Sampling Requirements for Nonlinear System Identification

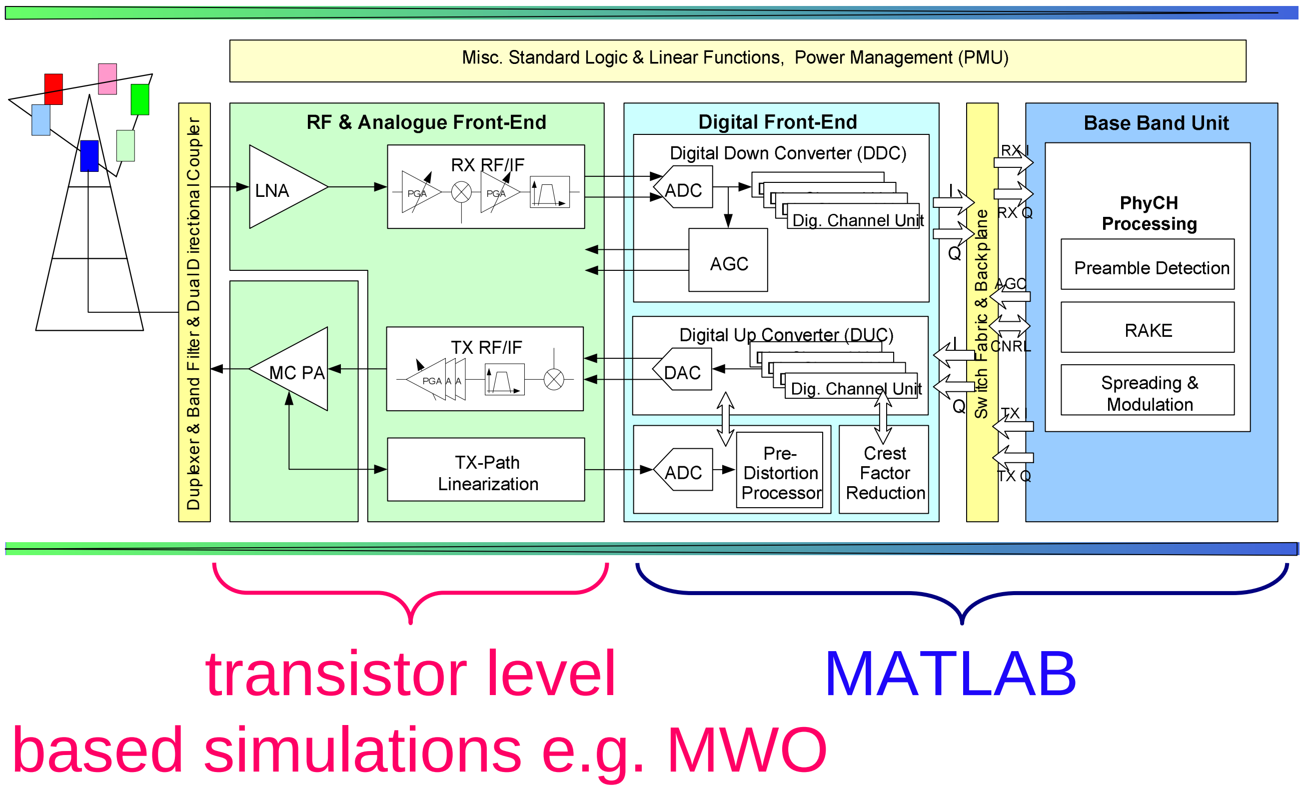
Peter Singerl

Christian Doppler Laboratory for Nonlinear Signal Processing
Graz University of Technology

Outline

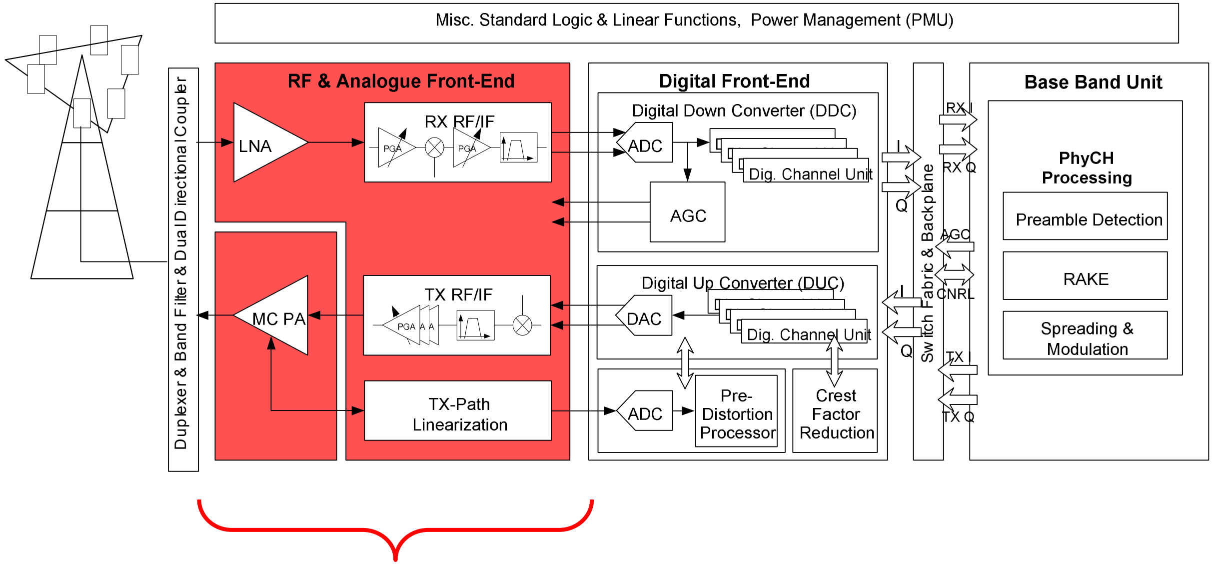
- Motivation
- Equivalence of Passband and Baseband Systems
- Nonlinear System Identification Setup (Systemmodeling, Predistortion)
- Complex Baseband Volterra Series Modelling
- Nyquist Sampling Regarding the Output Signal Bandwidth
- Nyquist Sampling Regarding the Input Signal Bandwidth
- Volterra Kernel Interpolation

