output and noise at instant $k\Delta$ respectively. It can be noticed from the scheme of over-sampling that the corresponding intermediate input has the form as

$$u(k) = u_m \qquad for \quad k \in [mp+1, \overline{m+1}p] \quad (2)$$

In the Hammerstein identification problem, $u(k)$ is not available, somtimes even $r(k)$ can not be observed directly, *eg.*, in blind communication systems. Since the nonlinear element $f(\cdot)$, the parameters $\{a_i, b_i\}$ of the linear dynamic model, and the input $u(k)$ are all unknown, therefore, it will be considered as a blind identification problem. In this paper, we present a new approach for a transfer function model in the presence of noise, and clarify how to attain the consistency of the parameter estimates. Furthermore, by estimating the unknown input, we can identify a nonlinear function which is not necessarily a finite polynomial expansion.

For this kind of estimation problem, we have the following assumptions as

A1. The signal $r(t)$ is a white random signal with duration T. As a consequence, the intermediate input $u(t)$ is also a white signal with duration T.

A2. The discrete-time linear part can be described by (1), where its order n is known as *a priori.*

A3. $v(k)$ is a stationary white noise with zero mean, finite variance and is independent to $u(k)$.

3. IDENTIFICATION OF THE LINEAR DYNAMICS VIA OVER-SAMPLING

Let the model parameters $\{a_i\}$, $\{b_i\}$ in (1) be denoted by the following vectors as

$$\boldsymbol{\theta}_a = [a_1, \ \cdots \ a_n]^T, \quad \boldsymbol{\theta}_b = [b_1, \ \cdots \ b_n]^T$$

In the following, we describe how to estimate the model parameters of $\boldsymbol{\theta}_a$ and $\boldsymbol{\theta}_b$.

3.1. Estimation of $\{a_i\}$

Define the input and output regressor vectors as

$$\begin{aligned}
\boldsymbol{y}_j &= [y(j), \ \ y(p+j), \ \ \cdots \ \ y(Mp+j)]^T \\
\boldsymbol{u}_j &= [u(j), \ \ u(p+j), \ \ \cdots \ \ u(Mp+j)]^T \\
\boldsymbol{v}_j &= [v(j), \ \ v(p+j), \ \ \cdots \ \ v(Mp+j)]^T \\
\boldsymbol{\Phi}_1 &= [\boldsymbol{y}_p, \ \ \boldsymbol{y}_{p-1}, \ \ \cdots \ \ \boldsymbol{y}_2]^T \\
\boldsymbol{\Phi}_2 &= [\boldsymbol{y}_{p-1}, \ \ \boldsymbol{y}_{p-2}, \ \ \cdots \ \ \boldsymbol{y}_1]^T \\
\boldsymbol{\Psi}_1 &= [\boldsymbol{u}_p, \ \ \boldsymbol{u}_{p-1}, \ \ \cdots \ \ \boldsymbol{u}_2]^T \\
\boldsymbol{\Psi}_2 &= [\boldsymbol{u}_{p-1}, \ \ \boldsymbol{u}_{p-2}, \ \ \cdots \ \ \boldsymbol{u}_1]^T
\end{aligned}$$

whrer $j = 1, \cdots, p+1$. From the feature described in (2), we notice that $\boldsymbol{\Psi}_1 = \boldsymbol{\Psi}_2$. Then the input-output description is given by

$$\boldsymbol{y}_{p+1} - \boldsymbol{y}_p = (\boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\boldsymbol{\theta}_a + \boldsymbol{v}_{p+1} - \boldsymbol{v}_p \qquad (3)$$

