Echo Cancellation in DSL Systems

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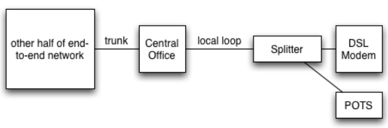

Fig. 1. Typical DSL Network

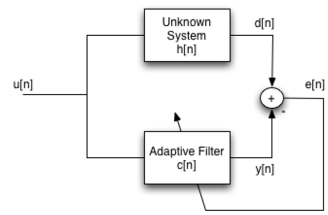

Fig. 2. Adaptive System

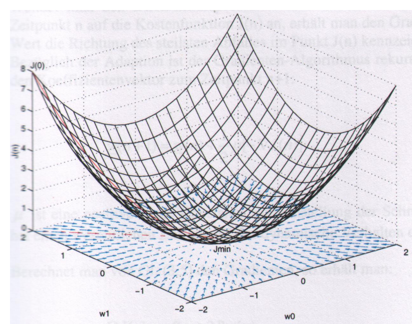

Fig. 3. Mean Squared Error Surface ($N = 2$), taken from [6]

***Abstract*— This paper gives a short review about echo cancellation techniques used in DSL systems and a short comparison between them. It should also give an overview about the basics and algorithms of echo cancellation and why it is used in Digital Subscriber Line systems.**

I. Introduction - Adaptive Systems and DSL

Digital Subscriber Line Technology - short DSL - uses the local loop of a telephone network, which was originally built up for POTS (Plain Old Telephone System) communication and signaling, to transport data at high data rates. This is possible by using the frequency band above the speech band ($300\,Hz$ to $3400\,Hz$).
A typical DSL network consists of the Central Office, where the data is prepared to transfer it over the local loop and a DSL modem at the telephone subscriber side. Using a higher frequency band causes an increase of damping, therefore the available data rate depends on the length of the local loop. A typical DSL network can be viewed in figure 1

As the conditions of the telephone network may change over the time and also depends on the location of the modem, we prefer an adaptive filter to have the possibility to adjust the filter parameters in case of a change.
The principle of an adaptive filter is to start at a specified condition (a specified impulse response) and then to find a good approach of the system by computing the impulse response iteratively. Several adaptive algorithms were developed, each of them with a special performance for a special field of applications. You can see a typical adaptive filter in figure 2

The most common implementation of echo cancelers uses the LMS algorithm. It adapts to a solution minimizing the mean square error $J(c) = e[n]^2$. The mean squared error surface is a $N+1$-dimensional paraboloid (N represents the order of the impulse response). The optimal solution can be found at the bottom of the bowl, as you can see in 3.

To get the optimal solution, we have to initialize the coefficient vector $c[n]$ with initial values. Then we can go an iteration step to the optimal solution by computing the negative gradient of the mean square error, which points to the minimum of $J(c)$. If we do these iterations long enough, the coefficient vector converges to an optimal solution, the wiener hopf solution. For more informations please refer to [6]. The equations of the steepest descent are the following:

$$e[n] = y[n] - d[n] \tag{1}$$

$$c[n+1] = c[n] + \frac{1}{2} * \mu * (-\nabla J(n)) \tag{2}$$

is the so called step-size parameter. It is responsible for the speed of the adaption algorithm.

II. The Hybrid and the Echo Source

The local link from the DSL modem to the central office is a bidirectional two-wire loop. If we think of a sattelite link, it uses two separate unidirectional channels, which lead to a four-wire transmission. To convert between the two-wire and the four-wire system, a so called hybrid is used. Ideally a hybrid should translate directly from the one system to the other. Because of impedance mismatching problems the hybrids do not operate optimal. Unfortunately this leads to a coupling of some energy from the incoming branch to the outgoing branch and this leads to an echo on the other side of the communication loop (shown in figure 4).