Motivation



- No standardized interface between Baseband and RF
- Long simulation times
- Feedback loops difficult to handle

Motivation



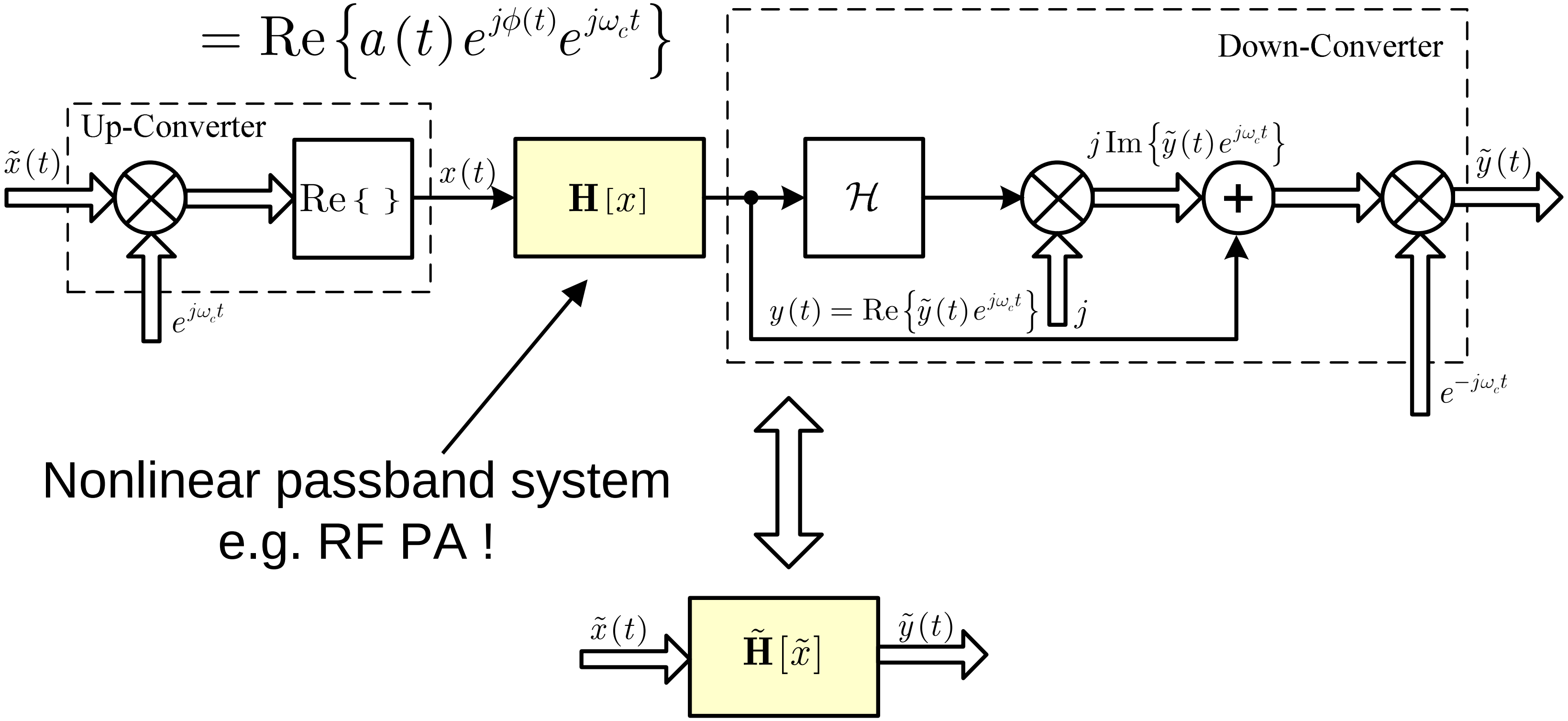
- Behavioral modelling (e.g. in MATLAB) for fast system simulations !

Efficient performance tests
(EVM, spectral masks, bit-error rate,...)

Equivalence of Passband and Baseband Systems

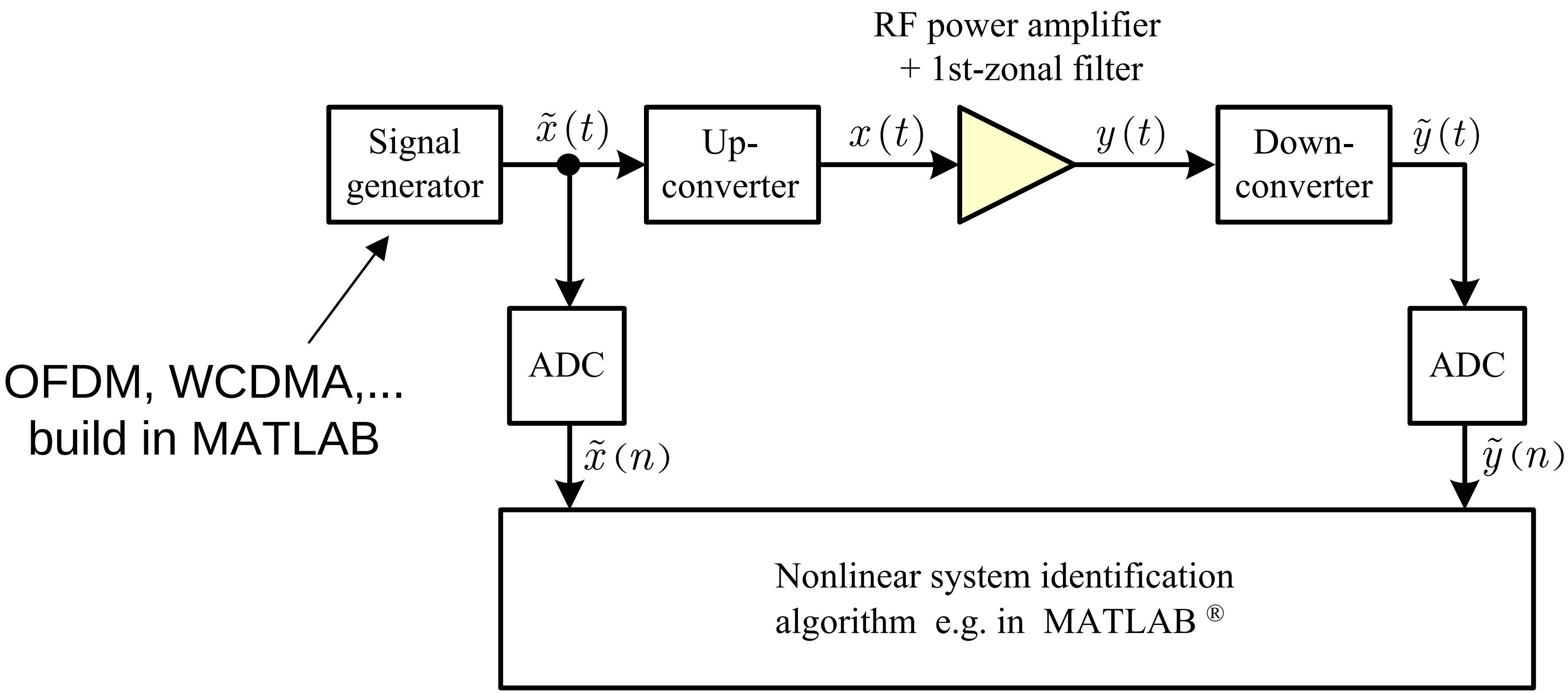
$$x(t) = a(t)\cos\left(\omega_c t + \phi(t)\right)$$

