Noncoherent Detection and Differential Detection

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Outline

- Introduction
- **Noncoherent Detection**
    - Optimal Noncoherent Detection of PSK Signals
- **Differential Detection**
    - Multiple Symbol Differential Detection (MSDD)
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    - Decision Feedback Differential Detection (DFDD)
- Summary

Introduction

Motivations :

- Coherent receivers require exact knowledge of the channel phase.
- The need for noncoherent detection if osscilator phase instability, uncertain and rapid changes propagation delay, fading, etc.

Selected Solutions :

- **Noncoherent Detection** is detection technique that does not use carrier recovery.
- For uncoded M-ary phase-shift keying (MPSK) signals, **noncoherent detection** is restricted to differential phase-shift-keying (DPSK) signals, and is called **Differential Detection**.

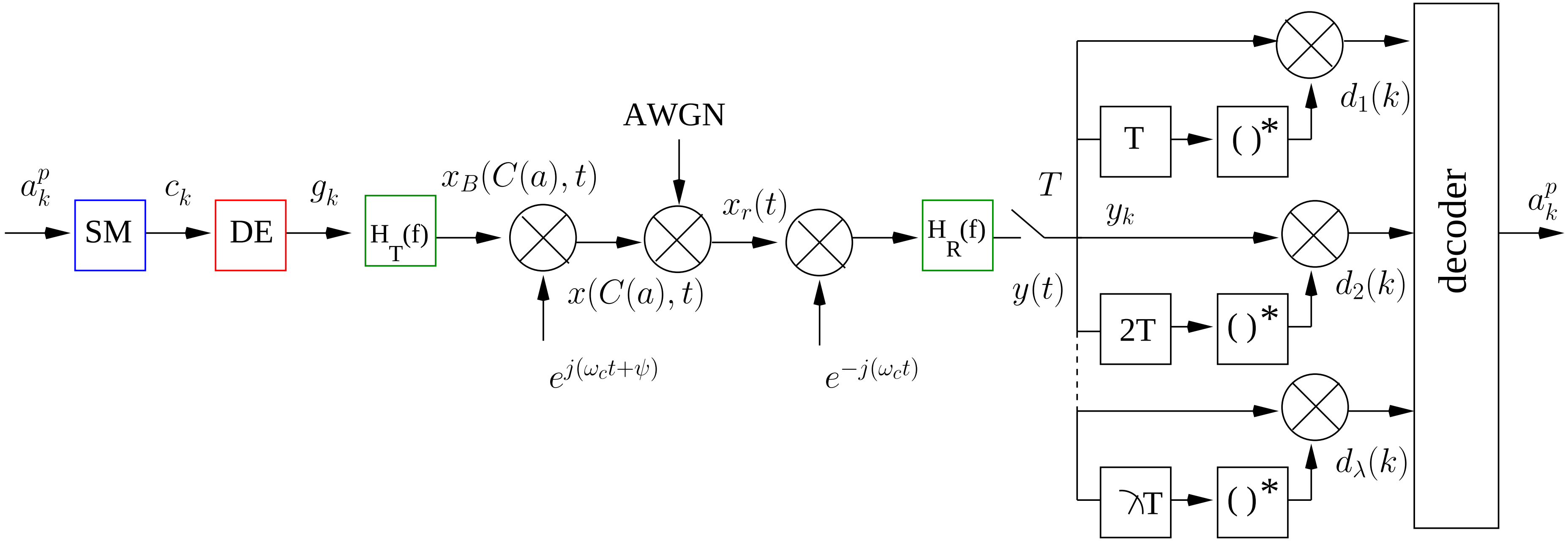
Noncoherent Detection

Motivations :

- Coherent receivers require exact knowledge of the channel phase.
- Thus, complex carrier phase and frequency synchronization circuits (e.g. PLLs) have to be implemented in the receiver.
- However, in fading environments or when low-cost local oscillators are employed acquisition and tracking of the carrier phase **may be difficult or even impossible**.
- In this case, **noncoherent detection** is a favorable choice.
- **Noncoherent detection** is a detection technique that can be implemented without using carrier recovery.
- Example: *Optimal Noncoherent Detection of PSK Signals*

Noncoherent Detection PSK — Block Diagram of Communications System

$$p = 2 \Rightarrow \pi/4 - QPSK; \; p = 3 \Rightarrow 8 - PSK$$



- $a_k^i \in \{0,1\}; 1 \le i \le p$; input p-bit words.
- $c_k = e^{j\Delta\Phi_k}$; symbol output of SM.
- $\Delta\Phi_k = (\pi/2^p)[\sum_{i=1}^{p} 2^{i-1}(2a_k^i - 1)]$.
- g_k; transmitted symbols;$g_k = c_k g_{k-1}$.
- $H_T(t)$ premodulation filter ($h_T(t)$).
- $x_B(C(a),t) = \sum_{i=0}^{L} c_i h_T(t - iT)$.
- $C(a) = [c_0, c_1, \cdots, c_L]; 0 \le t \le LT$.
- $x_B(C(a),t) \Rightarrow e^{j(\omega_c(t)+\psi)} \Rightarrow x(C(a),t)$.
- $\psi =$ initial phase of the modulator.
- $x_r(t) = x(C(a),t) + n(t)$; received signal.

Optimal Noncoherent Detection of PSK Signals (Cont'd)

Since, one is concerned with **noncoherent detection**,
so ψ is considered unknown to the receiver
and equally distributed in the $(0, 2\pi]$ interval.

Optimal Noncoherent Detection of PSK Signals (Cont'd)

The decoder should base its decision on the maximisation

$$f[x_r(t)|C(a)] = \int_{-\infty}^{\infty} f[x_r(t)|C(a),\psi]p(\psi)\,d\psi$$

$$= \Gamma \,\text{exp}\,\{-[\frac{1}{2N_0}\int_{-\infty}^{\infty}|x_r(t)|^2]\}\,dt \times \text{exp}\{-(\frac{1}{2N_0}\int_{-\infty}^{\infty}|x_B(C(a),t)|^2)\,dt\}$$

$$\times\, I_o\{|\frac{\int_{-\infty}^{\infty}[x_r(t)e^{-j(\omega_c t)}]x_B^*[C(a),t]\,dt|}{N_0}\}.$$

- I_o is zeroth order Bessel function ; Γ is a constant.
- Two exponential terms can be eliminated from the maximisation process.
- $y(t) = \int_{-\infty}^{\infty}[x_r(\tau)e^{-j(\omega_c\tau)}]h_T^*(t-\tau)d\tau$; output of filter $H_R(f)$ matched to $H_T(f)$.
- $\mathcal{R}_1[y,C(a)] = (|\sum_{k=0}^{L} y_k c_k^*|)$ with $y = [y_0, y_1, \cdots, yL]$; $y_k = y(kT)$.
- The (**monotically increasing funct.**) of I_o and the positive nature of $N_0 \Rightarrow$ **optimal Noncoherent Detection**

  $\simeq$ max $\mathcal{R}_1[y,C(a)] \Rightarrow (\mathcal{R}_1[y,C(a)])^2 = (|\sum_{k=0}^{L} y_k c_k^*|)^2$.

