

A GENERALIZED ARRAY MANIFOLD MODEL FOR LOCAL SCATTERING IN WIRELESS COMMUNICATIONS

David Asztély¹, Björn Ottersten¹, A. Lee Swindlehurst²

¹ Dept. of Signals, Sensors & Systems, Royal Institute of Technology, S-100 44 Stockholm, Sweden

² Dept. of Elec. & Comp. Engineering, Brigham Young University, Provo, UT 84602, USA

ABSTRACT

In wireless communication scenarios, local scatterers in the vicinity of the mobile sources cause angular spreading. As a result, the spatial signatures will not belong to the conventional array manifold parameterized by direction of arrival (DOA) alone. In this paper, a parameterized model for spatial signatures applicable in scenarios with local scattering is presented. Several algorithms that exploit this model are proposed, and the performance of signal waveform estimators using the model is investigated via simulations. It is demonstrated that considerable gain may result as compared with using the conventional plane wave model.

1. INTRODUCTION

The use of antenna arrays as a tool for improving coverage, reducing interference, and increasing capacity in wireless communications systems has recently attracted significant interest [6]. For the uplink (remote to base) portion of the system, signals can be separated at the array based on knowledge of their spatial or temporal "signatures". The array can also be used on the downlink (base to remote) channel to transmit energy towards one user and not at another. Such systems can reduce radiated power requirements, allow for multiple mobile co-channel users, and reduce signal contamination from adjacent cells.

In this work, scenarios with multipath propagation confined to local scatterers in the vicinity of the mobile sources are considered. Fast fading is usually attributed to local scattering, and local scattering models have been used to derive channel models including the spatial dimension [6, 9, 15]. By assuming a spatial distribution for the multipath components, it is possible to derive the fading correlation between the elements of an array. In many cellular radio systems, the base station antenna(s) are mounted on a tower away from potential multipath reflectors. If the scatterers are local to the mobile and the base station is some distance away, the scattered signals from a given user will be coherent and confined to a relatively small angular region.

Traditional direction of arrival (DOA) estimation techniques rely on the fact that the spatial signature, or channel, is a known function of the DOA. Due to multipath propagation, the spatial signature will not belong to the array manifold parameterized by DOA alone. However, under the assumption of *local* scattering, use of the DOA will still make sense for determining spatial signatures.

In [13, 15], a statistical model is proposed under which the spatial signature is approximately Gaussian with zero mean and covariance matrix parameterized by the nominal direction and angular spread of the scatterers. In this paper, a deterministic approach is taken using a *generalized* array manifold consisting of a linear combination of the nominal steering vector and its gradient. Instead of estimating the parameters of the distribution from multiple realizations of the spatial signatures as in [13], the coefficient of the gradient vector is estimated along with each user's DOA for each realization. Also, in contrast to [13], the estimation of the extra parameter is linear, leading to a computationally simpler solution.

The use of array manifold derivatives has been studied as a way of making minimum variance beamformers more robust by widening their main beam response [1, 3, 14]. Compared with these methods and the stochastic approaches in [13, 15], the technique described here seeks to identify the actual spatial signatures of each user. The paper aims at presenting algorithms for parameterized estimation of spatial signatures in scenarios with local scatterers, and the focus is on uplink signal waveform estimation.

2. GENERALIZED ARRAY MANIFOLD MODEL

It is assumed that d mobile sources are present emitting narrowband signals. The scenario is assumed to be time invariant, and the time dispersion introduced by the multipath propagation is assumed to be small in comparison with the reciprocal of the bandwidth of the emitted signals. Thus, for an array of m antennas, the following low rank model results:

$$\mathbf{x}(t) = \mathbf{V}\mathbf{s}(t) + \mathbf{n}(t) \in \mathcal{C}^m. \quad (1)$$