$$= \mathrm{Re}\left\{a(t)\, e^{j\phi(t)} e^{j\omega_c t}\right\}$$

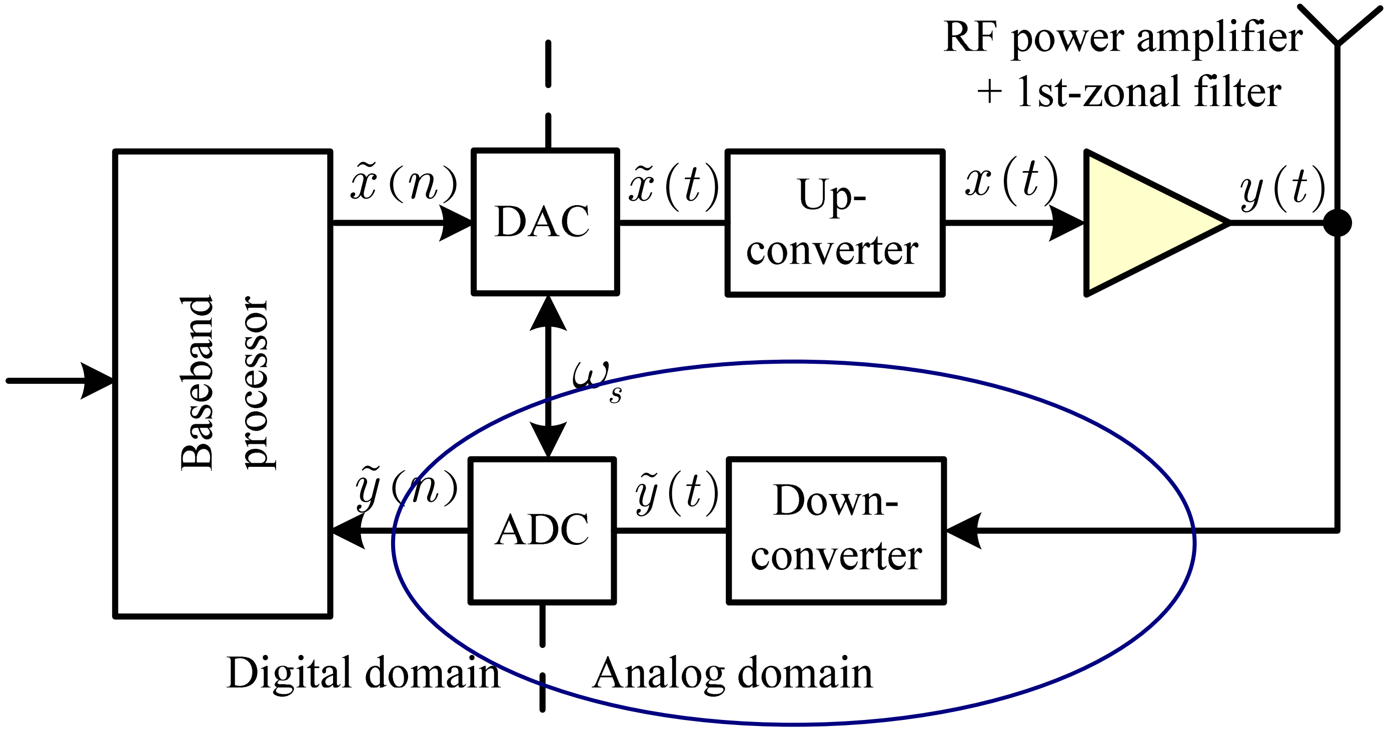


No carrier must be considered for simulation !

