# BLIND ADAPTIVE EQUALIZATION AND DIVERSITY COMBINING *

Ye Li †
AT&T Labs-Research
791 Holmdel-Keyport Road
Holmdel, NJ 07733-0400
liye@research.att.com

K. J. Ray Liu
Electrical Engineering Department
University of Maryland
College Park, MD 20742, USA
kjrliu@src.umd.edu

**Abstract:** *Spatial-temporal equalizer can be used in the wireless communication systems with antenna arrays to improve the performance. In this article, we introduce two blind adaptive algorithms for spatial-temporal equalization, and present their convergence. Computer simulation demonstrates that the new algorithms converge faster than fractionally spaced constant-modulus algorithm (FS-CMA).*

## 1 PROBLEM FORMULATION

To improve the quality of wireless communication systems, antenna arrays are used for diversity reception. To make full use of the information contained in each sensor, spatial-temporal equalizer is used to remove intersymbol interference and mitigate the additive channel noise. The wireless communication systems with antenna arrays can be modeled as *single-input/multiple-output systems* shown as in Figure 1. Since the channels are time-varying and training sequences are not always available, blind adaptive techniques have to be used in these systems.

In Figure 1, the input sequence $\{s[n]\}$ is sent through $M$ different linear channels with impulse response $\{h_m[n]\}$ for $m = 1, 2, \cdots, M$. Hence, the channel outputs can be written in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n], \text{ or } \mathcal{X}_K[n] = \mathcal{H}_K \mathcal{S}_K[n], \quad (1.1)$$

where we have used the definitions

$$\mathcal{H}_K \triangleq \begin{pmatrix} \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] \end{pmatrix}, \quad (1.2)$$

$$\mathcal{X}_K[n] \triangleq (\mathbf{x}_K[n], \cdots, \mathbf{x}_K[n+N-1]), \quad (1.3)$$

$$\mathcal{S}_K[n] \triangleq (\mathbf{s}_K[n], \cdots, \mathbf{s}_K[n+N-1]), \quad (1.4)$$

with

$$\mathbf{s}_K[n] \triangleq (s[n-L+1], \cdots, s[n+K-1])^T,$$

$$\mathbf{h}[n] \triangleq (h_1[n], \cdots, h_M[n])^T,$$

$$\mathbf{x}_K[n] \triangleq (\mathbf{x}^T[n], \cdots, \mathbf{x}^T[n+K-1])^T,$$

and

$$\mathbf{x}^T[n] \triangleq (x_1[n], \cdots, x_M[n]).$$

The integer $K$ in the above equations determines the dimensions of the matrices and the vectors.

In this article, we will assume that the SIMO channels are of finite impulse response (FIR) with length $L$, and furthermore, they satisfy the *length-and-zero condition* [2], which makes $\mathcal{H}_K$ for any $K \geq L-1$ to be of full column rank. Hence, from (1.1), there exists a $KM \times (K+L-1)$ matrix $\mathcal{F}$ (not unique) such that

$$\mathbf{s}_K[n] = \mathcal{F}^H \mathbf{x}_K[n], \quad \mathcal{F} \triangleq (\mathbf{f}_1, \ \mathbf{f}_2, \cdots, \ \mathbf{f}_{K+L-1}), \quad (1.5)$$

*The work was supported in part by the NSF grants MIP9309506 and MIP9457397.

†The work was done while he was at the University of Maryland.

where $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ are column vectors with $KM$ elements.

The task of blind adaptive equalization of SIMO channels is to find algorithms to adaptively adjust the parameters $\mathbf{f}_k$ such that

$$\mathcal{F}^H \mathcal{H}_K = c I_{K+L-1}, \qquad (1.6)$$

for some non-zero constant $c$.