Since $\boldsymbol{\Phi}_2$ is not correlated with $\boldsymbol{v}_{p+1}$ or $\boldsymbol{v}_p$, then the estimate of $\boldsymbol{\theta}_a$ can be given by

$$\hat{\boldsymbol{\theta}}_a = (\boldsymbol{\Phi}_2^T \Delta\boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}_2^T \Delta\boldsymbol{y} \qquad (4)$$

where $\Delta\boldsymbol{\Phi} = \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2$, $\Delta\boldsymbol{y} = \boldsymbol{y}_{p+1} - \boldsymbol{y}_p$. The variance σ_v^2 of the noise $v(k)$ can be given by

$$\hat{\sigma}_v^2 = (\Delta\boldsymbol{y} + \Delta\boldsymbol{\Phi} \cdot \hat{\boldsymbol{\theta}}_a)^T (\Delta\boldsymbol{y} + \Delta\boldsymbol{\Phi} \cdot \hat{\boldsymbol{\theta}}_a) \Big/ (2M) \qquad (5)$$

Remark: If the variance σ_v^2 of the noise $v(k)$ is known, then the consistent estimate of $\boldsymbol{\theta}_a$ can also be given by

$$\hat{\boldsymbol{\theta}}_a = (\Delta\boldsymbol{\Phi}^T \cdot \Delta\boldsymbol{\Phi}/M)^{-1} (\Delta\boldsymbol{\Phi}^T \Delta\boldsymbol{y}/M + \boldsymbol{e}_1) \qquad (6)$$

where $\boldsymbol{e}_1 = [\sigma_v^2, \ 0, \ \cdots, \ 0]^T$.

3.2. Estimation of $\{b_i\}$

Substituting $\hat{\boldsymbol{\theta}}_a$ into (1), then we have

$$y_f(k) = b_1 u(k-1) + \ \cdots \ + b_n u(k-n) + v(k) \quad (7)$$

where $y_f(k) = y(k) + [y(k-1), \ \cdots, \ y(k-n)]\hat{\boldsymbol{\theta}}_a$. Now we define that

$$\begin{aligned}
\boldsymbol{y}_{f,j} &= [y_f(j), \ \ y_f(2p+j), \ \ \cdots, \ \ y_f(Mp+j)]^T \\
\bar{b}_{j,1} &= \sum_{i=1}^{\min(j,n)} b_i \quad \bar{b}_{j,2} = \sum_{i=j+1}^{n} b_i \\
\boldsymbol{u}_1 &= [u(0), \ \ u(p), \ \ \cdots \ \ u(Mp)]^T \\
\boldsymbol{u}_2 &= [0, \ \ u(0), \ \ \cdots \ \ u(\overline{M-1}p)]^T
\end{aligned}$$

whrer $j = 1, \cdots, p+1$, then we have

$$\boldsymbol{y}_{f,j+1} = \bar{b}_{j,1}\boldsymbol{u}_1 + \bar{b}_{j,2}\boldsymbol{u}_2 + \boldsymbol{v}_{j+1} \qquad (8)$$

It will lead to

$$\begin{aligned}
\bar{b}_{p,1}(\boldsymbol{y}_{f,j+1} - \boldsymbol{v}_{j+1}) &= \bar{b}_{j,1}(\boldsymbol{y}_{f,p+1} - \boldsymbol{v}_{p+1}) \\
&\quad + \bar{b}_{j,2}(\boldsymbol{y}_{f,1} - \boldsymbol{v}_1) \quad \mathit{for}\ j = 1, \cdots, \ p-1 \qquad (9)
\end{aligned}$$

In order to determine the parameters uniquely, we assume that $b_1 = 1$, *i.e.*, $\bar{b}_{1,1} = 1$. Then $\bar{b}_{j,1}$, $\bar{b}_{j,2}$ can be determined by

$$\hat{\bar{\boldsymbol{b}}} = \lambda(\boldsymbol{\Phi}^T\boldsymbol{\Phi}/M - \hat{\sigma}_v^2 \boldsymbol{I})^{-1}(\boldsymbol{\Phi}^T \bar{\boldsymbol{y}}/M) \qquad (10)$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{y}_{f,1,p+1} & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & \boldsymbol{y}_{f,1,p+1} & 0 \\ 0 & \cdots & 0 & \boldsymbol{y}_{f,1,p} \end{bmatrix}$$

