

# Asymptotic Performance of Blind Antenna Array Receiver Algorithms for CDMA

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## ABSTRACT

This paper considers the performance of four channel identification techniques for code division multiple access (CDMA) antenna array receivers. These techniques are based on the assumption that the interference is spatially white: they provide a spatial "matched filter" solution. Perturbation formulae are presented for estimating the attainable signal to interference and noise ratios (SINR) for these techniques. Some simulation results are also presented to compare the convergence performance of these methods.

## 1. INTRODUCTION

Cellular mobile communications may achieve higher cell capacities through the use of CDMA techniques, one example being the US IS-95 standard [1]. All users on the mobile-to-base station (reverse) link operate asynchronously on the same RF bandwidth, so that system capacity is eventually limited by multiple access interference (MAI). One promising method to suppress MAI and improve capacity is to employ an antenna array receiver at the base station [2]. An important aspect for antenna arrays is the choice of algorithm to operate the receiver. This paper considers the performance of a number of matched filter algorithms, which operate on the assumption of spatially white MAI.

The structure of this paper is as follows. In section 2, the channel model used for this paper is described; section 3 then introduces the algorithms and provides theoretical estimates of the SINR performance. Section 4 presents and discusses simulation results for algorithms' performance: finally, conclusions to the paper are given in section 5.

## 2. THE CDMA CHANNEL MODEL

A single cell direct sequence spread spectrum system employing binary phase shift keying will be considered here. There are  $P$  users: the  $p$ th mobile will generate a binary data sequence  $d_p(t)$ , with each symbol having a period of time  $T_s$  seconds. This is multiplied by the user's PN code  $c_p(t)$ , which is a sequence of "chips", each of period  $T_c$ . The processing gain of the code is  $L = T_s/T_c$ : each chip of the code is an independent, identically distributed (iid) binary ( $\pm 1$ ) random variable with zero mean. The resulting signal is transmitted in the correct RF channel.

Assuming a frequency non-selective, plane wave channel model, the  $M \times 1$  received baseband vector for an  $M$ -element antenna array is:

$$r(t) = \sum_{p=1}^P \alpha_p d_p(t - t_p) c_p(t - t_p) \exp\{j\phi_p\} a(\theta_p) + z(t) \quad (1)$$

The scalars  $\{\alpha_p, \phi_p, t_p\}$  represents the received amplitude, phase and time delay for the  $p$ th user and the  $M \times 1$  vector  $a(\theta_p)$  represents the array steering vector for the  $p$ th user's bearing  $\theta_p$ . The array is a uniform linear array with antenna

spacing of  $1/2$  the RF carrier wavelength, so that the  $m$ th entry of  $a(\theta)$  is  $\exp\{j(m-1)\pi \sin \theta\}$ . The  $M \times 1$  vector  $z(t)$  represents additive noise.

Consider the antenna array receiver which is matched in time to user  $p = 1$ . Initially,  $r(t)$  is passed through a filter matched to the pulse shaping of  $c_1(t)$ , which in this paper is assumed to be rectangular. The output is sampled at the chip rate, so that the  $M \times 1$  vector  $x(l, n)$  for the  $l$ th chip sample of the  $n$ th symbol for user 1 may be written as:

$$x(l, n) = \sum_{p=1}^P \alpha_p \zeta_p(l, n) \exp\{j\phi_p\} a(\theta_p) + z_1(l, n) \quad (2)$$

The scalar  $\zeta_p(l, n)$  denotes the pulse shape filter output for the  $p$ th user's data modulated PN code  $d_p(t - t_p) c_p(t - t_p)$ . For  $p = 1$ ,  $\zeta_1(l, n) = c_1(l, n) d(n)$ , where  $c_1(l, n)$  denotes the  $l$ th code chip transmitted for the first user's  $n$ th symbol  $d(n)$ . Finally the  $M \times 1$  vector  $z_1(l, n)$  denotes zero-mean temporally and spatially white Gaussian noise of power  $\sigma^2$ .