The i th column of $\mathbf{V}$, denoted $\mathbf{v}_i$, represents the spatial signature of the signal $s_i(t)$ transmitted by user i . The vector $\mathbf{s}(t) = [s_1(t), \dots, s_d(t)]^T$ contains the signals of all users at time t , and $\mathbf{n}(t)$ denotes additive noise. The term *low rank* here refers to the fact that it is assumed that $d < m$. The spatial signature $\mathbf{v}_i$ may be written as

$$\mathbf{v}_i = \sum_{k=1}^{N_i} \alpha_{ki} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}), \quad (2)$$

where α_{ki} is the (complex) amplitude of the k th scattered signal, $\mathbf{a}(\theta)$ is the response of the array to a single unit

amplitude signal with DOA θ , and N_i is the total number of local scatterers for the i th source. The quantities θ_i and $\theta_i + \tilde{\theta}_{ik}$ represent respectively the nominal DOA of the i th user, and the arrival angle of the k th scattered signal. The phenomenon is illustrated in Figure 1. The assumption of

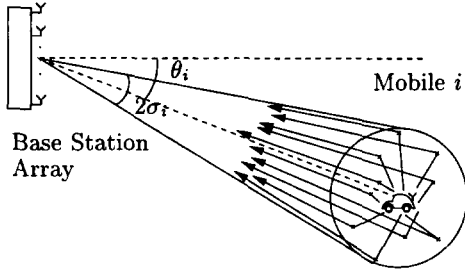


Figure 1: Local scattering

local scattering near each user means that σ_i in Figure 1 is small. Define the gradient $\mathbf{d}(\theta) = \partial \mathbf{a}(\theta) / \partial \theta$, so that a first order Taylor series expansion of (2) yields

$$\begin{aligned} \mathbf{v}_i &\simeq \sum_{k=1}^{N_i} \alpha_{ik} \left[\mathbf{a}(\theta_i) + \tilde{\theta}_{ik} \mathbf{d}(\theta_i) \right] \\ &= \left(\sum_{k=1}^{N_i} \alpha_{ik} \right) \mathbf{a}(\theta_i) + \left(\sum_{k=1}^{N_i} \alpha_{ik} \tilde{\theta}_{ik} \right) \mathbf{d}(\theta_i) \\ &= \mathbf{a}(\theta_i) + \phi_i \mathbf{d}(\theta_i), \end{aligned} \quad (3)$$

where it is assumed that the spatial signature is scaled so that $\sum_k \alpha_{ik} = 1$, and ϕ_i is defined as $\phi_i = \sum_{k=1}^{N_i} \alpha_{ik} \tilde{\theta}_{ik}$. Substituting (3) into $\mathbf{V}$ leads to the following compact matrix notation:

$$\mathbf{V} \simeq \mathbf{A}(\theta, \phi) = \mathbf{A}(\theta) + \mathbf{D}(\theta) \Phi(\phi) \quad (4)$$

where

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_d)], \quad \mathbf{D}(\theta) = [\mathbf{d}(\theta_1), \dots, \mathbf{d}(\theta_d)],$$

$$\Phi(\phi) = \text{diag} \{ \phi_1, \dots, \phi_d \},$$

$\theta = [\theta_1, \dots, \theta_d]^T$ and $\phi = [\phi_1, \dots, \phi_d]^T$. Both $\mathbf{a}(\theta)$ and its gradient $\mathbf{d}(\theta)$ are assumed to be known (calibrated) functions of θ , and the problem addressed in this paper is the estimation of θ and ϕ given N observations of the array output.

Under the assumption that the derived model is valid, a natural question is under what conditions the parameters are identifiable. For the methods considered here, it is assumed that the parameters are uniquely determined by the column span of $\mathbf{A}(\theta, \phi)$. This holds if an $m \times (d+1)$ matrix $\mathbf{A}(\theta, \phi)$ has full rank for any collection of distinct parameters $\theta_1, \dots, \theta_{d+1}$ and arbitrary $\phi_1, \dots, \phi_{d+1}$. As in [12], necessary conditions may be derived. If the $m \times 2(d+1)$ matrix $[\mathbf{A}(\theta) \ \mathbf{D}(\theta)]$ has full rank for all $\theta \in \mathbb{R}^{d+1}$ with distinct elements, then, except for a set of zero measure, the parameters are identifiable if $d \leq m - 2$.