## 2 ALGORITHEM DEVELOPEMENT

From the above definitions, we have

$$\mathbf{f}_k^H \mathbf{x}_K[n] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1], \qquad (2.1)$$

for $k = 1, \cdots, K+L-2$, and every integer $n$. Hence, using the above identity, we may obtain $\mathbf{f}_k$. If the channel noise is considered, $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ can be estimated by minimizing the cost function

$$\mathcal{C} = \sum_{k=1}^{K+L-2} E|\mathbf{f}_k^H \mathbf{x}_K[n] - \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1]|^2 \qquad (2.2)$$

subject to

$$\sum_{k=1}^{K+L-1} |\mathbf{f}_k^H \mathbf{x}_K[n]|^2 = c_o,$$

where $c_o$ is a non-negative constant. The constraint is added here to prevent from the trivial solution of $\mathbf{f}_k = \mathbf{0}$ for all $k = 1, \cdots, K+L-1$.

From (2.2), a direct calculation yields that

$$\begin{aligned} \mathcal{C} = & \sum_{k=1}^{K+L-2} \{\mathbf{f}_k^H R_x[0]\mathbf{f}_k - \mathbf{f}_k^H R_x[1]\mathbf{f}_{k+1} \\ & -\mathbf{f}_{k+1}^H R_x^H[1]\mathbf{f}_k + \mathbf{f}_{k+1}^H R_x[0]\mathbf{f}_{k+1}\}, \end{aligned} \qquad (2.3)$$

where we have used the definition and identity

$$R_x[m] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n-m]\}, \; R_x[-m] = R_x^H[m].$$

Hence,

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}_k} = \begin{cases} R_x[0]\mathbf{f}_1 - R_x[1]\mathbf{f}_2, \text{ if } k = 1, \\ 2R_x[0]\mathbf{f}_k - R_x[1]\mathbf{f}_{k+1} - R_x^H[1]\mathbf{f}_{k-1}, \\ \qquad \text{if } k = 2, \cdots, K+L-2, \\ R_x[0]\mathbf{f}_{K+L-1} - R_x^H[1]\mathbf{f}_{K+L-2}, \\ \qquad \text{if } k = K+L-1. \end{cases} \qquad (2.4)$$

Let

$$\mathbf{f} \triangleq \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_{K+L-1} \end{pmatrix}, \qquad (2.5)$$

then, (2.4) can be written as

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}} = R\mathbf{f}, \qquad (2.6)$$

where $R$ is a $KM(K+L-1) \times KM(K+L-1)$ matrix defined as

$$R \triangleq \begin{pmatrix} R_x[0] & -R_x[1] & \cdots & \mathbf{0} \\ -R_x^H[1] & 2R_x[0] & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & R_x[0] \end{pmatrix}.$$

Using the gradient-based approach, we can obtain an adaptive algorithm to estimate $\mathbf{f}$ as following:

$$\widehat{\mathbf{f}}^{(n+1)} = \mathbf{f}^{(n)} - \mu R \mathbf{f}^{(n)}, \qquad (2.7)$$

$$p^{(n+1)} = \left( \sum_{k=1}^{K+L-1} (\widehat{\mathbf{f}}_k^{(n+1)})^H R_x^{(n+1)}[0] \widehat{\mathbf{f}}_k^{(n+1)} \right)^{1/2}, \qquad (2.8)$$

$$\mathbf{f}^{(n+1)} = \frac{c_o}{p^{(n+1)}} \widehat{\mathbf{f}}^{(n+1)} \qquad (2.9)$$

and $R_x[m]$ in $R$ can be estimated using

$$R_x^{(n+1)}[m] = \lambda R_x^{(n)}[m] + (1-\lambda)\mathbf{x}_K[n]\mathbf{x}_K^H[n-m], \qquad (2.10)$$

where $\mu$ is a step-size and $\lambda \in [0, \; 1]$ is a forgetting factor.

The blind adaptive algorithm for SIMO channel equalization defined by (2.7)-(2.10) is called *second-order statistics based algorithm*, or SOSA.

