

# REMOTE CALIBRATION OF ACTIVE PHASED ARRAY ANTENNAS FOR COMMUNICATION SATELLITES

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## ABSTRACT

This work describes an algorithm called the control circuit encoding (CCE) algorithm that is effective for the remote calibration of an  $N_e$  element active phased array antenna. The algorithm involves transmission of orthogonal encoded signals. CCE is ideally suited for analog beamformers as it requires no additional encoding hardware. The CCE method encodes phased array elemental signals using a Hadamard matrix to control the switching of intrinsic phase shifter delay circuits. The CCE algorithm can reduce the average measurement integration times for the complete set of calibration parameters by  $\sim N_e$  relative to the corresponding values for single-element calibration procedures.

## I. INTRODUCTION

This work describes an algorithm called the control circuit encoding (CCE) algorithm that uses time multiplexed orthogonal encoded signals to remotely calibrate transmitting and/or receiving active phased array systems [1,2]. The CCE calibration technique has been implemented and experimentally validated with a L-band active phased array testbed developed at the University of Texas at Austin [3].

Active phased array systems belong to the class of *smart antenna systems* that possess the ability to perform programmable changes of the amplitude and phase of the elemental phased array signals in order to accommodate different beamforming scenarios. The applications featured in this work relate to future generation geostationary (GEO) communication satellite systems that will deploy analog beamforming active phased array transmitting antennas rather than present day state-of-the-art reflector antennas. Extensions of the calibration algorithms to receiving and digital beamforming systems are straightforward.

In an analog phased array transmitting antenna

with a  $p$  bit beamformer, there are  $p$  independent delay circuits that can be switched into the elemental electrical path to ideally provide  $2^p$  quantized phase levels corresponding to phase shifts of  $2\pi m/2^p$  for  $m = 0, 1, \dots, 2^p - 1$ . The gain and phase (complex gain) of the coherent elemental signals at the receiver is a function of the complex gain of the phase shifter delay circuits, the power amplifier, and the transmission path to the receiver. A calibration system must be capable of accurately measuring the complex gains associated with all of the  $N_e p$  delay circuits as well as both the relative gains and phases associated with the *straight through* path to the receiver with no delay circuits switched in.

Ideally, remote on-line calibration should be performed with minimal additional remote hardware; no error other than possible unavoidable propagation and/or receiver noise; and maximized effective SNR<sup>1</sup> (ESNR) per measurement in order to minimize the required signal measurement integration times. This is important in satellite applications as reliable calibration measurements have to be performed in time window short enough that the parameters can be treated as being stationary.

There is virtually no known published literature describing practicable remote calibration procedures that can measure the relative *ground truth* of the full set of elemental complex gains for an  $N_e$  element array. The suggested methods[4] are predominantly variations on the theme of coherent detection of a single element (SE) under test while all the other elements are turned off.

The SE methods are conceptually simple, but unfortunately have fundamental problems that make their usefulness questionable for the satellite mission calibration requirements. The first problem deals with the necessity of implementing a multi-pole microwave switch at the front end of the elemental electrical path for the purpose of di-

<sup>1</sup>The ESNR is defined as the ratio of the square of the mean to the variance of the parameter estimate.

recting the calibration signals to a single element at a time. This switch must be implemented in a manner that the relative complex gains (insertion phase) of the straight-through paths for all the elements *are not altered* by the switching process. The second problem arises from the fact that the SE procedures are relatively low ESNR calibration procedures, which translates into relatively long measurement integration times. At practicable phased array satellite link budget power levels, the integration times required to extract the calibration data for the SE procedures could be too long to satisfy the quasi-stationarity time window criteria referred to above.

The ESNR can be substantially enhanced by using  $N \geq N_e$  coherent transmission of orthogonal transform encoded signals from  $N_e$  elements of the array. This procedure enhances the ESNR by a factor  $\sim N$  over single-element transmission because all elements are transmitting simultaneously. The encoding and decoding provide a vehicle for extracting the parameters of the individual elemental signals.

The CCE achieves the above mentioned ESNR enhancement by using controlled switching of the delay phase control circuits themselves to effectively generate a *perfect orthogonal transform encoding* of the signal vectors, even though the control circuits may be imperfect - *no additional encoding hardware is required*. The switching is dictated by matrix elements of an  $N \times N$  Hadamard matrix. The coherent signals are decoded with the inverse of the same Hadamard matrix used in the control circuit encoding.