In order pick out the desired user's signal, a digital matched filter containing the PN code  $c_1$  is applied to  $x$ . If  $c_1$  is delayed by  $l_1$  chips compared to the signal  $x(l, n)$ , the symbol rate PN code filter output  $y(n, l_1)$  is given by:

$$y(n, l_1) = \sum_{p=1}^P \alpha_p \psi_p(n, l_1) \exp\{j\phi_p\} a(\theta_p) + n(n, l_1) \quad (3)$$

where the  $M \times 1$  vector  $n(n, l_1)$  is the filter output for the input noise sequence  $\{z_1(l, n)\}$  only. The scalar  $\psi_p(n, l_1)$  is filter output for the scalar input sequence  $\{\zeta_p(l, n)\}$ .

The statistical properties of  $\psi_p(n, l_1)$  and  $\{\zeta_p(l, n)\}$  are of use in later sections, so the following points are noted here. From the definition of  $c_p(t)$ , it follows that  $E[\zeta_p(l, n)] = 0$  and  $E[\zeta_p(l, n)^2]$  is the same for all chip positions  $l$ . The notation  $E[\cdot]$  is the time expectation operator. The sequence  $\psi_p(n, l_1)$  is then zero mean and has variance  $LE[\zeta_p(l, n)^2]$ , unless  $p = 1$  and  $l_1 = 0$  whereupon it is  $L^2 E[\zeta_p(l, n)^2]$ . The central limit theorem (CLT) may be invoked here to show that the asymptotic distribution (as  $L \rightarrow \infty$ ) of  $\psi_p(n, l_1)$  is Gaussian when  $l_1 \neq 0$ . Furthermore,  $\psi_p(n_1, l_1)$  is uncorrelated with  $\psi_p(n_2, l_2)$ , when  $n_1 \neq n_2$  or if the relative delay of  $l_1$  and  $l_2$  is two chips or more (and  $n_1 = n_2$ ).

## 2.1. Signal Characterisation

The necessary assumptions for the parameters described above will now be made. Relative to the time delay  $t_1$  for user 1, all other time delays  $\{t_p, p \neq 1\}$  are uniformly distributed over the interval  $[0, T_s]$ . The phase terms  $\{\phi_p\}$  are assumed to be uniformly distributed over  $[0, 2\pi]$  and the amplitude coefficients  $\{\alpha_p\}$  have all been set equal to 1 (perfect power control). No Doppler effects are considered, so that each realisation of the channel is stationary. A single  $120^\circ$  coverage cell sector is considered, with the bearing of user 1 set to  $\theta_1 = 0^\circ$  (array broadside). The bearings of all other users are uniformly distributed over  $[-60^\circ, 60^\circ]$ .

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One common method for characterising the received signal  $y(n, l_1)$  is to estimate its  $M \times M$  covariance matrix  $\hat{R}_y(l_1)$  from  $K$  consecutive snapshots of data:

$$\hat{R}_y(l_1) = \frac{1}{K} \sum_{k=1}^K y(n_k, l_1) y(n_k, l_1)^H \quad (4)$$

where  $y^H$  denotes the Hermitian transpose operator and  $n_k$  denotes the  $k$ th symbol for averaging. For a given realisation of the simulation parameters  $\{\alpha_p, t_p, \phi_p\}$ , the mean of  $\hat{R}_y(l_1)$  for  $l_1 \neq 0$ , denoted as  $\bar{R}_y(l_1)^{\dagger}$ , may be obtained from eqn (3) as:

$$\bar{R}_y(l_1) = \sum_{p=1}^P \alpha_p^2 E[\psi_p(n, l_1)^2] a(\theta_p) a^H(\theta_p) + L\sigma^2 I = L S + \bar{Q} \quad (5)$$

where the signal matrix  $S = \alpha_1^2 a(\theta_1) a^H(\theta_1)$ ,  $I$  denotes the identity matrix and  $\bar{Q}$  denotes the total interference and noise covariance.