3. PARAMETERIZED ESTIMATE OF SPATIAL SIGNATURE

In this section, two algorithms for estimating θ and ϕ are proposed that take advantage of the special structure of the spatial signatures in (4), which we refer to as a *generalized array manifold* (GAM). The basic idea behind the algorithms comes from similar GAMs that arise in situations involving diversely polarized antenna arrays [5, 10, 12]. The key advantage is that a search is required only for the DOA parameters; the gradient coefficients are separable and solved for explicitly given the resulting DOA estimates.

The algorithms use the orthogonality between the noise subspace and the signal subspace. The eigenvectors associated with the $m - d$ smallest eigenvalues of the sample covariance matrix of the N observations are used as an estimated basis of the noise subspace. The collection of estimated noise subspace eigenvectors is denoted $\hat{\mathbf{E}}_n$.

3.1. A MUSIC-Based Approach

In the standard MUSIC algorithm [10], the DOAs are estimated by searching one by one for values of θ that make $\mathbf{a}(\theta)$ nearly orthogonal to $\hat{\mathbf{E}}_n$. The measure of orthogonality for MUSIC is defined to be

$$V_{\text{MU}}(\theta) = \frac{\mathbf{a}^*(\theta) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{a}(\theta)}{\mathbf{a}^*(\theta) \mathbf{a}(\theta)},$$

and the d minima of $V_{\text{MU}}(\theta)$ are taken to be the estimates of the DOAs. With a GAM, $\mathbf{a}(\theta)$ must be replaced with

$$\mathbf{a}(\theta) + \phi \mathbf{d}(\theta) = \bar{\mathbf{A}}(\theta) \bar{\phi}$$

where $\bar{\mathbf{A}}(\theta) = [\mathbf{a}(\theta) \ \mathbf{d}(\theta)]$ and $\bar{\phi} = [1 \ \phi]^T$. For this case the MUSIC cost function becomes

$$V_{\text{MU}}(\theta, \phi) = \frac{\bar{\phi}^* \bar{\mathbf{A}}^*(\theta) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \bar{\mathbf{A}}(\theta) \bar{\phi}}{\bar{\phi}^* \bar{\mathbf{A}}^*(\theta) \bar{\mathbf{A}}(\theta) \bar{\phi}}.$$

The MUSIC criterion is seen to be a ratio of quadratic forms in $\bar{\phi}$, and thus minimizing $V_{\text{MU}}(\theta, \bar{\phi})$ with respect to $\bar{\phi}$ is equivalent to finding, as a function of θ , the following minimum generalized eigenvalue and eigenvector:

$$\bar{\mathbf{A}}^*(\theta) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \bar{\mathbf{A}}(\theta) \mathbf{z}_{\min} = \lambda_{\min} \bar{\mathbf{A}}^*(\theta) \bar{\mathbf{A}}(\theta) \mathbf{z}_{\min}.$$

As proposed in [10], the DOA estimates can then be found by viewing $\lambda_{\min}$ as a function of θ , and searching for its minima. The gradient coefficient ϕ can be determined from the eigenvector associated with $\lambda_{\min}(\hat{\theta}_i)$.