Optimal Noncoherent Detection of PSK Signals (Cont'd)

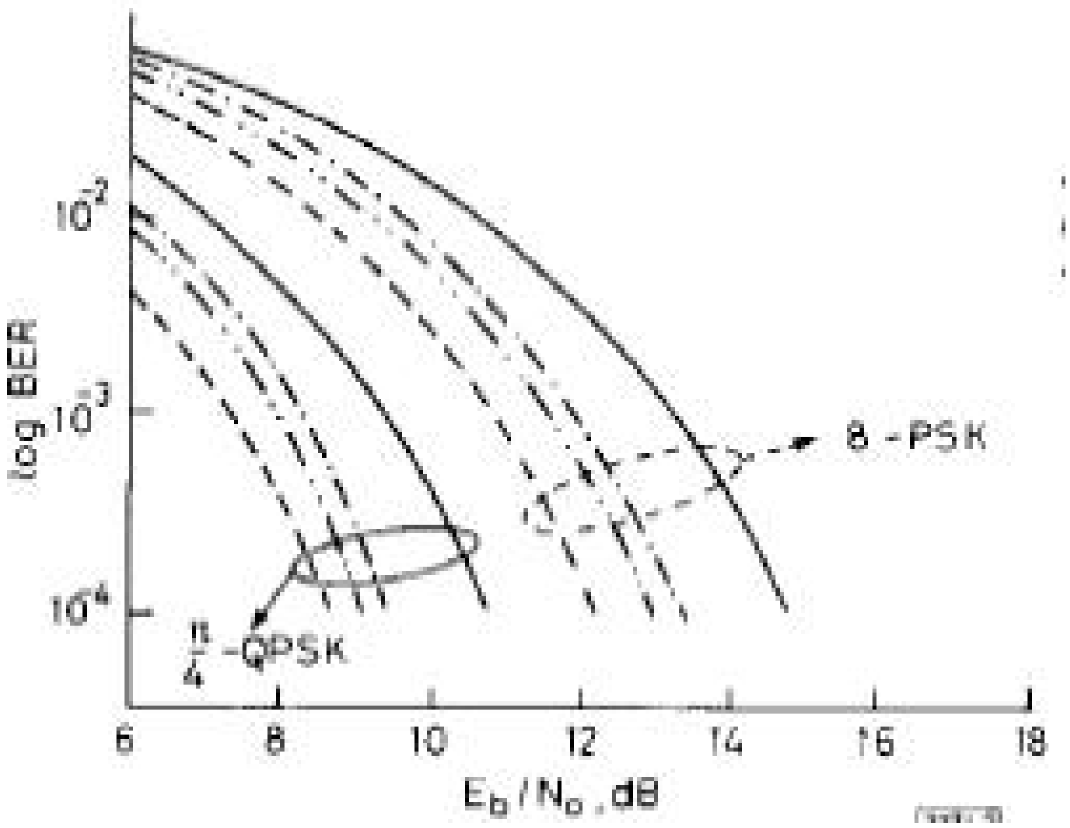
- Since l takes values between 1 and L, more than one (multiple) differential detectors are required for the implementation to the **optimal Noncoherent Detection**.
- The reduced complexity structure $\Rightarrow$ truncate the number of differential detectors to maximum of λ.

$$\varphi[y, C(a)] = \sum_{k=1}^{L} \sum_{l=1}^{\lambda} \{d_l^I(k) \cos[\Delta\Theta_l(k)] + d_l^Q(k) \sin[\Delta\Theta_l(k)]\}$$

- $d_l(k) = y_k y_{k-1}^*;\ d_l^I(k) = \text{Re}\ \{d_l(k)\};\ d_l^Q(k) = \text{Im}\ \{d_l(k)\}$ represents differential detector having delay element equal to lT second.
- $\Delta\Theta_l(k) = \text{Arg}\ [c_k c_{k-1}^*] = \Delta\Theta_k \oplus \Delta\Theta_{k-1} \oplus \cdots \oplus \Delta\Theta_{k-l}.$
- The transmission of λ symbols $c_0, c_1, \cdots, c_{\lambda-1}$, known to the receiver, before the transmission of the actual information. These symbols are used by the decoder as an initial reference.

Optimal Noncoherent Detection of PSK Signals (Cont'd)

- BER performance result for $\pi/4 - QPSK$ and $8 - PSK$.
- — $\Rightarrow \lambda = 1$ conventional differential receiver (symbol-by-symbol detector).
- $-.-. \Rightarrow \lambda = 2, 3$.
- $- - - \Rightarrow$ **coherent detection**.
- λ respresents truncated number of differential detectors used.



Courtesy of Makrikis et al (1990)

Differential Detection

Motivations :

- **Coherent receivers** requires **exact knowledge of the channel phase** for optimum performance.
- A **conventional (two symbol obervation) differential detector** uses the signal received in the previous symbol interval as a phase reference for the received signal in the current interval.
- For uncoded M-ary phase-shift keying (MPSK) signals, **noncoherent detection** is restricted to differential phase-shift keying (DPSK) signals, and is called **differential detection**.

Differential Detection Cont'd

- *Although differential detection **eliminates the need for carries acquisition and tracking in the receiver**, it suffers from a performance penalty (additional required SNR at a given bit error rate) when compared to ideal (perfect carrier phase reference) coherent detection.*

Questions :

- Is there a way of enhancing conventional differential detection technique so as to recover a portion of performance lost relative to that of coherent detection ?
- What is the the tradeoff between the amount of performance recovered and additional complexity ?