As the distribution of  $\psi_p(n, l_1)$  is approximately Gaussian for large  $L$ , the distribution of  $\hat{R}_y(l_1)$  (for  $l_1 \neq 0$ ) may be approximated by the complex Wishart distribution [3]. Also the vectors  $y(n_k, l_1)$  are iid, so the errors in the estimate of  $\bar{R}_y(l_1)$ , denoted as  $\Delta \bar{R}_y(l_1) = \hat{R}_y(l_1) - \bar{R}_y(l_1)$ , are asymptotically (as  $K \rightarrow \infty$ ) Gaussian distributed, with zero mean and variance [4]:

$$E[x_1^H \Delta \bar{R}_y(l_1) x_2 x_3^H \Delta \bar{R}_y(l_1) x_4] = \frac{x_1^H \bar{R}_y(l_1) x_4 x_3^H \bar{R}_y(l_1) x_2}{K} \quad (6)$$

where  $x_1 - x_4$  are arbitrary  $M \times 1$  complex vectors.

For the algorithms in this paper, the most commonly used form of  $\bar{R}_y$  is for the delay  $l_1 = 0$ , where it is given by  $L^2 S + \bar{Q}$ . The moments of  $\hat{R}_y(0)$  are slightly different, because the desired signal has a constant amplitude of  $L\alpha_1$  and is no longer approximately Gaussian. In this case, the distribution of the error term  $\Delta \bar{R}_y(0) = \hat{R}_y(0) - \bar{R}_y(0)$  may be approximated by the non-central complex Wishart distribution [3]. The mean of the error is again zero and its asymptotic variance is:

$$E[x_1^H \Delta \bar{R}_y(0) x_2 x_3^H \Delta \bar{R}_y(0) x_4] = \frac{x_1^H \bar{R}_y(0) x_4 x_3^H \bar{R}_y(0) x_2}{K} - \frac{L^4 x_1^H S x_4 x_3^H S x_2}{K} \quad (7)$$

For a fixed value of  $l$ , it is possible to define the mean covariance matrix  $\bar{R}_x(l)$  of the pre-correlation vector  $x(l, n)$ :

$$\bar{R}_x(l) = \sum_{p=1}^P \alpha_p^2 E[\zeta_p(l, n)^2] a(\theta_p) a^H(\theta_p) + \sigma^2 I = S + \frac{1}{L} \bar{Q} \quad (8)$$

Again  $\bar{R}_x(l)$  may be estimated from  $K$  snapshots by replacing  $y(n_k, l_1)$  with  $x(l, n_k)$  in eqn (4). For large  $P$ , the MAI present in  $x(l, n_k)$  will be approximately Gaussian distributed. As the vectors  $x(l, n_k)$  are iid, the CLT indicates that the error matrix  $\Delta \bar{R}_x(l) = \hat{R}_x(l) - \bar{R}_x(l)$  is asymptotically Gaussian distributed as  $K \rightarrow \infty$ . The mean of  $\Delta \bar{R}_x(l)$  is 0 and its variance is:

$$E[x_1^H \Delta \bar{R}_x(l) x_2 x_3^H \Delta \bar{R}_x(l) x_4] = \frac{x_1^H \bar{R}_x(l) x_4 x_3^H \bar{R}_x(l) x_2}{K} - \frac{x_1^H S x_4 x_3^H S x_2}{K} \quad (9)$$

The second term on the RHS of eqn (9) again occurs due to the constant amplitude of the signal term in  $x(l, n_k)$ .

<sup>†</sup>The overbar notation for  $R_y$  denotes the fact that the matrices are a function of the  $P$  users' signal parameters.