$$\begin{aligned}
\boldsymbol{y}_{f,1,p+1} &= [\boldsymbol{y}_{f,p+1}, \ \ \boldsymbol{y}_{f,1}] \\
\bar{\boldsymbol{b}} &= [b_{1,1}, b_{1,2}, b_{2,1}, b_{2,2}, \cdots b_{n-1,2}, b_{n,1}]^T \\
\bar{\boldsymbol{y}}^T &= [\boldsymbol{y}_{f,2}^T, \cdots, \boldsymbol{y}_{f,p+1}^T]
\end{aligned}$$

where λ is chosen as $\hat{\bar{b}}_{1,1} = 1$. From the definition of $\bar{b}_{j,1}$ and $\bar{b}_{j,2}$, we also have

$$\bar{\boldsymbol{b}} = \boldsymbol{Q}\boldsymbol{\theta}_b \qquad (11)$$

where

$$\boldsymbol{Q} = \begin{bmatrix} \mathbf{1}_1 & \mathbf{0}_1 & \cdots & \mathbf{1}_{n-1} & \mathbf{0}_{n-1} & \mathbf{1}_n \\ \mathbf{0}_{n-1} & \mathbf{1}_{n-2} & \cdots & \mathbf{0}_1 & \mathbf{1}_1 & \mathbf{0}_0 \end{bmatrix}^T$$

$$\mathbf{0}_i = [0, \cdots, 0]_{1\times i}^T \quad \mathbf{1}_i = [1, \cdots, 1]_{1\times i}^T$$

Thus we can obtain the estimate of $\boldsymbol{\theta}_b$ by

$$\hat{\boldsymbol{\theta}}_b = (\boldsymbol{Q}^T\boldsymbol{Q})^{-1}\boldsymbol{Q}^T \hat{\bar{\boldsymbol{b}}} \qquad (12)$$

4. ESTIMATION OF INTERMEDIATE INPUT AND NONLINEARITY

Define matrix $\boldsymbol{A}$ and vectors $\boldsymbol{b}$, $\boldsymbol{c}$ as

$$\boldsymbol{A} = \begin{bmatrix} -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\boldsymbol{c} = [1, \ 0, \ \cdots, \ 0]$$

and give $A_T(z^{-1})$, $B_j(z^{-1})$ and $C(z^{-1})$ as follows

$$\begin{aligned} A_T(z^{-1}) &= \det(\boldsymbol{I} - \boldsymbol{A}^p z^{-p}) \\ &= 1 + a_{T,1}z^{-p} + \cdots + a_{T,n}z^{-np} \end{aligned}$$

$$\begin{aligned} B_j(z^{-1}) &= z^{-1}\boldsymbol{c} \cdot adj(\boldsymbol{I} - \boldsymbol{A}^p z^{-p}) \cdot \\ &\left(\sum_{i=1}^{j} \boldsymbol{A}^{i-1}\boldsymbol{b} + \sum_{i=j+1}^{p} \boldsymbol{A}^{i-1}\boldsymbol{b} z^{-p} \right) \\ &= z^{-1}(b_{j,0} + \cdots + b_{j,n}z^{-np}) \end{aligned}$$

$$C(z^{-1}) = \frac{A_T(z^{-1})}{A(z^{-1})} = c_0 + c_1 z^{-1} + \cdots + c_l z^{-l}$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$

where $c_0 = 1$, $l = (p-1)n$. Then the input-output relationship can be rewritten by

$$y(k) = \frac{B_j(z^{-1})}{A_T(z^{-1})}u(k) + \frac{1}{A(z^{-1})}v(k) \tag{13}$$

where $j = mod(k-1, p) + 1$. Furthermore, let $q^p = z$, it can be represented by

$$\begin{aligned} A_T(q^{-1})y_j(m) &= \bar{B}_j(q^{-1})\bar{u}(m) \\ &+ \sum_{s=1}^{p} C_s(q^{-1})v_{j,s}(m) \end{aligned} \tag{14}$$