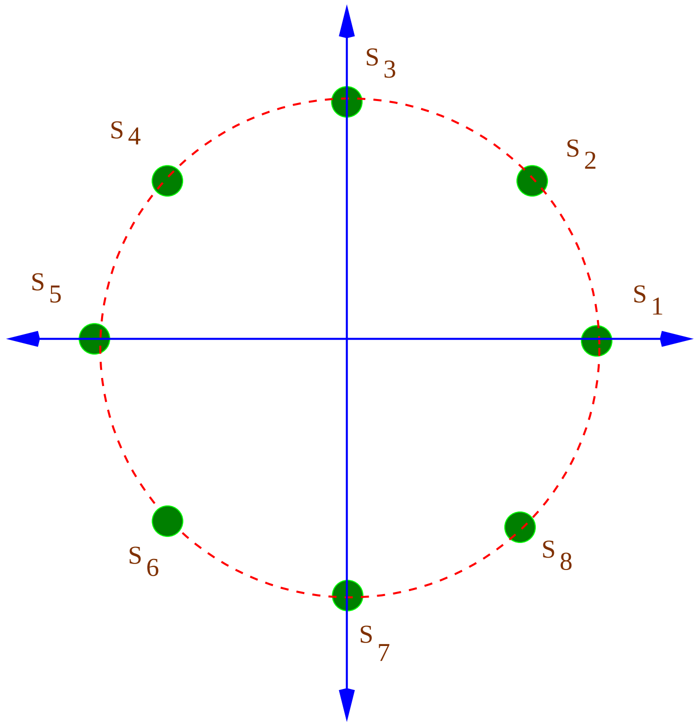
Differential Detection

PART 1

Multiple Symbol Differential Detection (MSDD) for Uncoded MPSK

MSDD for Uncoded MPSK

- Divsalar & Simon (1990)
- MPSK signals over an AWGN channel.
- MSDD performs *maximum-likelihood detection* of a block of information symbols based on a corresponding observation interval.
- **Transmitted signal** :
  - $s_k = \sqrt{2P} e^{j\phi_k}; \; kT \leq t \leq (k+1)T$
  - $P =$ constant signal power
  - $T =$ MPSK symbol interval
  - $\phi_k =$ transmitted phase (M unit circle)
- **Received signal** :
  - $r_k = s_k e^{j\theta_k} + n_k$
  - $n_k =$ zero-mean complex Gaussian noise
  - $\theta_k =$ phase introduced by channel $(-\pi, \pi)$



M-PSK modulation; M=8

ϕ_k takes on one of M values

$\beta_m = 2\pi m/M$

m=0,1,$\cdots, M-1$

MSDD for Uncoded MPSK (Cont'd)

N -length of received signal with assumption that θ_k is independent of k

$$\mathbf{r} = \mathbf{s}\mathbf{e}^{\mathbf{j}\theta} + \mathbf{n}$$

AWGN channel, a posteriori probability of $\mathbf{r}$ given $\mathbf{s}$ and θ

$$p(\mathbf{r}|\mathbf{s}, \theta) = \frac{\mathbf{1}}{(\mathbf{2}\pi\sigma_{\mathbf{n}}^{\mathbf{2}})^{\mathbf{N}}} \mathbf{exp}\,\{-\frac{\|\mathbf{r} - \mathbf{s}\mathbf{e}^{\mathbf{j}\theta}\|^{\mathbf{2}}}{\mathbf{2}\sigma_{\mathbf{n}}^{\mathbf{2}}}\}$$

where

$$\|\mathbf{r} - \mathbf{s}\mathbf{e}^{\mathbf{j}\theta}\|^{\mathbf{2}} = \sum_{\mathbf{i}=\mathbf{0}}^{\mathbf{N}-\mathbf{1}} |\mathbf{r}_{\mathbf{k}-\mathbf{i}} - \mathbf{s}_{\mathbf{k}-\mathbf{i}}\mathbf{e}^{\mathbf{j}\theta}|^{\mathbf{2}}$$

MSDD for Uncoded MPSK (Cont'd)

θ has been assumed to be uniformly distributed

$$p(r|s) = \int_{-\pi}^{\pi} p(r|s,\theta)p(\theta)\, d\theta$$

$$= \frac{1}{(2\pi\sigma_n^2)^N} \exp\Big(-\frac{1}{(2\sigma_n^2)} \sum_{i=0}^{N-1} [|r_{k-i}|^2 + |s_{k-i}|^2]\Big)$$

$$\times I_o\Big(-\frac{1}{\sigma_n^2}\Big| \sum_{i=0}^{N-1} r_{k-i} s_k^* - i\Big|\Big).$$

$\max_i |\sum_{i=0}^{N-1} r_{k-i}\ s_{k-i}^*|^2 \Rightarrow$ choose $\hat{\phi}$ if $|\sum_{i=0}^{N-1} r_{k-i} e^{-j\hat{\phi}_{k-i}}|^2$ is maximum

$$\eta = |\sum_{i=0}^{N-1} r_{k-i} e^{-j(\phi_{k-i} - \phi_{k-N+1})}|^2$$

- I_o is zeroth order Bessel function (**monotically increasing funct.**).
- Note that for MPSK, $|s_k|^2$ is **constant for all phases**.

MSDD for Uncoded MPSK (Cont'd)

- The decision statistic can be expressed as $\eta = |\sum_{i=0}^{N-1} r_{k-i} e^{-j(\phi_{k-i} - \phi_{k-N+1})}|^2$
- To resolve the problem of phase ambiguity, one should **differentially encode the phase information at the transmitter**

$$\phi_k = \phi_{k-1} + \Delta\phi_k ; \; \phi_{k-i} - \phi_{k-N+1} = \sum_{m=0}^{N-i-2} \Delta\phi_{k-i-m}$$

$\Delta\phi_k$ = input data phase corresponding to the kth transmission interval.

ϕ_k = the differentially encoded version.