If we examine the identities (1.4) and (1.5) carefully, we will see that

$$\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] = \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2], \qquad (2.11)$$

for $k_1, \; k_2 = 1, 2, \cdots, K+L-1$ and all integer $n$. Hence, we can modify the cost function (2.2) as

$$\tilde{\mathcal{C}} = \sum_{k_1, k_2 = 1}^{K+L-1} E|\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] - \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2]|^2. \qquad (2.12)$$

From this cost function, we are able to derive a *modified second-order statistics based algorithm* (MSOSA) similar to (2.7)-(2.10), except that $R$ is substituted by

$$\tilde{R} = \begin{pmatrix} (K+L-2)R_x[0] & \cdots & -R_x[K+L-2] \\ \vdots & \ddots & \vdots \\ -R_x^H[K+L-2] & \cdots & (K+L-2)R_x[0] \end{pmatrix}.$$

The dimension parameter $K$ in the above two algorithms is usually between $L-1$ to $L+1$ to get good trade-off between the performance and the complexity.

Since the MSOSA exploits more information about the structure of $\mathbf{s}_K[n]$, as comfirmed by our computer simulations, it is more robust than the SOSA. However, the MSOSA requires a little bit more computation than the SOSA.

# 3 CONVERGENCE

Having developed the SOSA and the MSOSA in the previous section, we will investigate their convergence in this section.

A sequence $\{s[n]\}$ is said to be *$K$-th order persistently exciting* [3] if and only if $\mathcal{S}_K[n]$ defined in (1.4) is of full row rank for some $N$. Under this assumption, we can prove the following global convergence theorem.

**Theorem 1:** *For digital communication systems, assume that the channel input sequence $\{s[n]\}$ is $K+1$-th order persistently exciting for some $K \geq L-1$ and $N$ in (1.4), and the noiseless channel satisfies the length-and-zero condition. If for some $k$ and $n$*

$$\mathbf{f}_k^H \mathbf{x}_K[n] \neq 0, \tag{3.1}$$

*and for all $k = 1, 2, \cdots, K+L-2$ and $n = 1, 2, \cdots, N$*

$$\mathbf{f}_k^H \mathbf{x}_K[n+1] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n], \tag{3.2}$$

*then,*

$$\mathcal{F}^H \mathcal{H}_K = c I_{K+L-1} \tag{3.3}$$

*for some non-zero c, where $I_{K+L-1}$ is a $(K+L-1)\times(K+L-1)$ identity matrix.*

# 4 COMPUTER SIMULATION EXAMPLE

A Monte Carlo simulation example has been conducted to demonstrate the performance of the new algorithms used in 16-QAM digital communication systems. In our simulation, the channel input sequence $\{s[n]\}$ is i.i.d. and randomly distributed over $\{\pm 1 \pm \jmath\}$, $\{\pm 3 \pm \jmath\}$, $\{\pm 1 \pm 3\jmath\}$ and $\{\pm 3 \pm 3\jmath\}$. It is sent through two time-invariant linear channels with impulse responses $\{0.6662 - \jmath 0.8427, 1.6323 - \jmath 0.2503, -0.6617 - \jmath 0.4102\}$ and $\{0.4607 + \jmath 0.5789, 0.5855 - \jmath 2.6912, 1.3273 - \jmath 0.4184\}$ respectively. The complex white Gaussian noise, with zero-mean and variance making $SNR = 20dB$, is added at each channel output. The step-size $\mu$ and the forgetting factor $\lambda$ are chosen to optimize the performance of each equalization algorithm. Figure 4 (a), (b) and (c) are the eye patterns of the SOSA, the MSOSA and the orthogonal FS-CMA respectively after 500 iterations, and Figure 4 (d) illustrates the convergence of ISI of the three algorithms with respect to the number of iterations. From Figure 4, the performance of the SOSA and the MSOSA are much better than that of the orthogonal FS-CMA with the MSOSA being the best of the three algorithms.

In this article, we have proposed the SOSA and the MSOSA for blind adaptive equalization of SIMO channels using the correlation function of the channel outputs. The new algorithms converge faster than fractionally spaced CMA [2]. We are currently studying how to use the proposed algorithms in IS-54 systems to tracking the fast fading mobile radio channels.

## References

[1] Y. Li and K. J. R.Liu, "Fast blind adaptive algorithms for equalization and diversity reception in wireless communications using antenna array," *Technical Report: TR. 96-50*, Institute for Systems Research, University of Maryland at College Park, May 1996.