### 3. ALGORITHM PERFORMANCE

The performance metric used in this paper is the mean output SINR after the spatial filter  $\hat{w}$ , chosen by the algorithm using  $K$  snapshots of data, has been applied to the signal vector  $y(0, n)$ . This may be defined as:

$$\text{SINR} = E \left[ \frac{L^2 \hat{w}^H S \hat{w}}{\hat{w}^H \bar{Q} \hat{w}} \right] \quad (10)$$

In the following analysis, it is assumed that the matrices  $S$  and  $\bar{Q}$  are fixed over the expectation operation. The vector  $\hat{w}$  is a function of the received data and is assumed to be of the form  $w + \Delta w$ . The vector  $w$  denotes the mean spatial filter (i.e.  $K \rightarrow \infty$ ) obtained for each algorithm and the term  $\Delta w$  denotes an error term due to finite data averaging. Eqn (10) may then be written as:

$$\text{SINR} = L^2 E \left[ \frac{a + b}{c + d} \right] = L^2 \left( \frac{a + E[b]}{c} - \frac{aE[d] + E[bd]}{c^2} + \frac{aE[d^2]}{c^3} + \dots \right) \quad (11)$$

The RHS of eqn (11) represents the Taylor series expansion of the denominator term. The terms  $a$ ,  $b$ ,  $c$  and  $d$  may be defined as:

$$\begin{aligned} a &= w^H S w, \quad b = \Delta w^H S w + w^H S \Delta w + \Delta w^H S \Delta w \\ c &= w^H \bar{Q} w, \quad d = \Delta w^H \bar{Q} w + w^H \bar{Q} \Delta w + \Delta w^H \bar{Q} \Delta w \end{aligned} \quad (12)$$

The terms  $E[b]$ ,  $E[d]$ ,  $E[bd]$  and  $E[d^2]$  are evaluated below, retaining only those terms that involve the first and second order moments of  $\Delta w$ .

$$\begin{aligned} E[b] &= \sum_{j=1}^M \sum_{k=1}^M S_{jk} E[\Delta w \Delta w^H]_{kj} + 2\Re\{w^H S E[\Delta w]\} \\ E[d] &= \sum_{j=1}^M \sum_{k=1}^M [\bar{Q}]_{jk} E[\Delta w \Delta w^H]_{kj} + 2\Re\{w^H \bar{Q} E[\Delta w]\} \\ E[bd] &= \sum_{j=1}^M \sum_{k=1}^M (2\Re\{[S w w^H \bar{Q}]_{jk} E[\Delta w \Delta w^H]_{kj} \\ &\quad + [S^* w^* w^H \bar{Q}]_{jk} E[\Delta w \Delta w^T]_{kj}\}) \\ E[d^2] &= \sum_{j=1}^M \sum_{k=1}^M 2\Re\{[\bar{Q} w w^H \bar{Q}]_{jk} E[\Delta w \Delta w^H]_{kj} \\ &\quad + [\bar{Q}^* w^* w^H \bar{Q}]_{jk} E[\Delta w \Delta w^T]_{kj}\} \end{aligned} \quad (13)$$

The notation  $\Re$  indicates the real part of a complex value;  $[\bar{Q}]_{jk}$  denotes the  $j$ th row and  $k$ th column entry of the matrix  $\bar{Q}$ . The perturbation formulae will be most accurate when the error terms  $E[b]$  and  $E[d]$  are small. As a result, the terms  $E[bd]$  and  $E[d^2]$  are likely to be smaller still: it is possible to neglect the latter two terms and still retain an adequate approximation. The value of eqn (11) will now be considered for each algorithm in turn.

#### 3.1. Algorithm Description

This subsection will describe a number of algorithms to select the spatial filter  $\hat{w}$ :

**1. Eigenfilter Method:** This technique is based on the statistical principal components analysis method [5]. An eigenvalue decomposition of  $\hat{R}_y(0)$  is performed to obtain a set of  $M$  eigenvalues and associated eigenvectors. The eigenvector  $\hat{u}_1$  for the largest eigenvalue is assumed to provide a good estimate of the desired signal vector  $a(\theta_1)$  and is chosen as  $\hat{w}$ . After beamforming,  $d(n)$  may be estimated using DPSK demodulation [6].