where

$$\begin{aligned} &A_T(q^{-1}) = 1 + a_{T,1}q^{-1} + \cdots + a_{T,n}q^{-n} \\ &\bar{B}_j(q^{-1}) = b_{j,0} + b_{j,1}q^{-1} + \cdots + b_{j,n}q^{-n} \\ &C_s(q^{-1}) = c_{s-1} + c_{s+p-1}q + \cdots + c_{s+np-1}q^{-n} \\ &y_j(m) = y(mp + j + 1), \quad \bar{u}(m) = u(mp + j) \\ &v_{j,s}(m) = v(mp + j + 2 - s) \end{aligned}$$

Let $y_{f,j}(m) = A_T(q^{-1})y_j(m)$, then from (14), the variance of intermediate input can be determined by

$$\hat{\sigma}_u^2 = \left(cov(y_{f,j}(m)) - \hat{\sigma}_v^2 \sum_{i=0}^{(p-1)n} c_i^2 \right) \Big/ \sum_{i=0}^{n} b_{j,i}^2 \tag{15}$$

Let a stable polynimal $D_j(q^{-1})$ and scalar σ_ε^2 satisfy

$$\begin{aligned} \sigma_\varepsilon^2 D_j(q^{-1})D_j(q) &= \sigma_u^2 \bar{B}_j(q^{-1})\bar{B}_j(q) \\ &+ \sigma_v^2 \sum_{s=1}^{p} C_s(q^{-1})C_s(q) \end{aligned} \tag{16}$$

where $d_{j,0} = 1$. Then a white random sequence $\varepsilon_j(m)$ can be given by

$$\varepsilon_j(m) = \frac{A_T(q^{-1})}{D_j(q^{-1})}y_j(m) \tag{17}$$

Now we introduce the polynomial equation

$$\bar{B}_j(q^{-1}) = D_j(q^{-1})H_j(q^{-1}) + q^{-(N+1)}G_j(q^{-1}) \tag{18a}$$

$$H_j(q^{-1}) = h_{j,0} + h_{j,1}q^{-1} + \cdots + h_{j,N}q^{-N} \tag{18b}$$

where $N \geq 0$. The order of $G_j(q^{-1})$ is $n-1$. Then following the deconvolution approach given in [6], the intermediate input can be estimated by

$$\hat{u}_j(m|m+N) = \hat{\sigma}_u^2 \sum_{i=0}^{N} h_{j,i}\varepsilon_j(m+i)/\sigma_\varepsilon^2 \tag{19}$$

$$\hat{u}(m) = \sum_{j=1}^{p} \hat{u}_j(m|m+N)/p \tag{20}$$

Furthermore if the input signal $r(k)$ is available, then the nonparametric expression of nonlinearity $f(\cdot)$ can also be obtained through points of $\big(r(m) \sim \hat{u}(m)\big)$.

5. ANALYSIS OF THE ESTIMATION

The estimate error $\Delta\boldsymbol{\theta}_a$ can be given by

$$\Delta\boldsymbol{\theta}_a = \hat{\boldsymbol{\theta}}_a - \boldsymbol{\theta}_a = (\boldsymbol{\Phi}_2^T \Delta\boldsymbol{\Phi}/M)^{-1}(\boldsymbol{\Phi}_2^T \Delta\boldsymbol{v}/M) \tag{21}$$