[2] Y. Li and Z. Ding, "Global convergence of fractionally spaced Godard equalizer," *IEEE*

*Trans. on Signal Processing*, vol.44, pp818-826, April 1996.

[3] D. Yellin and B. Porat, "Blind identification of FIR systems excited by discrete-alphabet inputs," *IEEE Transactions on Signal Processing*, vol.41, pp1331-1339, March 1993

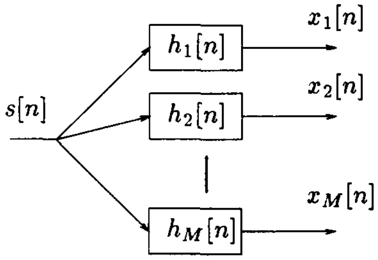


Figure 1: Single-input/multiple output systems

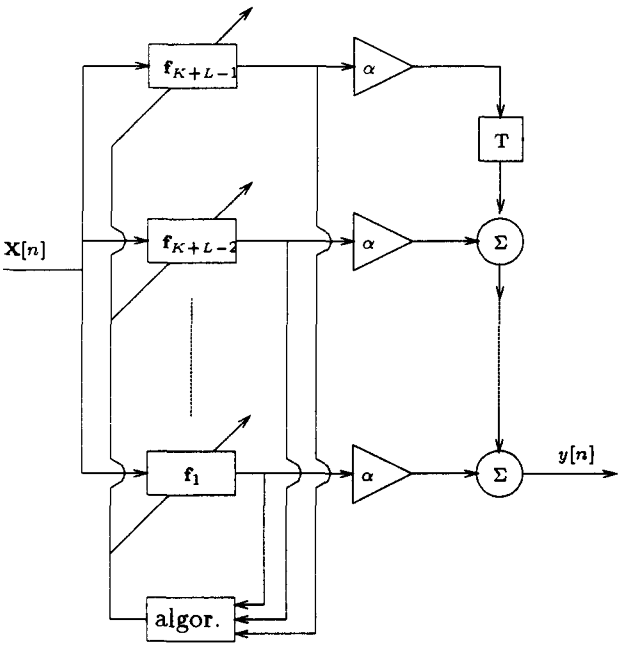


Figure 2: Blind adaptive equalizer with diversity combining.

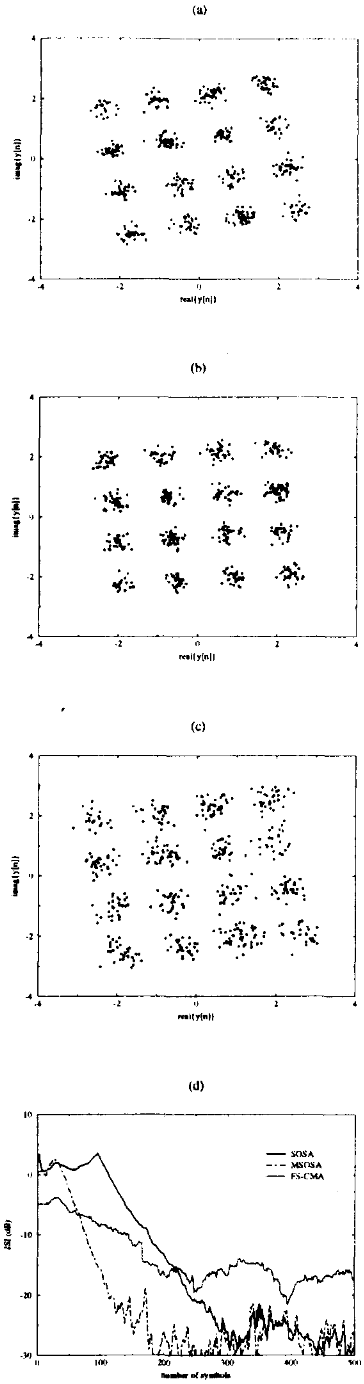


Figure 3: Comparison of the SOSA, the MSOSA, and the CMA when $SNR = 20db$, 500 symbol eye patterns of (a) the SOSA, (b) the MSOSA, and (c) the orthogonal FS-CMA after 500 iterations, and the convergence of the ISI for the three algorithms respectively.