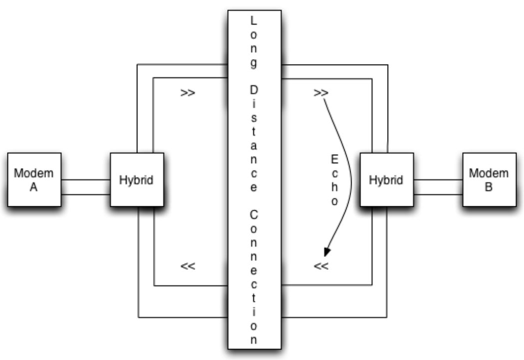

Fig. 4. The Echo Source

III. Motivation of Echo Cancellation

For full duplex operation, the passband can be split in two separate channels, one for upstream and one for downstream. This setup causes no echo because both channels are seperated. To increase the data transmission rates, it is useful to use the whole passband for both, transmission and reception, but therefore an echo cancellation has to be applied in the modem to remove the effects of the transmission on the reception. So echo cancellation is a duplexing method.

IV. Echo Path Model and Canceler Requirements

Data communication can be realized in full duplex mode with suiting echo cancellation techniques. At least $60\,dB$ of attenuation will be required to have an DSL transceiver working properly. For ADSL there is the need to implement different system at telephone subscriber side and central office side. The following subsection are following corresponding parts in [3].

A. The Echo Path

In case of frequency division duplex systems one may put passband filters in serial with the communication channel to filter out echo noise that is not in the same spectrum as the signal transmitted. Digital adaptive echo cancellers are put in parallel to the system echo path, as you can see in figure **??**.

The goal is to reproduce the echo and than subtract it from the received signal. For a better estimation one has to take into

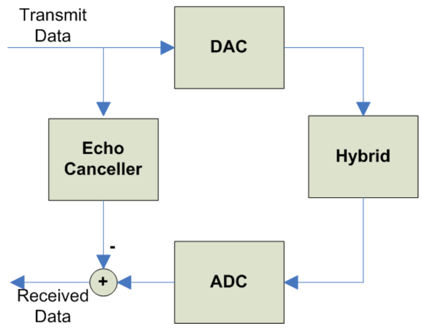

Fig. 5. Echo Canceller

account that there will be an echo path transfer function that is not constant over frequency. In the frequency range up to $10^5\,Hz$ the echo return loss varies between $-6\,dB$ $-24\,dB$. Low echo return loss values at low frequencies are caused by high impedances of twisted-pair loops. At high frequencies low echo loss values result out of leakage inductance in transformers.

The average echo return loss is given by:

$$K_{ep} = \frac{1}{f_c}\int_0^{f_c} |H_{ec}(f)|^2\, df \qquad (3)$$

$H_{ec}(f)$ ... echo path transfer function
f_c ... corner frequency is the bandwidth of interest.

Examples given in [3] find 18.5 dB for $f_c = 500\,kHz$ with the worst echo return at 6 dB as mentioned before.

Introducing filters in the model one might write as follows:

$$K_{ec} = \frac{1}{f_c}\int_0^{f_c} |G_t(f)H_{ec}(f)G_r(f)|^2\, df \qquad (4)$$

$G_t(f)$ ... transmit filter frequency response
$G_r(f)$ ... receiver filter frequency response

The echo path impulse response are quite different from system to system, but it is very likely to have a rapidly changing start and slowly decaying tails. This behavior is due to reflections that are caused by terminal impedance mismatches and bridged taps in the initial phase. Later on the primary inductance relates to the slowly changing tail. One should take care of having round trip times so that the initial reflections may die out.

RC-networks can be used to model twisted-pair loop impedances with a conductance in series to a resistor and a resistor parallel to these two. For such a model one is interested in th resonant frequency ω_0 and the damping factor α. The length of the tail can be reduced by a smaller primary inductance or a digital highpass $H(z) = 1 - z^{-1}$. It has to be taken into account that the removal of low frequency energy will influence the system performance in a NEXT dominated environment.