The moments of the beamformer error  $\Delta w$  have been obtained using the asymptotic approximations in Appendix A of [4]. In the results below, the eigenvector and eigenvalue pairs  $\{u_j, \lambda_j\}$  are those for the matrix  $\bar{R}_y(0)$ . If the MAI is

spatially coloured,  $\mathbf{u}_1$  may not be an unbiased estimator of  $\mathbf{a}(\theta_1)$ . Hence, the other eigenvectors  $\mathbf{u}_2 - \mathbf{u}_M$  may not be orthogonal to  $\mathbf{a}(\theta_1)$ : the product  $F_{jk} = L^2 \mathbf{u}_j^H \mathbf{S} \mathbf{u}_k$  is thus difficult to simplify. Then:

$$\begin{aligned} E[\Delta \mathbf{w}] &= \frac{1}{K} \sum_{j=2}^M \left[ -\frac{(\lambda_j \lambda_1 - F_{j1} F_{11})}{2(\lambda_1 - \lambda_j)^2} \mathbf{u}_1 \right. \\ &\quad \left. - \left( \frac{F_{j1} F_{11}}{(\lambda_1 - \lambda_j)^2} - \sum_{k=2}^M \frac{F_{j1} F_{kk}}{(\lambda_j - \lambda_1)(\lambda_k - \lambda_1)} \right) \mathbf{u}_j \right] \\ E[\Delta \mathbf{w} \Delta \mathbf{w}^T] &= -\frac{1}{K} \sum_{j=2}^M \sum_{k=2}^M \frac{F_{j1} F_{k1} \mathbf{u}_j \mathbf{u}_k^T}{(\lambda_1 - \lambda_k)(\lambda_1 - \lambda_j)} \\ E[\Delta \mathbf{w} \Delta \mathbf{w}^H] &= \frac{1}{K} \sum_{j=2}^M \sum_{k=2}^M \frac{(\lambda_1 \lambda_j \delta_{jk} - F_{11} F_{jk}) \mathbf{u}_j \mathbf{u}_k^H}{(\lambda_1 - \lambda_j)(\lambda_1 - \lambda_k)} \quad (14) \end{aligned}$$

where  $\delta_{jk}$  is the Dirac delta function.

**2. Stanford Method (a):** This algorithm [7, 8] is a modified version of the eigenfilter method; the steering vector is obtained as the  $M \times 1$  eigenvector  $\hat{\mathbf{u}}_1$  for the largest eigenvalue of the matrix  $\hat{\mathbf{X}}_a = \frac{1}{L}(\hat{\mathbf{R}}_y(0) - \hat{\mathbf{R}}_y(l_1))$ . Again, the filter output may be demodulated using DPSK techniques. In this paper, all simulations have used  $l_1 = 10$  chips.

The asymptotic formulae of [4] have again been used to estimate the moments of  $\Delta \mathbf{w}$ . The time expectation of  $\hat{\mathbf{X}}_a$  is proportional to the signal matrix  $\mathbf{S}$ : its eigenvector  $\mathbf{u}_1$  is simply the steering vector  $\mathbf{a}(\theta_1)$  and corresponds to the non-zero eigenvalue  $\lambda_1$  (equal to  $(L-1)M$  from eqn (5)ff). The other  $M-1$  eigenvectors  $\mathbf{u}_2 - \mathbf{u}_M$ , which all have zero eigenvalues, are orthogonal to  $\mathbf{S}$ . This means that any term of the form  $\mathbf{u}_j \bar{\mathbf{R}}_y(l) \mathbf{u}_k$  (for any  $l$ ) may be simplified to  $\mathbf{u}_j \bar{\mathbf{Q}} \mathbf{u}_k$ , if  $j$  or  $k$  is larger than 1. Thus:

$$\begin{aligned} E[\Delta \mathbf{w}] &= \frac{2}{KL^2 \lambda_1^2} \sum_{j=2}^M \left[ \left( \frac{C_{j1} D_{11}}{2} - \sum_{k=2}^M C_{j1} C_{kk} \right) \mathbf{u}_j - \frac{C_{jj} D_{11}}{4} \mathbf{u}_1 \right] \\ E[\Delta \mathbf{w} \Delta \mathbf{w}^T] &= \frac{1}{KL^2 \lambda_1^2} \sum_{j=2}^M \sum_{k=2}^M 2C_{j1} C_{k1} \mathbf{u}_j \mathbf{u}_k^T \\ E[\Delta \mathbf{w} \Delta \mathbf{w}^H] &= \frac{1}{KL^2 \lambda_1^2} \sum_{j=2}^M \sum_{k=2}^M C_{jk} D_{11} \mathbf{u}_j \mathbf{u}_k^H \quad (15) \end{aligned}$$

The terms  $C_{jk}$  and  $D_{jk}$  are defined as:

$$C_{jk} = \mathbf{u}_j^H \bar{\mathbf{Q}} \mathbf{u}_k \quad D_{jk} = \mathbf{u}_j^H (\bar{\mathbf{R}}_y(0) + \bar{\mathbf{R}}_y(l_1)) \mathbf{u}_k \quad (16)$$

**3. Stanford Method (b):** An alternative estimate for the spatial filter is the eigenvector  $\hat{\mathbf{u}}_1$  for the largest eigenvalue of the matrix  $\hat{\mathbf{X}}_b = (\frac{1}{L} \hat{\mathbf{R}}_y(0)) - \hat{\mathbf{R}}_x(L)$  [9]. The matrices  $\frac{1}{L} \bar{\mathbf{R}}_y(l_1)$  and  $\bar{\mathbf{R}}_x(L)$  have slightly different statistical properties (c.f. eqns (6) and (9)), but it turns out (by substituting  $L \bar{\mathbf{R}}_x$  for  $\bar{\mathbf{R}}_y(l_1)$  above) that the moments for  $\Delta \mathbf{w}$  in eqn (15) are the same for both algorithms.

**4. Maximal Ratio Combining (MRC):** The final approach is based on a form of maximal ratio combining [7]. The spatial filter is the  $M \times 1$  cross-correlation vector  $\hat{\mathbf{r}}$ , defined as:

$$\hat{\mathbf{r}} = \frac{1}{K} \sum_{k=1}^K \mathbf{y}(n_k, 0) \hat{d}(n_k) \quad (17)$$

In this paper, it will be assumed that the symbol sequence estimate  $\hat{d}(n)$  is always correct. The vector  $\hat{\mathbf{r}}$  provides an estimate of the carrier, so that it performs coherent data demodulation. Thus, the current symbol may be estimated directly from the beamformer output, unlike the

other methods. For a bit error ratio (BER) of  $10^{-3}$ , perfectly-coherent PSK requires 1-1.5 dB less SINR than for DPSK[6]. As with the Stanford methods, it is asymptotically unbiased. The moments of the error term are:

$$E[\Delta \mathbf{r}] = 0, \quad E[\Delta \mathbf{r} \Delta \mathbf{r}^T] = 0, \quad E[\Delta \mathbf{r} \Delta \mathbf{r}^H] = \frac{\bar{\mathbf{Q}}}{K} \quad (18)$$

It is of interest to compare the asymptotic moments obtained for the eigenfilter and Stanford methods. A simple case will be considered here, with no MAI ( $P = 1$ ). In this case, the mean eigenvector for all three techniques would be the same, i.e.  $\mathbf{u}_1 = \mathbf{a}(\theta_1)$  (except for a complex scaling factor). The other  $M-1$  eigenvectors are not uniquely defined, but will span the same subspace  $I - \mathbf{u}_1 \mathbf{u}_1^H$ .