3.2. Noise Subspace Fitting

As an alternative, consider the *noise subspace fitting* (NSF) approach outlined in [4, 11]. Under the GAM model, the NSF algorithm estimates θ and ϕ as the minimizing arguments of the following cost function:

$$V_{\text{NSF}}(\theta, \phi) = \text{Tr} \left(\mathbf{A}^*(\theta, \phi) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{A}(\theta, \phi) \mathbf{W} \right),$$

where $\mathbf{W} = \mathbf{W}^* > 0$ is a $d \times d$ weighting matrix. Using arguments similar to those in [12], the cost function may be written as

$$\{\hat{\theta}, \hat{\phi}\} = \arg \min_{\theta, \phi} \left[\mathbf{e}^T \phi^* \right] \mathbf{M}(\theta) \begin{bmatrix} \mathbf{e} \\ \phi \end{bmatrix}, \quad (5)$$

where $\mathbf{e}$ is a column vector composed of d ones and

$$\begin{aligned} \mathbf{M}(\theta) &= \begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ad} \\ \mathbf{M}_{da} & \mathbf{M}_{dd} \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{A}^* \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{A}) \odot \mathbf{W}^T & (\mathbf{A}^* \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{D}) \odot \mathbf{W}^T \\ (\mathbf{D}^* \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{A}) \odot \mathbf{W}^T & (\mathbf{D}^* \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{D}) \odot \mathbf{W}^T \end{bmatrix}. \end{aligned}$$

Here, $\odot$ denotes the element-by-element product. The cost function in (5) may be rewritten as

$$\begin{aligned} V_{\text{NSF}} &= (\phi + \mathbf{M}_{dd}^{-1} \mathbf{M}_{da} \mathbf{e})^* \mathbf{M}_{dd} (\phi + \mathbf{M}_{dd}^{-1} \mathbf{M}_{da} \mathbf{e}) \\ &\quad + \mathbf{e}^T (\mathbf{M}_{aa} - \mathbf{M}_{ad} \mathbf{M}_{dd}^{-1} \mathbf{M}_{da}) \mathbf{e}. \end{aligned}$$

From this it follows that the estimation of ϕ is separable from that of the DOAs, and is given by

$$\hat{\phi} = -\mathbf{M}_{dd}^{-1} \mathbf{M}_{da} \mathbf{e}. \quad (6)$$

The concentrated cost function then becomes

$$V_{\text{NSF}}(\theta) = \mathbf{e}^T (\mathbf{M}_{aa} - \mathbf{M}_{ad} \mathbf{M}_{dd}^{-1} \mathbf{M}_{da}) \mathbf{e}. \quad (7)$$

Thus, $V_{\text{NSF}}(\theta)$ is the sum of the elements of the Schur complement of $\mathbf{M}(\theta)$. The algorithm is implemented as follows

1. Estimate $\hat{\theta}$ as the argument that minimizes the sum of the elements of $\mathbf{M}_{aa} - \mathbf{M}_{ad} \mathbf{M}_{dd}^{-1} \mathbf{M}_{da}$.
2. Solve for $\hat{\phi}$ by using $\hat{\theta}$ in (6)

The GAM model follows from an approximation of (2). If the model of (4) is valid, the weighting matrix $\mathbf{W}$ can be chosen so that the above NSF method yields asymptotically efficient parameter estimates (i.e., the asymptotic variance of the estimates attains the Cramér-Rao bound). This follows directly from the results of [12]. The optimal $\mathbf{W}$ is parameter dependent, so the NSF approach must be preceded by a step where θ and ϕ are estimated consistently (e.g., using the MUSIC approach described earlier). It is well known that this procedure has no effect on the asymptotic properties of the estimates.

For small angular spread, it may be reasonable to neglect the scattering when estimating the DOAs. The estimated DOAs may be used in (6) to solve for $\hat{\phi}$. Simulations indicate that such a decoupled approach performs well for small angular spreads.

3.3. Uniform Linear Arrays

For a Uniform Linear Array (ULA) with elements separated by Δ wavelengths, it is easily established that the gradient is $\partial \mathbf{a}(\theta) / \partial \theta = j\pi \Delta \cos(\theta) \Psi_m \mathbf{a}(\theta)$, where Ψ_m is a diagonal matrix, $\Psi_m = \text{diag}\{- (m-1), -(m-3), \dots, (m-1)\}$.