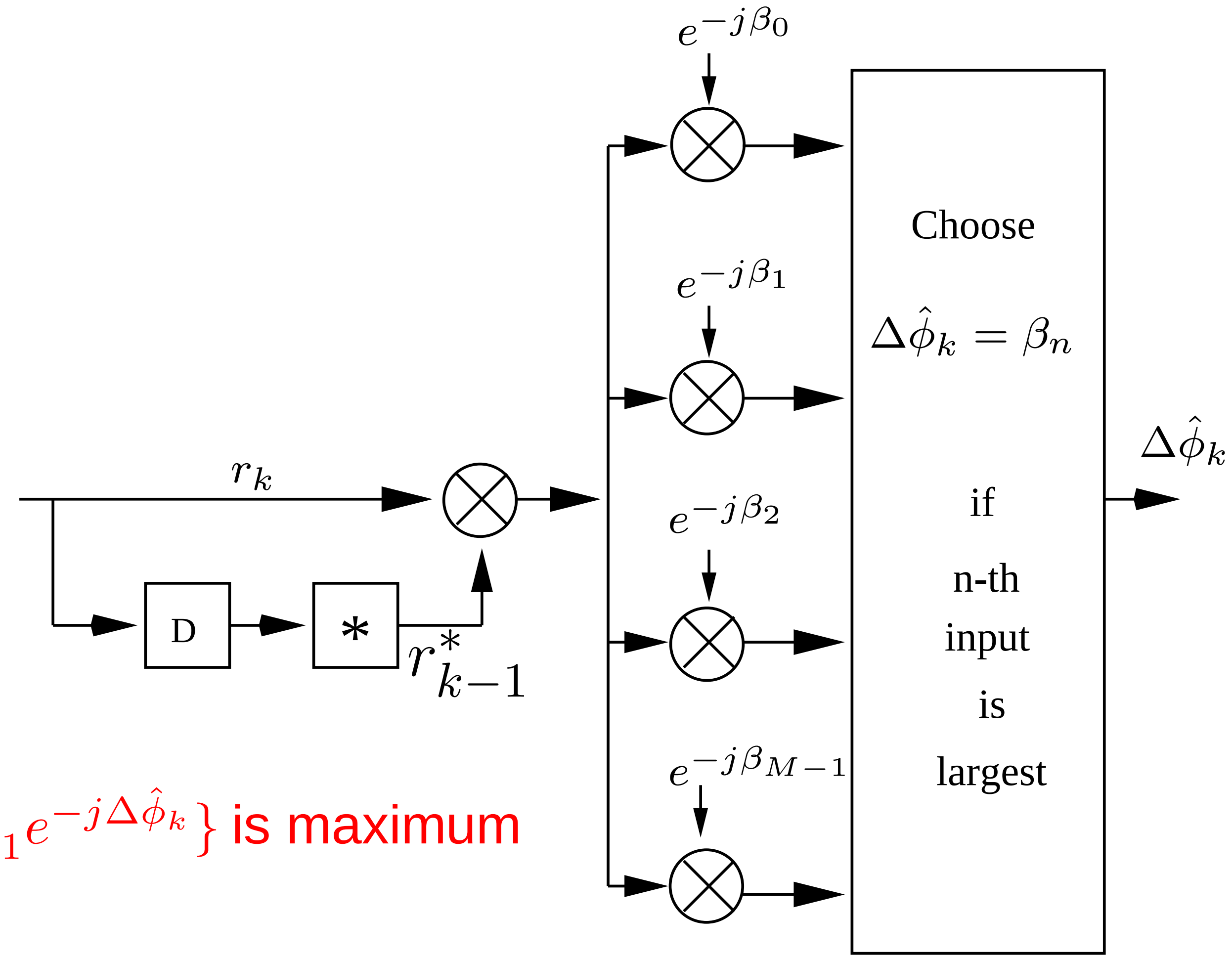
- **The decision statistics** :

$$\eta = |\mathbf{r}_{k-N+1} + \sum_{i=0}^{N-2} \mathbf{r}_{k-i} \mathbf{e}^{-\mathbf{j} \sum_{m=0}^{N-i-2} \Delta\phi_{k-i-m}}|^2$$

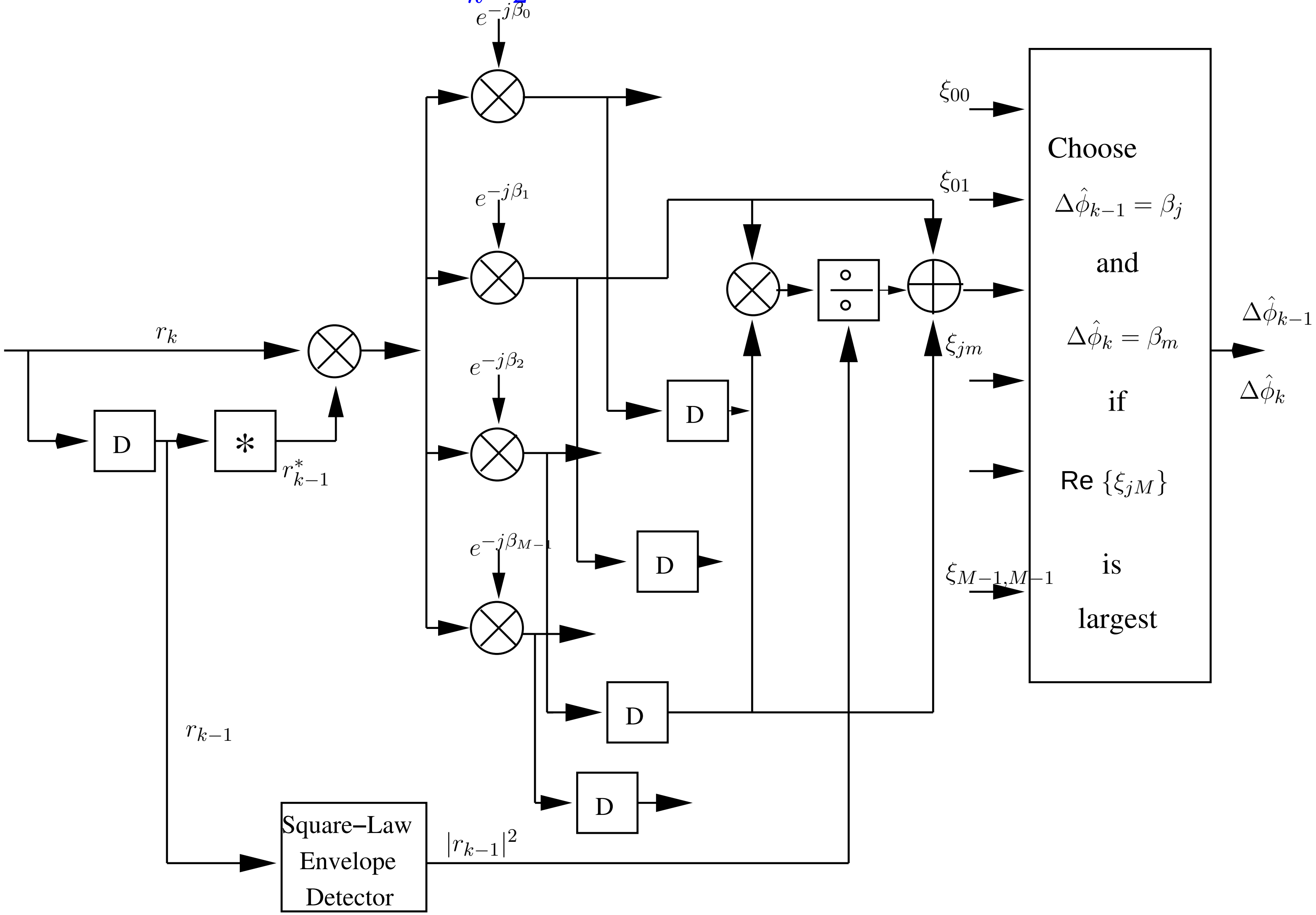
- MSDD is detector that makes a decision about a block of N consecutive PSK symboles based on $N+1$ received samples.
- The complexity **grows exponentially** with the sequence length !!