Let  $\{s(n, i)\}$  represent the individual coherent straight-through-path signals that have been received at a single earth station receiver, demodulated, coherently detected, and sampled at times  $t_i$ . The algorithmic operations always involve coherent signals that are sampled at the same time point in their respective coherent bursts. This allows us to simplify the nomenclature by suppressing the sampling index  $i$  with the understanding that all operations refer to the same sampled time points. Accordingly, we use the notation,

$$\mathbf{S} = [s(1), s(2), \dots, s(N)]^T, \quad (1)$$

to represent the received, demodulated, sampled signal vector at the receiver when all the elements are simultaneously transmitting their straight-through-path signals.

The calibration process is based on the following beamformer model. Calibration signal powers are assumed to be in the linear regime with respect to

the beamformer such that the effect of switching in the  $\mu$ th delay circuit of  $n$ th array element with complex gain  $d_\mu(n)$  imposes a complex gain,

$$x(n) \longrightarrow \boxed{d_\mu(n)} \longrightarrow d_\mu(n)x(n) \quad (2)$$

on the input elemental signal  $x(n)$ . The effect of switching in multiple circuits generates the product of the complex gains,

$$x(n) \longrightarrow \boxed{d_\mu(n)} \longrightarrow \boxed{d_\nu(n)} \longrightarrow d_\mu(n)d_\nu(n)x(n). \quad (3)$$

This model implicitly assumes narrow band signals.

The delay circuits corresponding to the different bits can be conveniently represented as a diagonal matrix,  $\mathbf{d}_\mu = \text{diag}[d_\mu(1), \dots, d_\mu(N)]$ .

Conceivably, the calibration parameters of interest could be extracted by coherently detecting the complex beam pattern signals at  $N_e$  different ground stations located at spatial points  $\{\vec{R}_i\}$ . The received signals,  $\{A(\vec{R}_i)\}$ , are generated from the sum of the complex weighted elemental signals,  $\{s_n(\vec{R}_i)\}$ , at  $A(\vec{R}_i) = K_i \sum_n a_n s_n(\vec{R}_i)$ . The sampling points would have to be selected to provide  $N_e$  linearly independent simultaneous equations. In principle these equations can be solved for the  $a_n$ 's. In practice this procedure is not feasible as the different sampling point positions  $\{\vec{R}_i\}$  and the relative values of the different complex propagation constants  $\{K_i\}$  have to be known precisely. Orthogonal transform coding of the elemental signals provides a dramatic simplification as all the encoded interference patterns are sampled at *a single receiver point*. The orthogonal transform assures that the simultaneous equations are linearly independent (invertible). As there is only one propagation constant  $K$ , its value need not be known to determine the relative elemental complex gains. Also, in the far field, the parameters of interest are obtained *with no knowledge of the distance to the single receiver point*.

## II. THE CCE ALGORITHM

The CCE algorithm encodes coherent signals from the elements of an analog beamformed phased array system using controlled switching of the phase delay circuits. In general, CCE switching is dictated by matrix elements of an  $N \times N$  bipolar invertible matrix. The class of Hadamard matrices are optimal for this application as they are the only bipolar matrices that satisfy the minimum variance optimality criterion[1].

The overall CCE process requires two sets of transmissions. The first set of transmissions uses  $\mathbf{H}$  to control the switching. The second set of transmissions uses  $-\mathbf{H}$  to control the switching. The difference of the two signal vectors associated with the two transmissions is proportional to the bipolar matrix  $\mathbf{H}$ .

The first pair of encoded coherent signals is based on CCE with the  $\mu$ th delay circuit, with all other delay circuits switched out. These received transmissions are conveniently expressed as vectors  $\mathbf{Y}_{\mu 0}^F, \mathbf{Y}_{\mu 0}^R$ , with components,  $\{y_{\mu 0}^F(m), y_{\mu 0}^R(m)\}$ . The zero subscript on these vectors and their components reflects the fact that all delay circuits other than the  $\mu$ th are switched out. Here  $N$  bursts of coherent encoded signals corresponding to the  $N$  components of these vectors are transmitted and received for each calibration measurement. The  $m$ th received transmission of the first pair of CCE signals is represented by,

$$\begin{aligned} y_{\mu 0}^F(m) &= \sum_{n=1}^N D_{\mu}^F(m, n) s(n), \\ y_{\mu 0}^R(m) &= \sum_{n=1}^N D_{\mu}^R(m, n) s(n). \end{aligned} \quad (4)$$