Nonlinear System Identification Setup (Off-line)



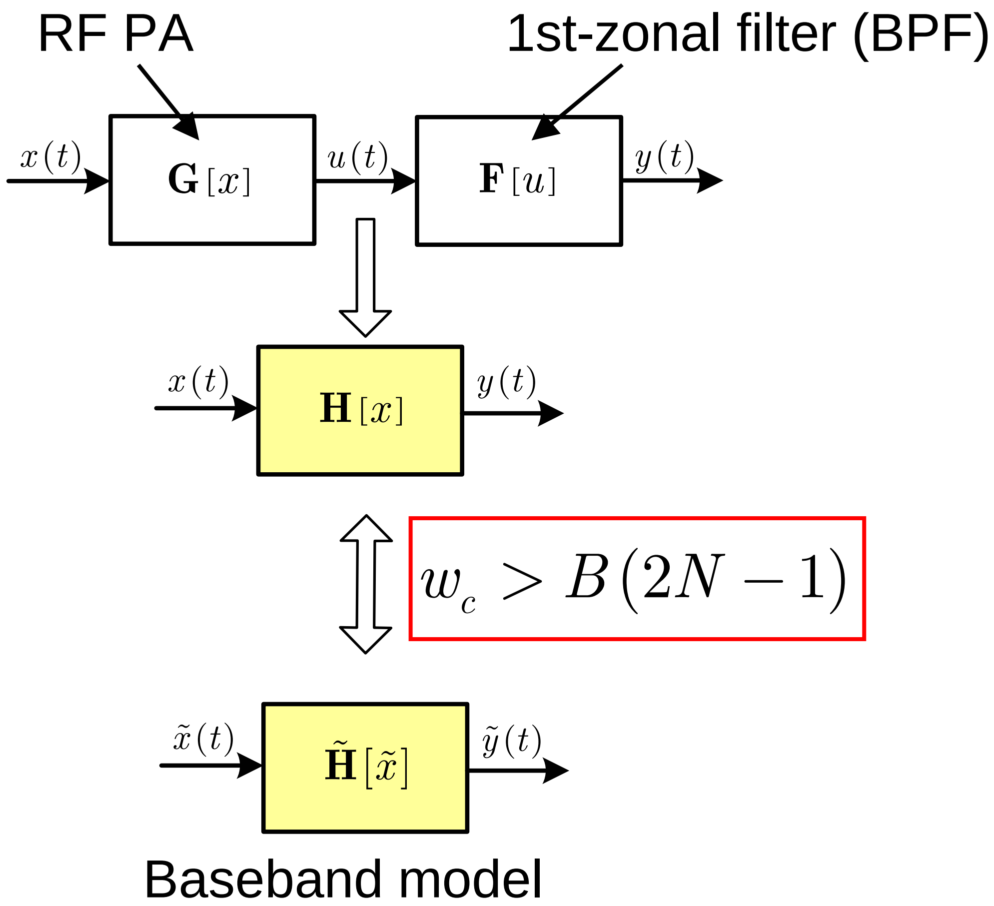
Nonlinear System Identification Setup

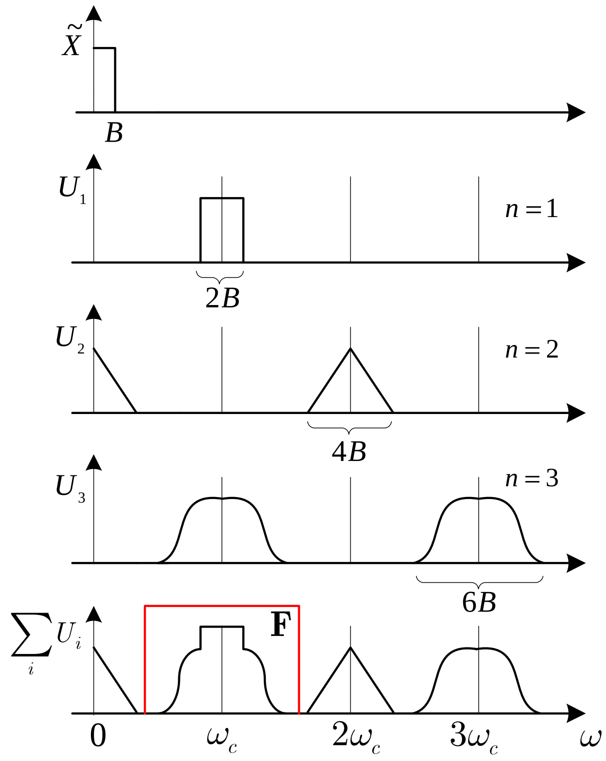


Feedback path (receiver) for system identification !

The sampling frequency of the RF PA model (predistorter) must be at least twice the output signal bandwidth of the RF PA !

Complex Baseband Volterra Series Modelling





$$\tilde{y}(t) = \sum_{k=0}^{\lceil N/2 \rceil - 1} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \tilde{h}_{2k+1}(\tau_1, \ldots, \tau_{2k+1}) \prod_{i=1}^{k+1} \tilde{x}(t-\tau_i) \prod_{i=k+2}^{2k+1} \tilde{x}^*(t-\tau_i)\, d\tau_1 \ldots d\tau_{2k+1}$$