The asymptotic mean and covariance terms for the error  $\Delta \mathbf{u}_1$  in the eigenfilter method become:

$$\begin{aligned} E[\Delta \mathbf{u}_1] &= \frac{-(M-1)(LM + \sigma^2)\sigma^2}{2K(LM)^2} \mathbf{u}_1 \\ E[\Delta \mathbf{u}_1 \Delta \mathbf{u}_1^H] &= \frac{(LM + \sigma^2)\sigma^2}{K(LM)^2} (I - \mathbf{u}_1 \mathbf{u}_1^H) \quad (19) \end{aligned}$$

Both results are the same for the Stanford methods, but are multiplied by  $L^2((L+1)M + 2\sigma^2)/((L-1)^2(LM + \sigma^2))$ . This indicates that both methods are asymptotically unbiased as  $E[\Delta \mathbf{u}_1] \propto \mathbf{u}_1$ . However, the eigenfilter method can be a biased estimator when the MAI is spatially coloured – both Stanford methods are always asymptotically unbiased as  $K \rightarrow \infty$ . On the other hand, the eigenfilter method provides lower variance estimates of the steering vector than the Stanford methods in this case, particularly when the output SINR is low (i.e.  $\sigma^2$  is large).

#### 4. SIMULATION WORK AND RESULTS

The first simulation to be performed considered the SINR performance of all the algorithms, operating with white Gaussian noise only. The array size is  $M = 8$  antennas and the noise power  $\sigma^2$  is set so that the matched filter bound (MFB) – with  $\hat{\mathbf{w}} = \mathbf{a}(\theta_1)$  – for the desired user is 0, 5 or 10 dB. The processing gain of CDMA system was  $L = 64$ , with all PN sequences obtained using the generator polynomial  $h(x) = x^{64} + x^4 + x^3 + 1$ . The SINR of the desired user for each of the algorithms at the output of the spatial filter has been calculated. These values have been averaged over 1000 Monte Carlo simulations: the results are plotted against number of snapshots  $K$  in figure 1. Simulation results are shown for all four algorithms as points: theoretical curves for MRC, eigenfilter and both Stanford methods are shown as lines. Comparing the curves in figure

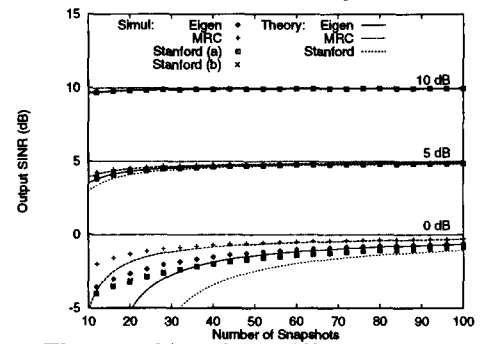


Figure 1. Theoretical/simulated SINR performance of the four algorithms, plotted against No of snapshots  $K$ . MFB SINRs are 0, 5 and 10 dB.

1, there is little to choose between the matched filter methods at reasonable SINRs. The 0 dB results show the Stanford methods have slightly worse SINR results than for the eigenfilter method – this is attributable to the larger variance term in eqn (19)ff for the Stanford methods. Perfect

knowledge of the data sequence  $d(n)$  is probably the reason for the improved performance of the MRC method over the others here. The fact that an algorithm's convergence time is large at low SINRs, such as 0 dB, is probably not significant as the receiver BER will be poor anyway. The theory curves provide a good approximation to the SINR performance for reasonable values of  $K$ . However, the curves tend to diverge as  $K$  reduces and the error terms become larger – higher order terms in the Taylor expansion will then become non-negligible. That the Stanford theory curve provides a slightly poorer fit than the others is probably again attributable to the larger variance of the beamformer error  $\Delta w$ .

The next simulation considered the SINR performance of all the algorithms, operating in a cell sector with  $P = 100$  mobiles. The SINR of the desired user for each of the algorithms at the output of the spatial filter has been calculated from 1000 Monte Carlo simulations, each with the same random positioning of the mobiles. The MFB SINR, ignoring MAI, has been set to 10 dB; otherwise, simulation conditions are the same as for figure 1. The results are plotted against number of snapshots  $K$  in figure 2, with simulation results shown as points and theoretical curves are shown as lines.