The encoding coefficients  $D_{\mu}^F(m, n)$ ,  $D_{\mu}^R(m, n)$  are dictated by the status of the delay circuits that are switched according to rules, referred to herein as *Hadamard control rules*, based upon the matrix elements of an  $N$ th order Hadamard matrix:

$$\begin{aligned} D_{\mu}^F(m, n) &= \begin{cases} +1 & \mu\text{th circ, } n\text{th elem open} \\ & \text{if } H(m, n) = +1 \\ d_{\mu}(n) & \mu\text{th circ, } n\text{th elem closed} \\ & \text{if } H(m, n) = -1 \end{cases} \\ D_{\mu}^R(m, n) &= \begin{cases} +1 & \mu\text{th circ, } n\text{th elem open} \\ & \text{if } H(m, n) = -1 \\ d_{\mu}(n) & \mu\text{th circ, } n\text{th elem closed} \\ & \text{if } H(m, n) = +1 \end{cases} \end{aligned} \quad (5)$$

The differences of the encoding matrices are represented in component and matrix form as,

$$\begin{aligned} D_{\mu}^F(m, n) - D_{\mu}^R(m, n) &= H(m, n)(1 - d_{\mu}(n)); \\ \mathbf{D}_{\mu}^F - \mathbf{D}_{\mu}^R &= \mathbf{H}(\mathbf{I} - \mathbf{d}_{\mu}). \end{aligned} \quad (6)$$

Take the difference of received signal vectors  $\mathbf{Y}_{\mu 0}^F, \mathbf{Y}_{\mu 0}^R$  and decode by premultiplying by the inverse of the *same Hadamard matrix* that was used in the control switching at the remote site. In the

absence of noise, the decoded vector  $\mathbf{Z}_{\mu 0}$  is obtained,

$$\begin{aligned} \mathbf{Z}_{\mu 0} &\stackrel{\text{def}}{=} \mathbf{H}^{-1}(\mathbf{Y}_{\mu 0}^F - \mathbf{Y}_{\mu 0}^R) \\ &= \mathbf{H}^{-1}(\mathbf{D}_{\mu}^F - \mathbf{D}_{\mu}^R)\mathbf{S} = (\mathbf{I} - \mathbf{d}_{\mu})\mathbf{S}. \end{aligned} \quad (7)$$

The second transmission pair corresponds to CCE again with the  $\mu$ th delay circuit, while an additional delay circuit, say the  $\nu$ th, is permanently switched in on all of the elements. Here,

$$y_{\mu\nu}^{F,R}(m) = \sum_{n=1}^N D_{\mu}^{F,R}(m, n) d_{\nu}(n) s(n). \quad (8)$$

The resulting decoded signals in vector form are,

$$\mathbf{Z}_{\mu\nu} \stackrel{\text{def}}{=} \mathbf{H}^{-1}(\mathbf{Y}_{\mu\nu}^F - \mathbf{Y}_{\mu\nu}^R) = (\mathbf{I} - \mathbf{d}_{\mu})\mathbf{d}_{\nu}\mathbf{S}. \quad (9)$$

The  $N$  complex gains,  $\{d_{\nu}(n)\}$  are extracted from these decoded signals by taking the ratio of the decoded vector components,  $\{d_{\nu}(n) = [z_{\mu\nu}(n)]/[z_{\mu 0}(n)]\}$ . Repeating the above process for all of the  $\nu \neq \mu$  delay circuits determines the subset of  $N(p-1)$  complex gains,  $\{d_{\nu}(n)\}$ .

The  $N$  values of  $\{d_{\mu}(n)\}$  have yet to be determined. These are determined by repeating the procedure described above with the  $\mu$ th circuit permanently switched in on all of the elements, while any other circuit is used for the CCE. For example, if the  $\zeta \neq \mu$ th delay circuit is used in encoding, the  $\{d_{\mu}(n)\}$ 's can be determined from,  $d_{\mu}(n) = [z_{\zeta\mu}(n)]/[z_{\zeta 0}(n)]$ . Now that all the  $\{d_{\mu}(n)\}$  are known, the straight through signals,  $\{s(n)\}$ , are determined from,  $s(n) = [z_{\mu 0}(n)]/[1 - d_{\mu}(n)]$ . Hence the complete data set of  $Np$  delay circuit complex gains plus the  $N$  straight through signals are obtained with  $N(p+2)$  CCE paired transmissions.

### III. VARIANCE OF PARAMETER ESTIMATES

The calibration algorithms considered herein depend on coherent detection of demodulated signals. The classical additive white Gaussian noise (AWGN) model that is typically used in performance analyses of statistical estimation problems is not appropriate for coherent detection analyses due to inherent analytical divergences. These difficulties are resolved herein by choosing a noise model that is both physically more appropriate for the coherent detection process, and is not hampered by analytical divergences. In the coherent detection process the demodulated signals are both bandlimited and energy limited. Our coherent detection systems noise model is referred

to as additive truncated Gaussian noise (ATGN). ATGN is represented by independent amplitude and phase random variables with probability density functions (pdf's) characterized respectively by a truncated Rayleigh amplitude distribution and a uniform phase distribution. In our simulations, the ATGN noise energy is truncated at  $\alpha E_{max}$ , with  $\alpha = 0.9$ , corresponding to 90% of the maximum deterministic signal energy.