The factor $j\pi \Delta \cos \theta_i$ is associated with ϕ_i , and $\hat{\mathbf{d}}(\theta) = \Psi_m \mathbf{a}(\theta)$ is used instead of the true gradient $\mathbf{d}(\theta)$. In [2] it was argued that for a ULA with small aperture and with

ϕ small, the spatial signature is approximately a Vandermonde vector,

$$\mathbf{a}(\theta) + \phi \mathbf{d}(\theta) \approx e^{-j \frac{m-1}{2} \omega} \begin{bmatrix} 1, e^{j\omega}, \dots, e^{j\omega(m-1)} \end{bmatrix}^T, \quad (8)$$

where $\omega = 2\pi \Delta (\sin \theta + \phi \cos \theta)$. Note that ω is complex since ϕ is. As outlined in [2], the ESPRIT algorithm [8] may be used to estimate the complex frequency ω , and hence $\mathbf{V}$. This may be viewed as an approximation of the GAM, applicable to ULAs.

4. NUMERICAL EXAMPLES

In this section, the accuracy of the estimated spatial signatures is evaluated by comparing their ability to perform signal separation (e.g., via beamforming). The signal waveform estimator considered is the so called stochastic maximum likelihood estimator [7], which uses a structured estimate of the array covariance. Given estimates of the spatial signatures, $\hat{\mathbf{V}}$, the estimated signals are given by

$$\hat{\mathbf{s}}(t) = \mathbf{W}^* \mathbf{x}(t) = \left((\hat{\mathbf{V}} \hat{\mathbf{R}}_{ss} \hat{\mathbf{V}}^* + \hat{\sigma}^2 \mathbf{I})^{-1} \hat{\mathbf{V}} \hat{\mathbf{R}}_{ss} \right)^* \mathbf{x}(t),$$

$$\hat{\mathbf{R}}_{ss} = \hat{\mathbf{V}}^\dagger (\hat{\mathbf{R}} - \hat{\sigma}^2 \mathbf{I}) \hat{\mathbf{V}}^\dagger$$

$$\hat{\sigma}^2 = \frac{1}{m-d} \text{Tr} \left\{ (\mathbf{I} - \hat{\mathbf{V}} \hat{\mathbf{V}}^\dagger) \hat{\mathbf{R}} \right\}.$$

Here $\hat{\mathbf{V}}^\dagger = (\hat{\mathbf{V}}^* \hat{\mathbf{V}})^{-1} \hat{\mathbf{V}}^*$ and $\hat{\mathbf{R}}$ is the sample covariance matrix. Another alternative is the Linearly Constrained Minimum Variance (LCMV) beamformer [1, 3]. The weight vector for estimating the i th signal $s_i(t)$, $\mathbf{w}_i$, attempts to minimize $E[\mathbf{w}_i^* \mathbf{x}(t)]^2 = \mathbf{w}_i^* \mathbf{R} \mathbf{w}_i$ subject to the linear constraints $\mathbf{w}_i^* \mathbf{C} = \mathbf{f}^*$. If the nominal DOA is θ_i , it follows from (3) that a reasonable set of constraints when angular spread is present is $\mathbf{w}_i^* \mathbf{a}(\theta_i) = 1$ and $\mathbf{w}_i^* \mathbf{d}(\theta_i) = 0$. This gives $\mathbf{C} = [\mathbf{a}(\theta) \mathbf{d}(\theta)]$ and $\mathbf{f} = [1, 0]^T$. The estimated signal is then given by

$$\hat{s}_i(t) = \mathbf{w}_i^* \mathbf{x}(t), \quad \mathbf{w}_i = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C}^* \mathbf{R}^{-1} \mathbf{C})^{-1} \mathbf{f}. \quad (9)$$

Note that this approach will use two degrees of freedom for each source. In (9), $\mathbf{R}$ and θ_i are replaced with estimates.