The required echo cancellation quality can be expressed in terms of cancellation level. The length of the echo canceller may also underly constraints. The cancellation level can be calculated as:

$$K_{EC} = SNR + K_{ch} - K_{ep} \tag{5}$$

K_{ch} ... average channel loss
K_{ep} ... average echo path loss
SNR ... required signal-to-residual noise floor ratio

Having computed the required echo cancellation, we may now determine the time that has to be spanned:

$$K_{EC} = -10log_{10}\frac{\int_{t_s}^{\infty} h_{EC}^2(t)dt}{\int_0^{\infty} h_{EC}^2(t)dt} \tag{6}$$

The number of taps at a given input/output rate with t_s is determinde via K_{EC}.
ie.: 128 taps, 400kHz, $t_s < 320\mu s$

Echo canceller filter coefficients are estimated and updated using the LMS algorithm.

$$H_{k+1} = H_k + \mu A_k(y(k) - H_k^T A_k) \tag{7}$$

They are trained in half duplex mode at startup for faster convergence.

Echo canceller precision can be a requirement. It might be an issue for processing in full duplex mode while working with small step sizes of in the adaption of filter coefficients. In a useful setup the noise (its variance) introduced by the filter must not be larger than that one introduced by devices like the ADC.

B. Asymmetrical Cancellation

For asymmetric setups there will be different conditions at the central office and the telephone subscriber side. This is due to asymmetrical throughput condition, transmit spectra and different sampling rates. ADSL is a typical setup where different sides of the system require different echo cancellers.

V. Cancellation Algorithms for Data Transmission

[2] compares AEC to echo cancellation in Data Transmission and is somewhat based on Material that can be found in [1]. The text below will focus on parts dealing with Data Transmission.

Data transmission is mainly done on two wire connections due to high cost of copper wire. For an overview of alternative technologies [4] talks about different solutions: Fiber To The Home (FTTH), Coax or Fiber-Coax-Hybrids, Wireless Systems and Satellite Services.

Motivation: Assuming $10\,dB$ of attenuation of feedthrough of the hybrid due to lacking knowledge of two-wire impedance. For $50\,dB$ channel loss and equal signal power

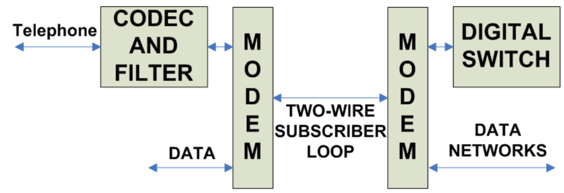

Fig. 6. Application

on both sides of the line the local feedthrough (echo) would be $40\,dB$ higher than the signal to be received. Aiming for $20\,dB$ SNR we are have to attenuate the echo by $60\,dB$.

Timing constraints - different sampling rates at input and output:
Transmitted signal:

$$s(t) = \sum_m C_m g(t - mT) \tag{8}$$

where $g(t)$ is the transmitted pulse shape and C_m are the transmitted data symbols. T is the time between two symbols. Passing the signal through a filter with a given echo response H_e we can rewrite $s(t)$ to $r(t)$ as the filter response to $s(t)$:

$$r(t) = \sum_m C_m h(t - mT) \tag{9}$$

For reconstruction of this signal in the echo canceller for an integer multiple of T as RT we define: $r_l(l) = r((i + \frac{l}{R})T)$ with i as the data symbol epoch while l from 0 to $R - 1$ represents the sample chosen in this epoch.

We can redefine the echo pulse response to generate the replica.

$$h_i(l) = h((i + \frac{l}{R})T) \;\; ... \;\; 0 \le l \le R - 1 \tag{10}$$

$r_i(l) = \sum_m h_m(l) C_{i-m}$ ... as samples of the received echo
$r_i = \sum_{m=0}^{n-1} C_{i-m} a_m$ ... $0 \le i < n - 1$ ... a as filter coefficients.
This FIR filter approximates the echo response.
Now there are R independent cancellers that will get the same reference input within an symbol epoch.
Example: Receive far-end data plus hybrid-echo. Decimate this input down to R signals at symbol data rate. Cancel the echo independently in each channel and then receive the far-end data after recombination of channels.

A. Design Consideration

There are two main goals for which we have to find a trade-off: Speed of Adaption vs. Accuracy of Cancellation These two goals typically counteract as a higher precision can be reached with some asymptotic behavior which could be called the inverse of Speed of Adaption. It has to be taken into account that it is the fast startup that has to be worked on - it is not that there are problems with a system already running a long time. The echo path transfer function will not vary too fast. Anyway the speed of echo cancellers might not be too critical. Ie.: it is possible to implement simple stochastic gradient descent algorithms.