MSDD for Uncoded MPSK (Cont'd)

- $N = 2$.
- Conventional differential detector of MPSK.
- $\eta = |r_{k-1} + r_k e^{-j\Delta\phi_k}|^2 = |r_{k-1}|^2 + |r_k|^2 + 2\text{Re}\{r_k r_{k-1}^* e^{-j\Delta\phi_k}\}$
- choose $\Delta\hat{\phi}_k$ if $\text{Re}\{r_k r_{k-1}^* e^{-j\Delta\hat{\phi}_k}\}$ is maximum



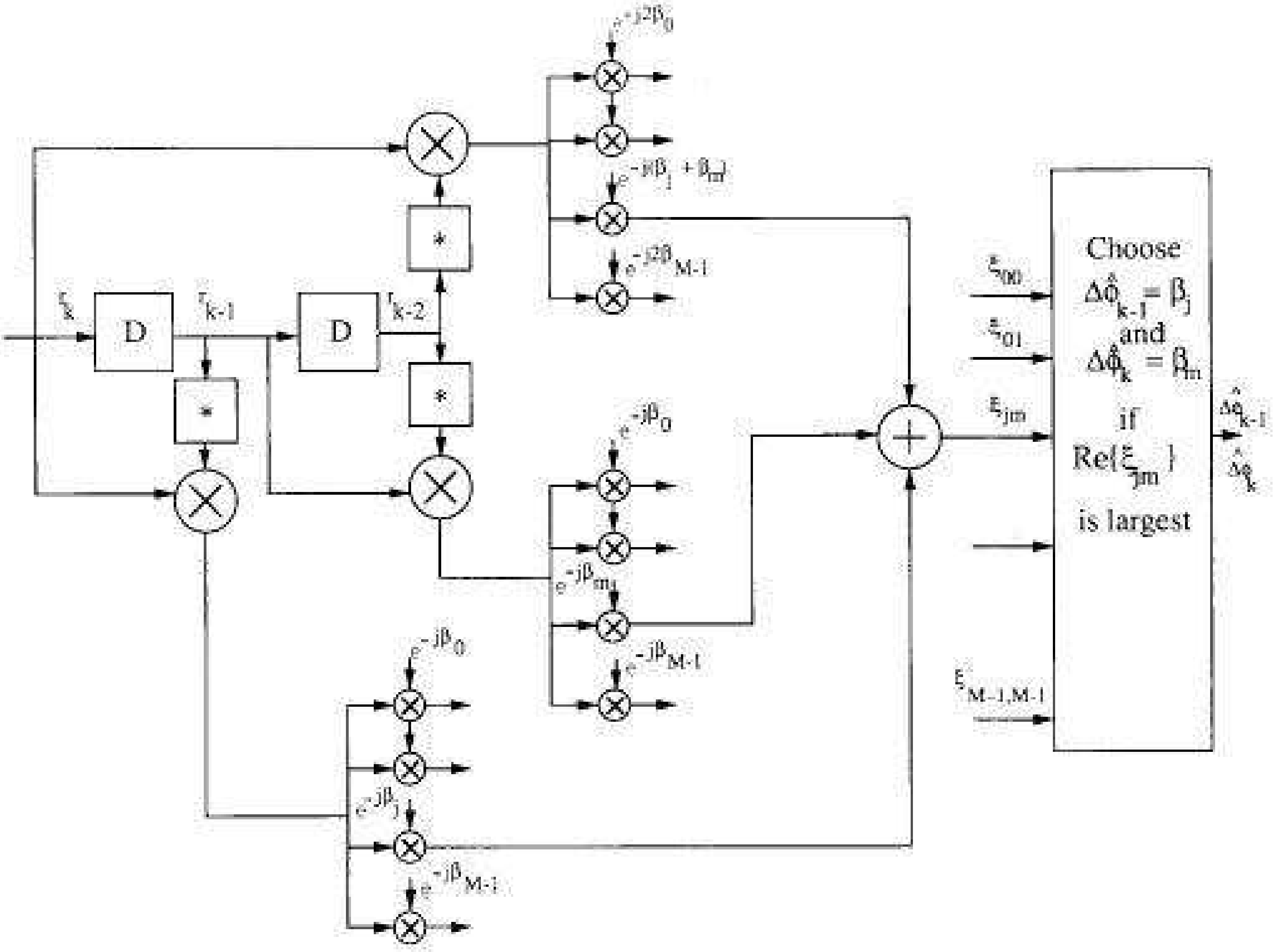
Choose $\Delta\hat{\phi}_k$ *and* $\Delta\hat{\phi}_{k-1}$ *if* $Re\{r_k r^*_{k-1} e^{-j\Delta\phi_k} + r_{k-1} r^*_{k-2} e^{-j\Delta\phi_k} + r_k r^*_{k-2} e^{-j(\Delta\phi_k + \Delta\phi_{k-1})}\}$ *is maximum.*