In the simulations, a ULA with elements separated by half a wavelength is used. In each trial, a spatial signature is generated by drawing 30 local scatterers from a uniform angular distribution of width 2σ . In each trial 100 snapshots are collected. The signals are estimated and the signal to interference plus noise ratio (SINR) is averaged over 2000 trials. The standard ESPRIT algorithm was used for determining DOA estimates used by the LCMV beamformer, and as initial estimates for MUSIC and NSF. The purpose of the simulations is to compare the performance using different models and methods for determining $\hat{\mathbf{V}}$. In the plots, DOA refers to ignoring the scattering and using $\hat{\mathbf{V}} = \mathbf{A}(\hat{\theta})$ with $\hat{\theta}$ estimated using the standard ESPRIT algorithm. MUSIC and NSF use $\mathbf{A}(\hat{\theta}, \hat{\phi})$ with $\hat{\theta}$ and $\hat{\phi}$ estimated with MUSIC and NSF respectively. VM refers to using the ESPRIT algorithm to estimate Vandermonde vectors as outlined in Section 3.3 and in [2].

In the first example two mobile sources with 20 dB and 40 dB SNR are present. The angular width of both sources

is $2\sigma = 4^\circ$ as seen from an array with six elements. In Figure 2 the average SINR of the weaker signal is shown for different angular separations. In the second example a

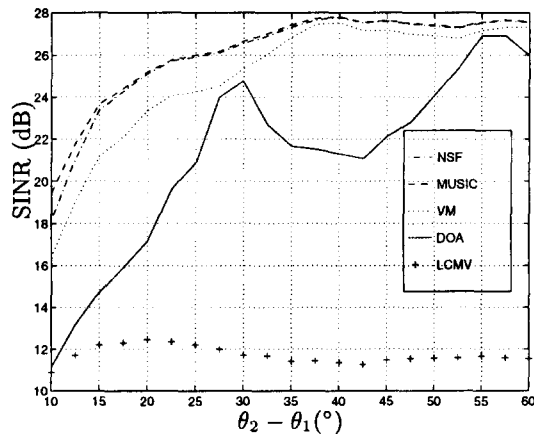


Figure 2: Different angular separations, $2\sigma = 4^\circ$.

ULA with eight elements is used and three well separated mobile sources with nominal DOAs -30° , 0° and 30° and SNR 40 dB, 20dB and 40dB are present. In Figure 3, the average SINR of the weaker signal is shown for various angular spreads. In both examples, the use of the GAM is seen to provide a significant improvement in SINR.

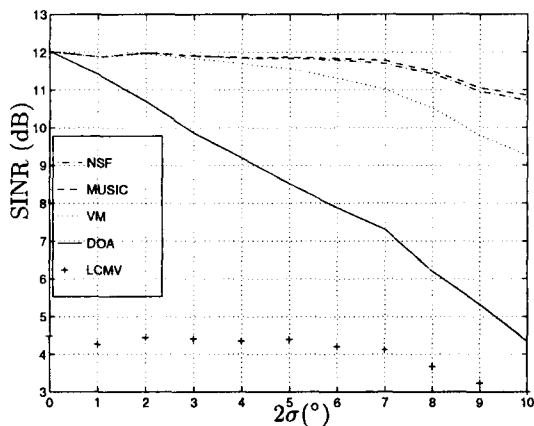


Figure 3: Different angular spreads.

5. CONCLUSIONS

A parameterized model, GAM, for spatial signatures in the presence of local scattering was presented. Two algorithms based on MUSIC and NSF were proposed for estimating the parameters. Numerical comparisons were made with the approach proposed in [2], with the conventional approach ignoring the scattering and with the LCMV beamformer

implementing a derivative constraint. For the cases considered, the simulations indicate that the performance gain compared to the mentioned conventional methods may be considerable for signal waveform estimators, especially in scenarios with relatively small angular spread and strong interference. The relatively poor performance of the LCMV method is probably due to signal cancellation effects.

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