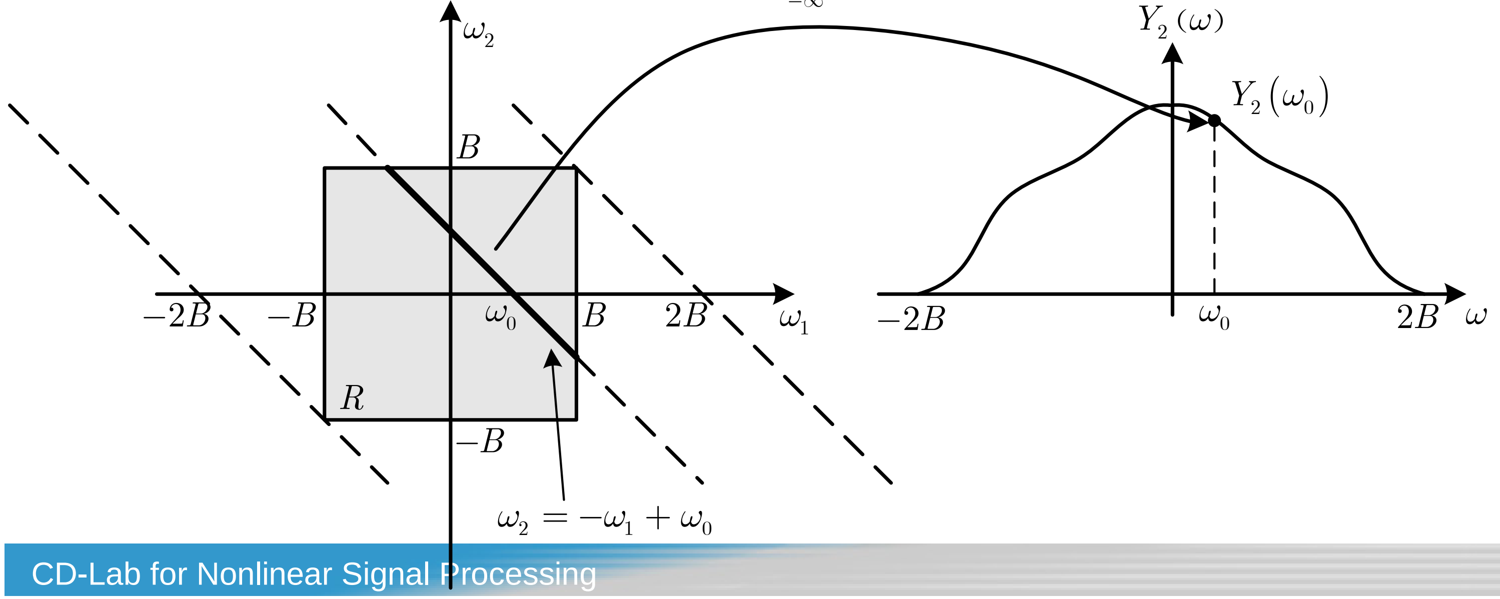
Frequency-domain Representation

$$\tilde{Y}_{2k+1}(\omega) = \frac{1}{(2\pi)^{2k}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \tilde{Y}_{(2k+1)}(\omega - v_1, v_1 - v_2, \ldots, v_{2k})\, dv_1 \ldots dv_{2k}$$

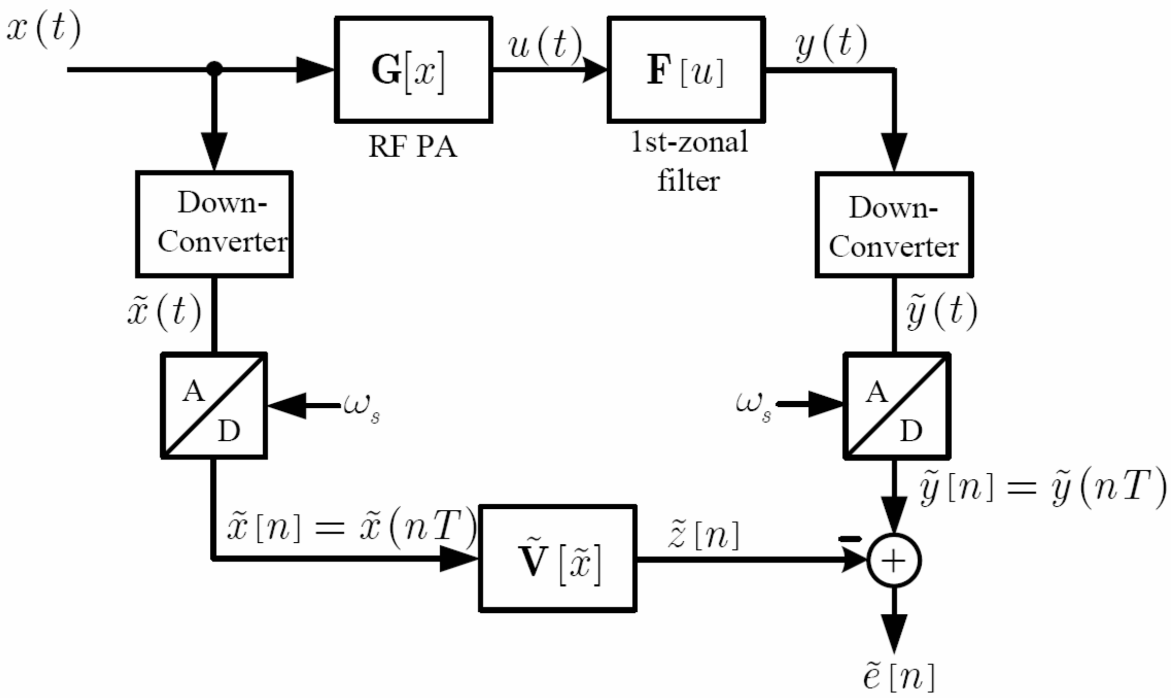
$$\tilde{Y}_{(2k+1)}(\omega_1, \ldots, \omega_{2k+1}) = \tilde{H}_{2k+1}(\omega_1, \ldots, \omega_{2k+1}) \prod_{i=1}^{k+1} \tilde{X}(\omega_i) \prod_{i=k+2}^{2k+1} \tilde{X}^*(-\omega_i)$$

Band-limited to [-B,B]

$$Y_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_{(2)}(\omega - \omega_2, \omega_2)\, d\omega_2$$



Nyquist Sampling Regarding the Output Signal



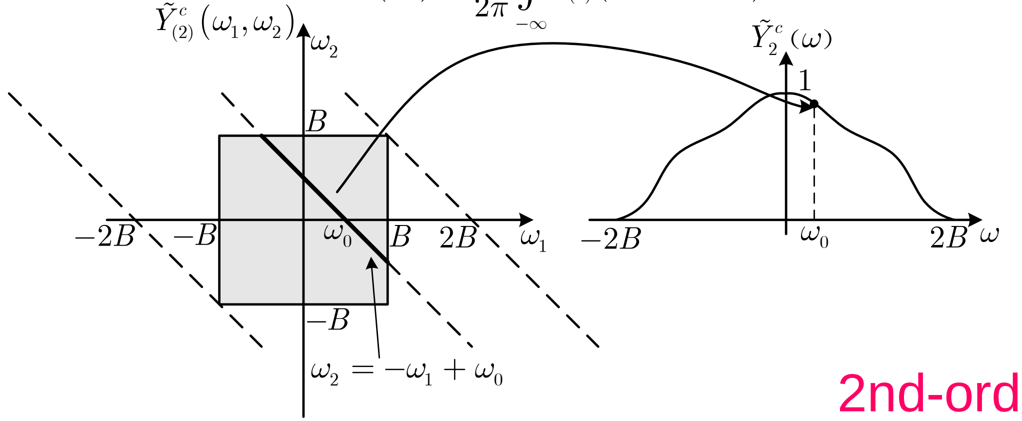
$$\begin{aligned}\tilde{z}[n] &= \tilde{\mathbf{V}}[\tilde{x}[n]] \\ &= \sum_{k=0}^{\lceil L/2 \rceil - 1} \sum_{n_1=0}^{N_{2k+1}} \cdots \sum_{n_{2k+1}=0}^{N_{2k+1}} \tilde{v}_{2k+1}[n_1, \ldots, n_{2k+1}] \\ &\quad \times \prod_{i=1}^{k+1} \tilde{x}[n-n_i] \prod_{i=k+2}^{2k+1} \tilde{x}^*[n-n_i],\end{aligned}$$