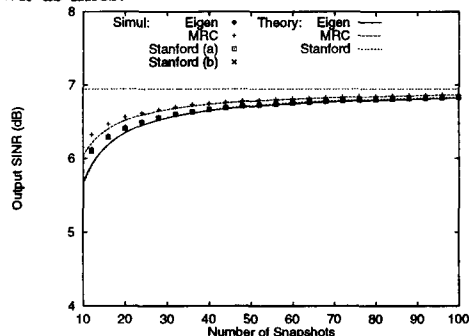


Figure 2. The Output SINR plotted against No of snapshots  $K$  for a sector with  $P = 100$  users.

The results show similar characteristics to those for figure 1. The eigen-decomposition methods all provide similar performance in this case, with MRC providing a slightly higher output SINR. As  $K \rightarrow \infty$ , there is a small bias present in the eigenfilter beamformer estimate, as the MAI is not spatially white. The maximum achievable SINR for the eigenfilter method is 6.93 dB, while for the other techniques it is 6.95 dB (shown as a horizontal line).

When the array size is doubled, it would be hoped that the SINR of the received signal at each antenna could be halved. It is important to assess how well the algorithms cope in this situation. The final simulation has been conducted with a single desired CDMA user and antenna sizes  $M$  of 2, 4, 8 and 16 elements. In all cases, the MFB SINR was maintained at 7 dB. Otherwise, simulation conditions are the same as for figure 1. The results in figure 3 show that for the eigenfilter method, the convergence times to achieve a given combiner loss at least double as the antenna size doubles. This is because the noise power at each antenna is also doubled each time. Similar trends have been observed in the performance of all the other algorithms.

The BER performance of the MRC method is difficult to compare with that of the eigen-decomposition algorithms, because different demodulation schemes are used. The MRC should perform slightly better than the other techniques because it uses coherent demodulation. However, if incorrect decisions are used in eqn (17), error propagation effects may occur. To mitigate these problems, periodic training symbols may be transmitted by the mobile.

Comparing the eigen-decomposition techniques, all three algorithms offer reasonably similar performance. The eigenfilter method has a lower asymptotic variance error term

in eqn (19) than the Stanford methods; however, the latter are always asymptotically unbiased estimators, unlike the eigenfilter method. The Stanford methods will thus perform better when the MAI is significantly spatially coloured. However, in this case, a method that maximises the SINR of the receiver [8, 9] is likely to perform better still.

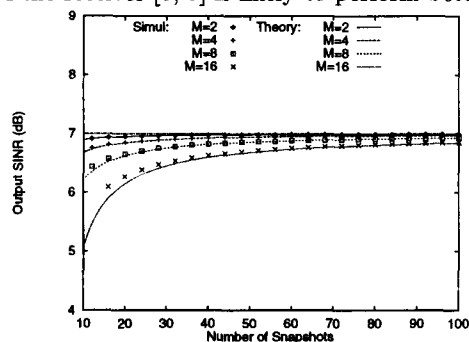


Figure 3. The eigenfilter method output SINR plotted against No of snapshots, for different antenna sizes  $M$ . There is one CDMA user and the MFB SINR is 7 dB.

In many cellular environments, the radio channel will be frequency selective with more than one resolvable channel tap. The receiver can apply the above algorithms to each tap and then combine the beamformer outputs with a RAKE filter: the so-called 2D-RAKE filter. The results from figure 1 show that for a fixed value of  $K$  the larger the matched filter bound SINR, the smaller the combining loss. If the total signal power is fixed, the receiver will have better performance for a channel where it is split equally between a small number of taps than where it is spread over a large number of taps.

## 5. CONCLUSIONS

In this paper, some asymptotic results have been presented for the performance of spatially matched filter algorithms for antenna array receivers, operating on the reverse link of CDMA cellular systems. Simulation results have been presented to compare with the theoretical results; these demonstrate that all the matched filter algorithms provide similar SINR performance.

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