Serial Implementation with N=3 *Courtesy of Simon (1990)*

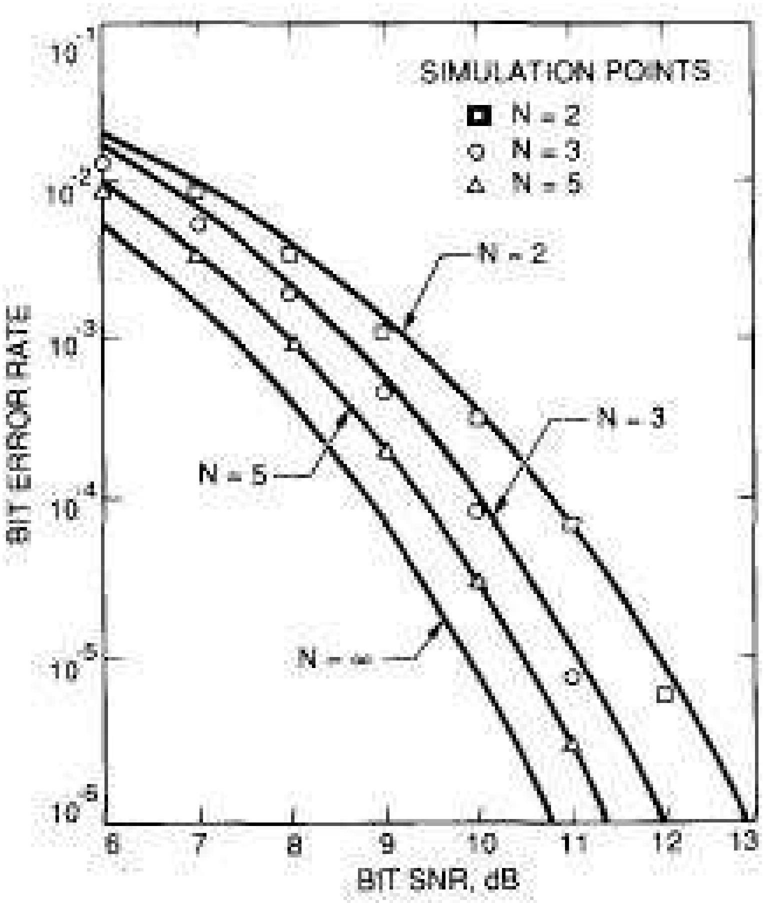
Choose $\Delta\hat{\phi}_k$ *and* $\Delta\hat{\phi}_{k-1}$ *if* $Re\{r_k r^*_{k-1} e^{-j\Delta\phi_k} + r_{k-1} r^*_{k-2} e^{-j\Delta\phi_k} +$

$r_k r^*_{k-2} e^{-j(\Delta\phi_k + \Delta\phi_{k-1})}\}$ *is maximum.*

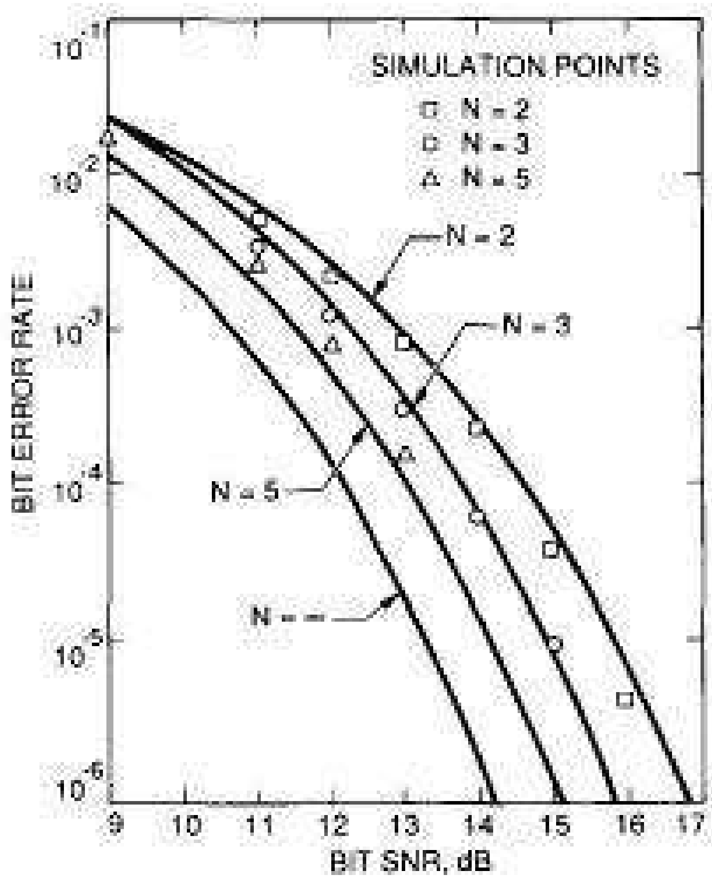


Parallel Implementation with N=3 *Courtesy of Simon (1990)*

MSDD for Uncoded MPSK (Cont'd)



BER versus E_b/N_0
for MSDD of MPSK; M = 4



M = 8
Courtesy of Simon (1990)

Differential Detection

PART 2

Fast Algorithm for MSDD

Fast Algorithm for MSDD

- Yingqun Yu and Zhaowu Chen (1999)
- To reduce the complexity of Simon's.
- The computational complexity **only grows linearly** with the lengh of the observed symbols.

Proposed solutions :

- *To define a decision statistic for each symbol based on the information of the past decided symbol sequence*.
- *A useful relation is established between successive symbol decision statistics, which not only makes symbol-by-symbol detection possible but also greatly reduced computational complexity*.

Fast Algorithm for MSDD (Cont'd)

- Assume the **carrier phase is constant** over the observation of the sequence.

$$\eta = |\mathbf{r_{k-N+1}} + \sum_{\mathbf{i=0}}^{\mathbf{N-2}} \mathbf{r_{k-i}} \mathbf{e}^{-\mathbf{j}\sum_{\mathbf{m=0}}^{\mathbf{N-i-2}} \boldsymbol{\Delta}\phi_{\mathbf{k-i-m}}}|^{\mathbf{2}}$$
$$\times |e^{j\sum_{m=1}^{N-2}\Delta\phi_{k-m}}|^2 \text{ (equal to 1)}$$

- if $N-1$ symbols, before the kth symbol have been determined to be $\Delta\hat{\phi}_k$ can be used to determined the kth symbol.
- A decision statistic for the kth symbol:
  $\eta = |\sum_{i=2}^{N-1} r_{k-i} e^{-j\sum_{m=1}^{i-1}\Delta\hat{\phi}_{k-m}} + r_{k-1} + r_k e^{-j\Delta\phi_k}|^2$

$$\lambda_{k,i} = \begin{cases} r_{k-i} e^{j\sum_{m-1}^{i-1}\Delta\hat{\phi}_{k-m}} & \text{for } 2 \le i \le N-1, \\ r_{k-1} & \text{for } i = 1. \end{cases}$$

$$\eta_k(\Delta\phi_k) = |\sum_{i=1}^{N-1} \lambda_{k,i} + r_k e^{-j\Delta\phi_k}|^2$$

Fast Algorithm for MSDD (Cont'd)

- A decision statistic for the kth symbol:

$$\eta \;=\; |\sum_{i=2}^{N-1} r_{k-i} e^{-j\sum_{m=1}^{i-1}\Delta\hat{\phi}_{k-m}} + r_{k-1} + r_k e^{-j\Delta\phi_k}|^2$$

$$\lambda_{k,i} = \begin{cases} r_{k-i} e^{j\sum_{m-1}^{i-1}\Delta\hat{\phi}_{k-m}} & \text{for } 2 \le i \le N-1, \\ r_{k-1} & \text{for } i = 1. \end{cases}$$

$$\eta_k(\Delta\phi_k) \;=\; |\sum_{i=1}^{N-1} \lambda_{k,i} + r_k e^{-j\Delta\phi_k}|^2$$