Notation:
filter coefficients: $a^T = [a_0, a_1, ..., a_{n-1}]$
reference input: $y_i^T = [y_i, y_{i-1}, ..., y_{i-n+1}]$
echo path impulse response: $h^T = [h_0, h_1, ..., h_{n-1}]$
received signal: x
uncancable error (echo with delay exceeding the number of canceller coefficients: $v_k = \sum_{m=n}^{\infty} h_m y_{k-m} + x_k$
canceller error: $e_k = \sum_{m=0}^{\infty} h_m y_{k-m} - \sum_{m=0}^{n-1} a_m y_{k-m} + x_k = (h-a)y_k + v_k$

Assuming the reference signal y_i and the near-end talker v_i to be jointly wide-sense stationary we define: $p = E[v_i y_i]$, $\Phi = E[y_i y_i^T]$, $R_j = E[y_i y_{i+j}]$
So the optimum coefficient vector that minimizes $E[e_i^2]$ is: $a_{opt} = h + \Phi^{-1} p$ $\Phi = R_0 I$ ... I is the identity matrix $p = 0$ ... with mutually uncorrelated reference samples, without correlation of reference and received signal: $a_{opt} = h$

One way to come close to having a echo impulse response equal to the filter coefficients is to choose the number of coefficients large enough to find a suiting approximation. The most common algorithm for cancellers are stochastic gradient algorithms (LMS).

The expectation is usually not known in practice - so we leave it out. The replacing quantity has the same Expectation, but it is a different random variable - so the estimation is unbiased but noisy.
Stochastic gradient algorithm:

$$a_i = a_{i-1} - \frac{\beta}{2} \nabla_a [e_i^2] \tag{11}$$

where β is the step size for convergence control.

Via coefficient vector trajectories one can define the evolution of the error over iterations. Evaluating this approach one will find that there are several constraints for the matrix $\Phi = R_0 I$ with $R_j = E[y_i y_{i+j}]$: symmetric, Toeplitz(Toeplitz matrix is a matrix which has constant values along negative-sloping diagonals), non-negative definite($\Re[x^* A x] > 0$).
The eigenvalue spread of Φ can be related to th power spectral density:

$$minS(\omega) < \lambda_j < maxS(\omega) \tag{12}$$

$\lambda_{max} \rightarrow maxS(\omega)$
$\lambda_{min} \rightarrow minS(\omega)$
A large ratio between maximum and minimum spectrum of the reference signal leads to slow convergence of average coefficient vector trajectories. A small ratio lets the trajectories converge fast. For the theory of fluctuation of trajectories about the average [1] gives a detailed analysis. There is a mixing condition that needs to be fullfilled so that convergence can be guaranteed. This condition is that the reference signal has a non-zero power spectrum up to half the sampling rate.

B. Further Adjustments

1) Nonlinear Echo Cancellation: The echo cancellation algorithm presented above deals with echo path impulse responses as linear combination of the reference signal. Converters are the main sources for nonlinear noise. Although the variance may be very small one has to take into account that we are targeting up to 60 dB of cancellation.

Detailed analysis of this problem can be found in [1]. Volterra series can be shown to be capable of providing perfect cancellation with a finite number of filter coefficients, but it assumes that the echo can be represented as a function of a finite number of past transmitted data symbols [2].

2) Adaption Speedup: Fast initial convergence can be achieved through setting β to a suitable large value.
Another way is to use Lattice filter structures in the echo canceller, but there is no guarantee to find good solutions for every case. The lattice filter whitens the reference signal and therefore introduces some dependency on the input that can be hard to deal with.
Kalman filters could also be a choice to find a fast convergence. One nice advantage using this approach result from dealing with uniformly distributed noise from the converters. That is exactly what kalman filters are supposed to deal with, but again one will have to check if all prerequisites are met for kalman filtering theory.

VI. Conclusion

The main focus in echo cancellation lies on cancellation in speech transmission. Many algorithms exist, one has to find the correct one for the correct approach, but only some theoretical background for this problem can be found in literature.

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