← Discrete-time Volterra model

$$\tilde{V}_{2k+1}(\exp(j\Omega_1), \ldots, \exp(j\Omega_{2k+1})) \equiv \tilde{H}_{2k+1}\left(\frac{\Omega_1}{T}, \ldots, \frac{\Omega_{2k+1}}{T}\right)$$

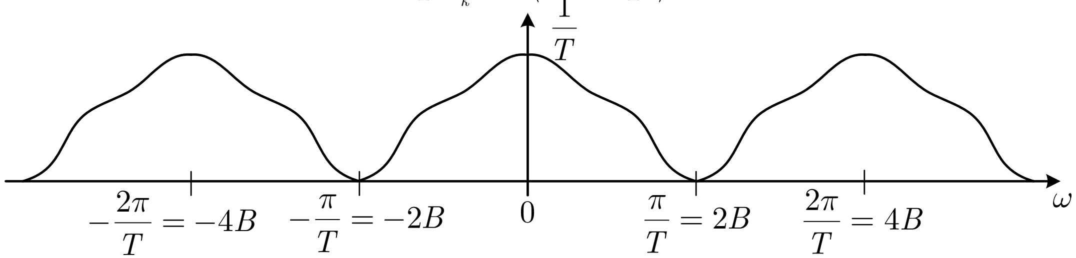
Nyquist Sampling Regarding the Output Signal

$$\tilde{Y}_2^c(\omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{Y}_{(2)}^c(\omega_0 - \omega_2, \omega_2)\, d\omega_2$$

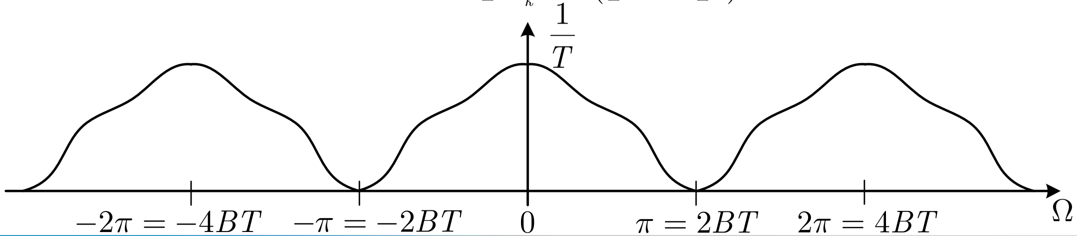


2nd-order nonlinear system !

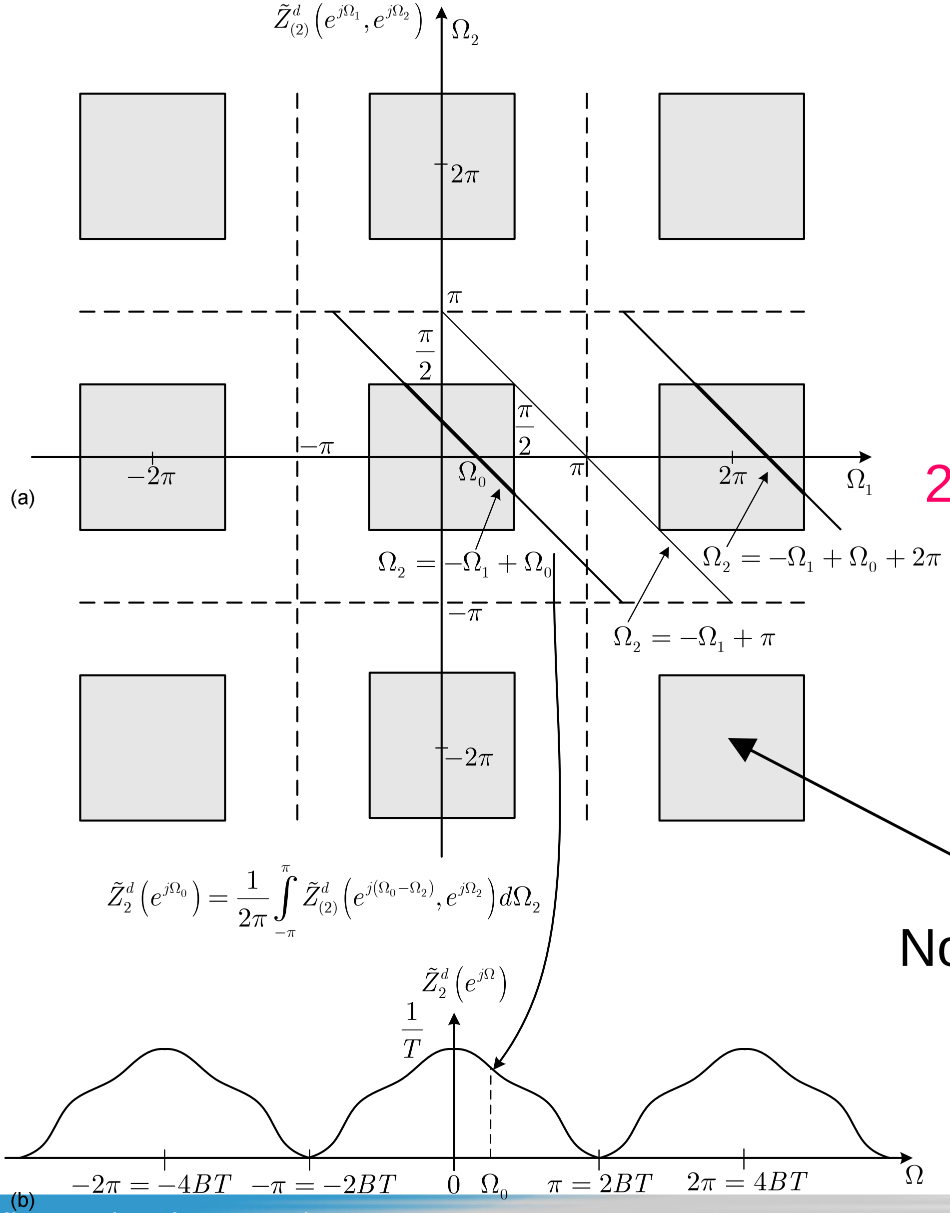
$$\frac{1}{T}\sum_k \tilde{Y}_2^c\left(\omega - k\frac{2\pi}{T}\right)$$