- Summary of decision algorithm:
  1. The initial value, i.e, $\lambda_{0,i}(i = 1, N-1)$ is 0.
  2. Use $\eta_k(\Delta\phi_k) \;=\; |\sum_{i=1}^{N-1} \lambda_{k,i} + r_k e^{-j\Delta\phi_k}|^2$ to compute the decision statistic for the kth symbol, and decide the kth symbol as $\Delta\hat{\phi}_k$ if $\eta_k(\hat{\phi}_k)$ is maximal.
  3. Use $\lambda_{k+1,i} = \lambda_{k,i-1} e^{j\Delta\hat{\phi}_k}$ for $2 \le i \le N-1$ and $\lambda_{k+1,i} = r_k$ to compute $\lambda_{k+1,i}$ $(i-1, \cdots, N-1)$ for the $(k+1)th$ symbol
  4. let $k = k+1$ and go to step 2.

Fast Algorithm for MSDD (Cont'd)

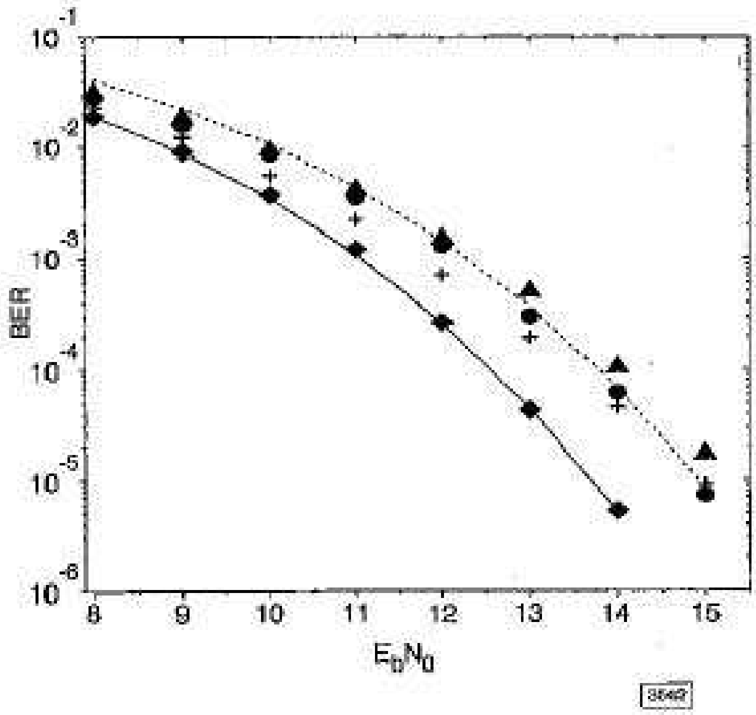


Fig. 2 *BER of uncoded 8PSK*

theoretical, constant phase ([2])
– – – – $N = 3$
——— $N = 10$
simulation, constant phase
● $N = 3$
◆ $N = 10$
Gaussian random walk phase, $\sigma^2 = 0.001$ rad^2
▲ $N = 3$
+ $N = 10$

- Results for $8PSK$
  - The carrier phase is either constant or given by the random-walk phase model.
  - For the **constant carrier phase** model, there is almost no E_b/N_0 performance loss compared with theoretical value.
  - For the random walk phase model some E_b/N_0 degradation is found, and there is more degradation for $N = 10$ than for $N = 3$.
- *Courtesy of Yingqun Yu (1999)*.

Differential Detection

PART 3

Decision Feedback Differential Detection (DFDD)

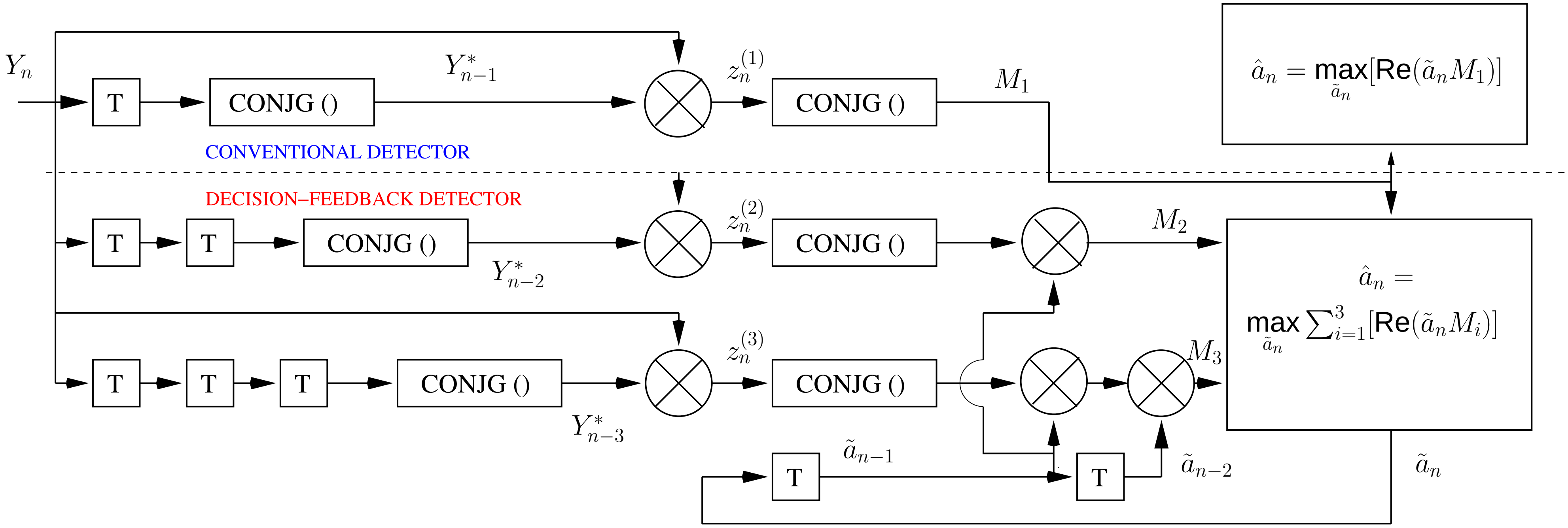
Decision Feedback Differential Detection (DFDD)

- Franz Edbauer (1992)
- Multiple differential feedback detection for binary ($2-DSPK$) and quaternary ($4-DSPK$).
- An improvement of BER performance can be achived if **symbol detectors with delay larger** than a symbol period are used and if **detected symbols are fed back** to the detection unit.
- The improvement is based on using L symbol detectors with delays of $1, 2, \cdots, L$ symbol periods and on feeding back detected PSK symbols.