In our analysis we have also shown that the expressions for the variances of the elemental complex gain estimates have asymptotic expansions that are valid in the SNR regime commensurate with typical satellite link budget power levels [1]. The asymptotic forms of the SE, and CCE variances, with  $E_{max}$  representing the maximum allowable single element power are given by,

$$\mathbf{V}^{SE}(d_\mu(k)) \sim \frac{\sigma^2}{E_{max}} + \left[ |d_\mu(k)|^2 + \frac{\sigma^2}{E_{max}} \right] \sum_{m=1}^{M_c} m! \left[ \frac{\sigma^2}{E_{max}} \right]^m;$$

$$\mathbf{V}^{CCE}(d_\mu(k)) \sim \frac{2\sigma^2}{NE_{max}|1-d_\nu(k)|^2} + \left[ |d_\mu(k)|^2 + \frac{2\sigma^2}{NE_{max}|1-d_\nu(k)|^2} \right] \sum_{m=1}^{M_c} m! \left[ \frac{2\sigma^2}{NE_{max}|1-d_\nu(k)|^2} \right]^m \quad (10)$$

These asymptotic series have to be truncated at an integral order  $M_c < \{E_{max}/\sigma^2\}$  in order to maintain local convergence within the errors indicated above. A cutoff value of  $M_c \sim 9$  associated with a 10dB single element SNR is quite effective for practicable system parameters. If  $M_c$  were extended to values  $\gg E_{max}/\sigma^2$ , the series would ultimately diverge. We note that the expansion parameter for the CCE variance has a factor of  $|1 - d_\nu(k)|^2$  in the denominator. As

$$|1 - d_\nu(k)|^2 = \begin{cases} 4 & \text{for } \pi \text{ phase shift} \\ 2 & \text{for } \pi/2 \text{ phase shift} \\ 0.04 & \text{for } \pi/16 \text{ phase shift} \end{cases}, \quad (11)$$

it is beneficial to perform the CCE encoding with the largest possible phase shifter circuit.

#### IV. COMPARISONS: THEORY AND SIMULATIONS

We now compare the asymptotic forms of the theoretical estimation variances for the SE and CCE algorithms given by Eq. (10), with MC simulations. For these examples, the errors induced by the truncation of the asymptotic expansions are so small that we truncated the expansions at their 9th order expansion terms. Fig. 1 illustrates these comparative results. Hadamard control matrices, as required, are used in the CCE simulations. For these simulations, we have used a value

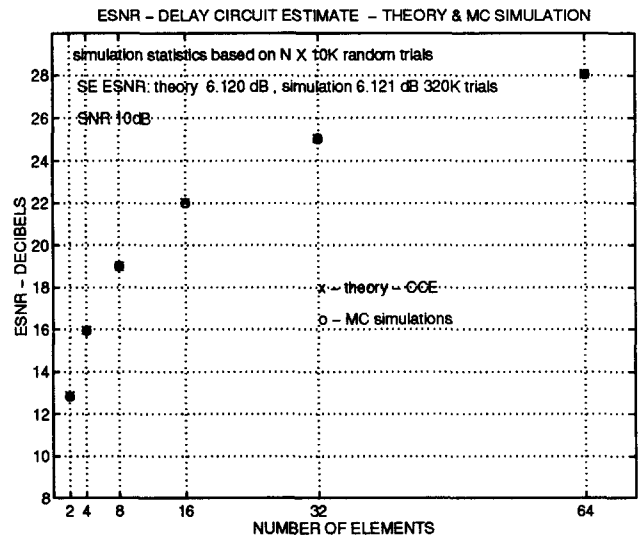


Figure 1: Theory vs. MC simulation of the effective estimation SNR (ESNR) for the estimate of the complex gain of the  $\pi/2$  phase shift circuit.

of the single element SNR which is defined as the ratio of received single element power to the receiver noise power,  $E_{max}/\sigma^2$  of 10 dB. Both the theoretical and simulation results given in the figure correspond to the ESNR for the complex gain estimates of the  $\pi/2$  phase shifter delay circuit. The *Monte Carlo* (MC) simulation and theoretical results for the single elements ESNR's of the  $\pi/2$  phase shifter are 6.121 and 6.120 dB respectively. These results illustrate the dramatic increase in the ESNR's that can be obtained using the orthogonal codes for control codes in the CCE process.

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