Since $\boldsymbol{\Phi}_2$ is not correlated with $\boldsymbol{v}_{p+1}$ or $\boldsymbol{v}_p$, then

$$\lim_{M\to\infty} \boldsymbol{\Phi}_2^T \Delta\boldsymbol{v}/M = 0 \tag{22}$$

moreover, we can give following expression from (14)

$$\lim_{M\to\infty} \boldsymbol{\Phi}_2^T \Delta\boldsymbol{\Phi}/M = \boldsymbol{\Sigma}(u) + \sigma_v^2 \boldsymbol{\Sigma}(v) \tag{23}$$

where

$$\boldsymbol{\Sigma}(u) = \boldsymbol{\Omega}(b)\boldsymbol{\Lambda}\boldsymbol{\Xi}^T(b), \quad \boldsymbol{\Sigma}(v) = \boldsymbol{\Pi}(c)\boldsymbol{\Gamma}\boldsymbol{\Pi}^T(c)$$

$$\boldsymbol{\Omega}(b) = \begin{bmatrix} b_{n-1,0} & b_{n-1,1} & \cdots & b_{n-1,n} \\ & \cdots & \cdots & \\ b_{1,0} & b_{1,1} & \cdots & b_{1,n} \\ 0 & b_{p,0} & \cdots & b_{p,n-1} \end{bmatrix}$$

$$\boldsymbol{\Xi}(b) = \begin{bmatrix} b_{n,0} & b_{n,1} & \cdots & b_{n,n} \\ & \vdots & \vdots & \\ b_{2,0} & b_{2,1} & \cdots & b_{2,n} \\ b_{1,0} & b_{1,1} & \cdots & b_{1,n} \end{bmatrix}_{n\times(n+1)}$$

$$\boldsymbol{\Pi}(c) = \begin{bmatrix} 0 & c_0 & \cdots & c_l & 0 & \cdots \\ \vdots & & \ddots & & \ddots & \\ 0 & \cdots & 0 & c_0 & \cdots & c_l \end{bmatrix}_{n\times(l+n)}$$

$$\boldsymbol{\Lambda} = E\left(\begin{bmatrix} \frac{\bar{u}(m)}{A_T(q^{-1})} \\ \vdots \\ \frac{\bar{u}(m-n)}{A_T(q^{-1})} \end{bmatrix} \begin{bmatrix} \frac{\bar{u}(m)}{A_T(q^{-1})} & \cdots & \frac{\bar{u}(m-n)}{A_T(q^{-1})} \end{bmatrix} \right)$$

$$\boldsymbol{\Gamma} = \sigma_v^2 \begin{bmatrix} -1 & 1 & 0 & \\ 0 & -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 \end{bmatrix}_{(l+n)\times(l+n)}$$

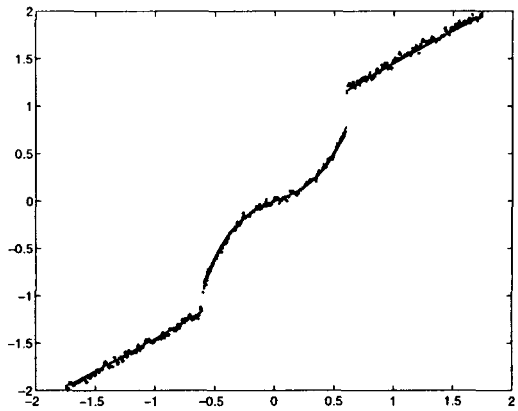

Figure 1. Estimated nonlinearity of case 1, solid line: true; dotted line: estimate

$\boldsymbol{\Lambda}$ and $\boldsymbol{\Gamma}$ have full rank if $\bar{u}(m)$ and $v(k)$ are white random. If $(\boldsymbol{A}, \boldsymbol{b})$ is completely controllable and $(\boldsymbol{A}, \boldsymbol{c_T})$ is observable, then $\boldsymbol{\Omega}(b)$, $\boldsymbol{\Xi}$ (b) and $\boldsymbol{\Pi}(c)$ are full column rank, therefore $\boldsymbol{\Sigma}(u)$ and $\boldsymbol{\Sigma}(v)$ have full rank. If

$$\sigma_v^2 \neq eig(-\boldsymbol{\Sigma}^{-1}(v)\boldsymbol{\Sigma}(u)) \tag{24}$$

then $\boldsymbol{\Sigma}(u)+\sigma_v^2\boldsymbol{\Sigma}(v)$ has full rank, *i.e.*, $\lim_{M\to\infty} \boldsymbol{\Phi}_2^T \Delta\boldsymbol{\Phi}/M$ exists and not singular, it means that $\lim_{M\to\infty} \Delta\boldsymbol{\theta}_a = 0$.