$$\tilde{Y}_2^d(e^{j\Omega}) = \frac{1}{T}\sum_k \tilde{Y}_2^c\left(\frac{\Omega}{T} - k\frac{2\pi}{T}\right)$$



Nyquist Sampling Regarding the Output Signal



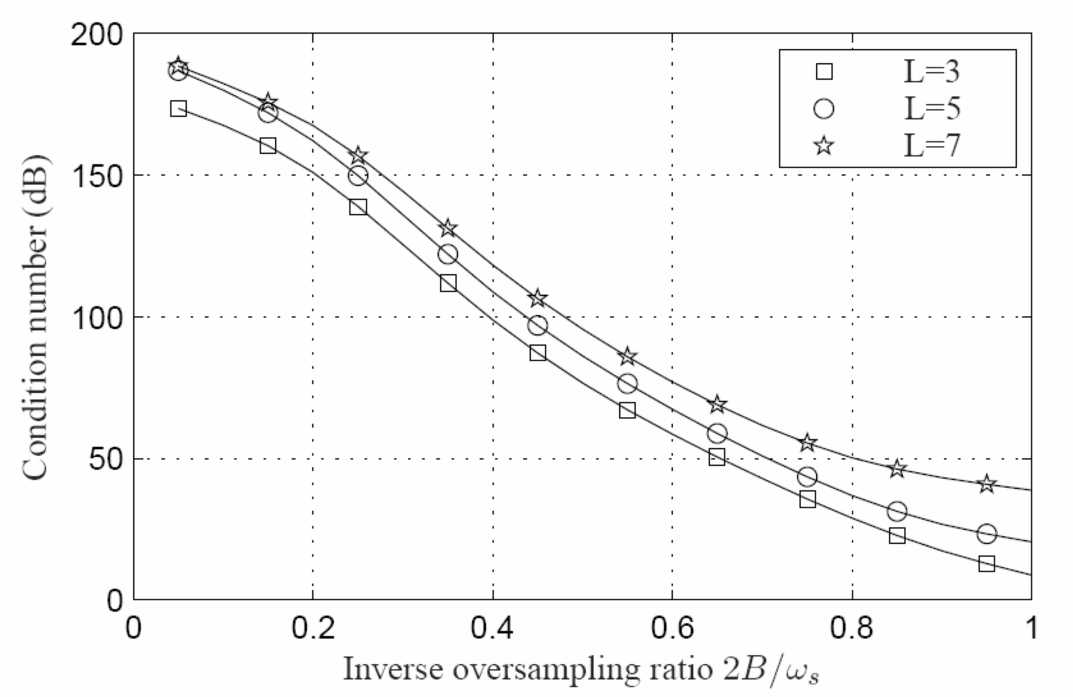
2nd-order nonlinear system !

Nonlinear system is purely determined on [- B,B]

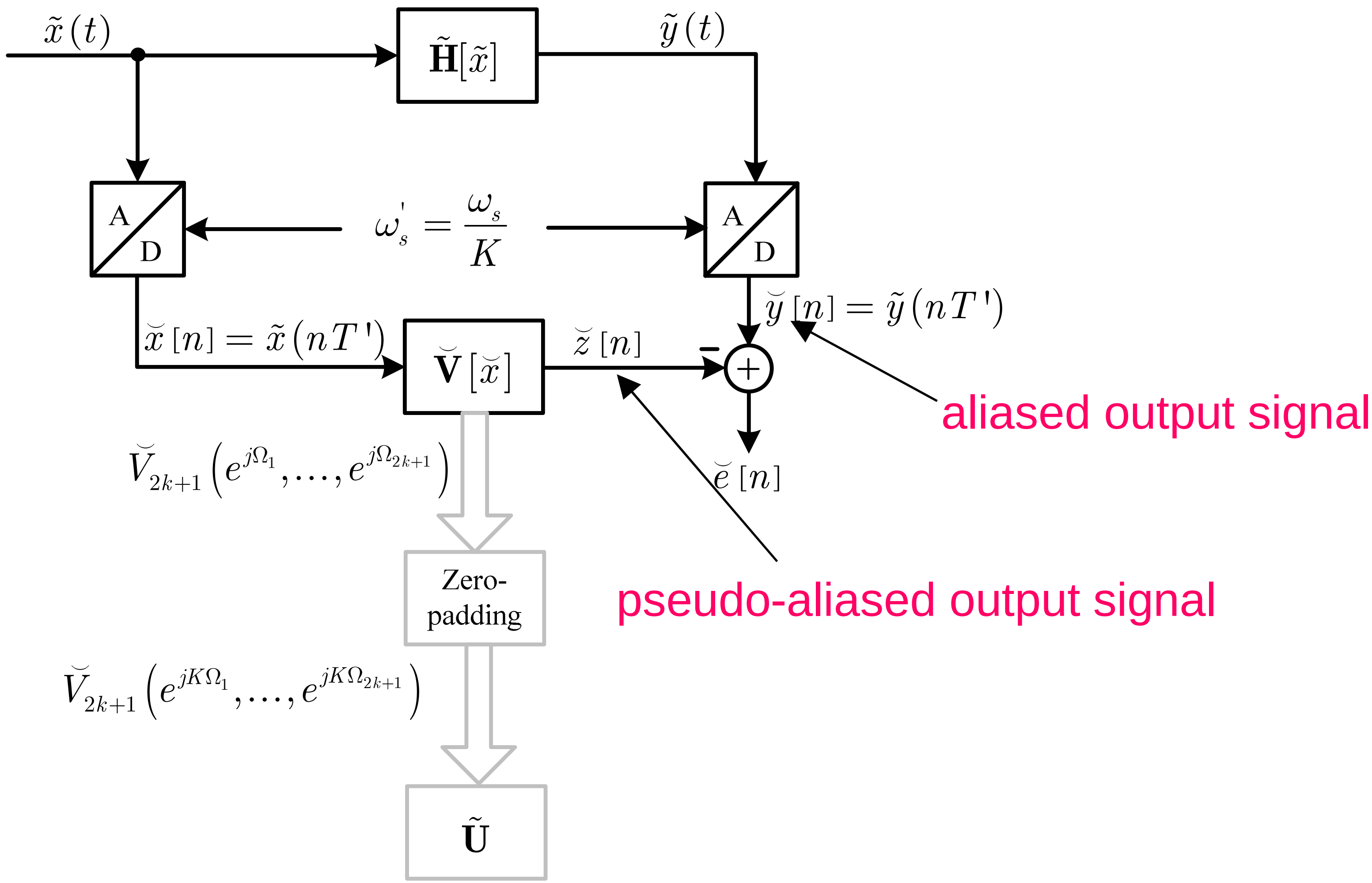
Nyquist Sampling Regarding the Output Signal