Decision Feedback Differential Detection (DFDD)

- **Transmitted signal** :
  - $s_t = \sqrt{2S}\sum_{i=-\infty}^{\infty} c_i p(t - iT)$
  - S is the signal power.
  - T is the symbol period.
  - $p(t) =$ unit pulse of duration T
  - **The differentially encoded** $M - DPSK$ **symbol** $c_n = c_{n-1}a_n$ is generated by adding the **phase of symbol** $a_n = \exp(j\phi_i); (n-1) < T \leq nT$ to the phase of the previous symbol c_{n-1}.
  - $\phi_i = i2\pi/M; i = 0, 1, \cdots, M-1$ defines M phases of the M-PSK signal.
- **Received signal** :
  - $r(t) = s(t)\exp[j(\omega t + \psi)] + n(t)$ is passed through the matched filter with impulse response $h(t) = (1/T)p(T-t)$ and sampled at time nT yielding $y_n = \sqrt{2S}c_n\exp(j\psi) + n_n$.
  - $n_k =$ zero-mean complex Gaussian noise
  - $\psi =$ phase offset

Decision Feedback Differential Detection (DFDD) Cont'd



- **Minimimizing the quadratic "errors"** of the outputs of L symbol detectors of orders $j = 1, 2, \cdots, L$ and **decision feedback**.

Decision Feedback Differential Detection (DFDD) Cont'd

- The symbol detector of order j uses the jth previous symbol y_{n-j} as phase reference and performs the operation $y_n y_{n-j}^*$

$$z_n^{(j)} = y_n y_{n-j}^* = 2S a_n a_{n-1} \cdots a_{n-j+1} + n_n^{(j)}$$

- The metric, *represents the quadratic error sum of the detector outpus under the hypothesis that at time n symbol* $\tilde{a}_n$ *has been sent*, is given by

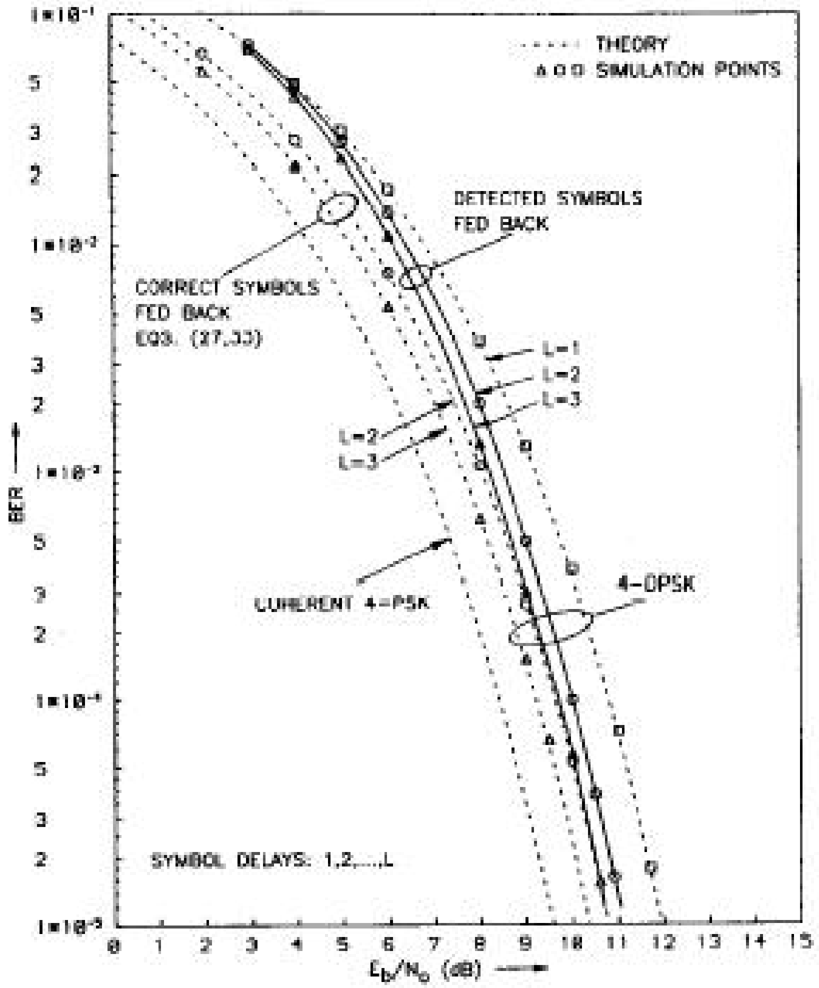
$$\eta = |z_n^{(1)} - 2S\tilde{a}_n|^2 + |z_n^{(2)} - 2S\tilde{a}_n\tilde{a}_{n-1}|^2 + \cdots + |z_n^{(L)} - 2S\tilde{a}_n\hat{a}_{n-1} \cdots \hat{a}_{n-L+1}|^2, \; L > 1$$

- **The decision rule** for $\tilde{a}_n$ is $\tilde{a}_n = \underset{\tilde{a}_n}{min}\{\eta\} = \underset{\tilde{a}_n}{max} \sum_{j=1}^{L} \{\tilde{a}_n M_j\}$

$$M_j = \begin{cases} z_n^{(1)*}, & j = 1, \\ \tilde{a}_{n-1}, \tilde{a}_{n-2}, \cdots, \tilde{a}_{n-j+1} z_n^{(j)*}, & j = 2, 3, \cdots, L. \end{cases}$$

- The conventional detector is a special case for $L = 1$.

Decision Feedback Differential Detection (DFDD) Cont'd



Bit error rate of the decision-feedback detector for 4-DPSK signals

with correct and detected symbols fed back $(L = 1, 2, 3)$. *Courtesy of Franz Edbauer (1992)*

Summary

- **Noncoherent detection** is a detection technique that can be implemented without using carrier frequency which the performance asymotically approached that of the coherent detection.
- **Differential detection** is an attractive alternative to coherent detection.
- MSDD does not require, however the ability to measure relative phase differences which eliminates the exact carrier phase recovery but the compuational complexity groys exponentially with the sequence length.
- Decision feedback differential detector is less complex than the MSDD based on maximum-likelihood detector.

References

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Thank you for your attention.

Questions ?