Review (8), when $\hat{\boldsymbol{\theta}}_a$ is consistent estimate, and $\boldsymbol{v}_j$ is not correlated with $\boldsymbol{u}_1$ or $\boldsymbol{u}_2$, then $\lim_{M\to\infty}(\hat{\bar{\boldsymbol{b}}} - \bar{\boldsymbol{b}}) = 0$ in (10), and we have $\lim_{M\to\infty}(\hat{\boldsymbol{\theta}}_b - \boldsymbol{\theta}_b) = 0$ by (12).

6. SIMULATION EXAMPLES

In the following numerical simulations, we deal with a discontinuous nonlinearity:

$$u = \begin{cases} 0.5r + 0.4\sqrt{r} + 0.55 & 0.6 < r < 1.8 \\ 0.5r - 0.2r^2 + 2.5r^3 & -0.6 \leq r \leq 0.6 \\ 0.5r - 0.4\sqrt{-r} - 0.55 & -1.8 < r < -0.6 \end{cases}$$

As for a linear dynamic system, we take a fourth-order transfer function model. The test signal duration $T = 1$. Further, we take the sampling interval $\Delta = T/5 = 0.2$, then the parameters of the unknown true linear part with the sampling interval Δ is

$$\begin{aligned} \boldsymbol{\theta}_a^T &= [-1.80 \quad 0.490 \quad 0.848 \quad -0.504] \\ \boldsymbol{\theta}_b^T &= [1.00 \quad 0.400 \quad -0.710 \quad 0.300] \end{aligned}$$

The variance σ_v^2 of white noise $v(k)$ is 0.1. The input signal is given in two cases. Case 1, r is chosen as a uniformly distributed white signal on interval $(-1.75, 1.75)$, and in case 2, $r(k)$ is not available but $r \in \{\pm 1.5, \pm 0.5\}$. The estimated linear part is

$$\begin{aligned} \hat{\boldsymbol{\theta}}_a^T &= [-1.7998 \quad 0.4945 \quad 0.8409 \quad -0.5011] \\ \hat{\boldsymbol{\theta}}_b^T &= [1.000 \quad 0.3981 \quad -0.7058 \quad 0.3073] \end{aligned}$$

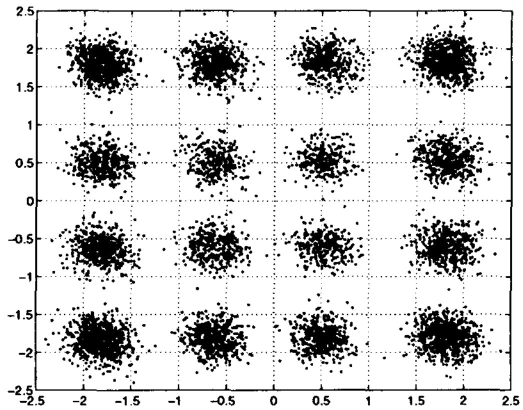

Figure 2. Estimated intermediate input $\bar{u}$ of case 2, x-axis is $\hat{u}(m)$, y-axis is $\hat{u}(m+1)$

and the estimated nonlilearity of case 1 is shown in Figure 1. The estimated intermediate input $\hat{u}(m)$ of case 2 is shown as Figure 2.

7. CONCLUSION

We have proposed a new identification algorithm for the Hammerstein model based on the over-sampling technique. If the test signal $r(k)$ is available, by over-sampling the output signal, we can identify an arbitrary nonlinear function, unlike other ordinary methods assuming a parametric nonlinearity such as a finite polynomial. Further, it is clarified that the estimates of the transfer function model are consistent if the noise is stationary zero-mean white noise independent of the test input and its varaince satisfies condition in (24).

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