Drawbacks

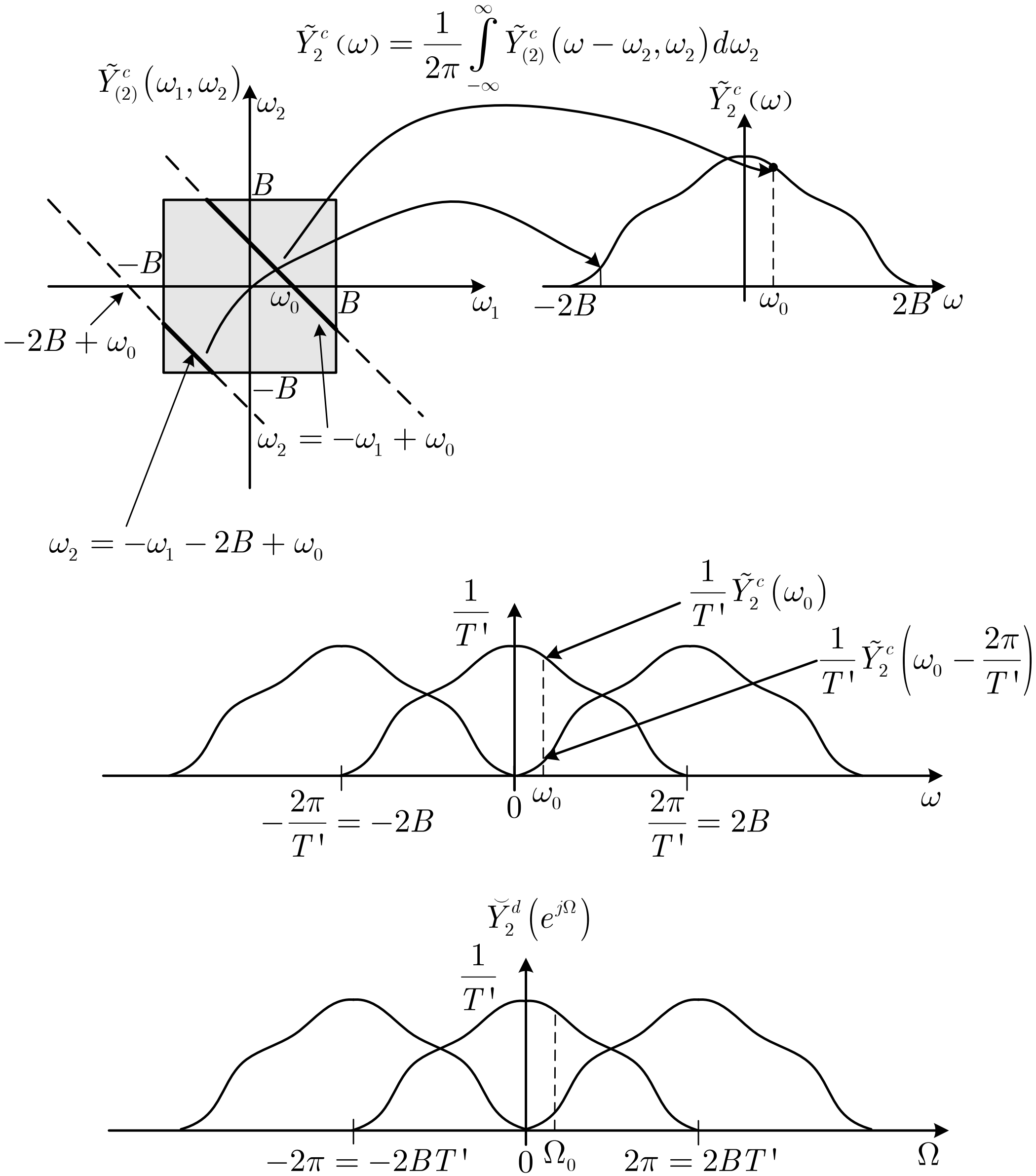
- high sampling frequency (depends on the order of the nonlinearity)!
- Ill-conditioned estimation problem → Numerical unstable solution, or low convergence speed !



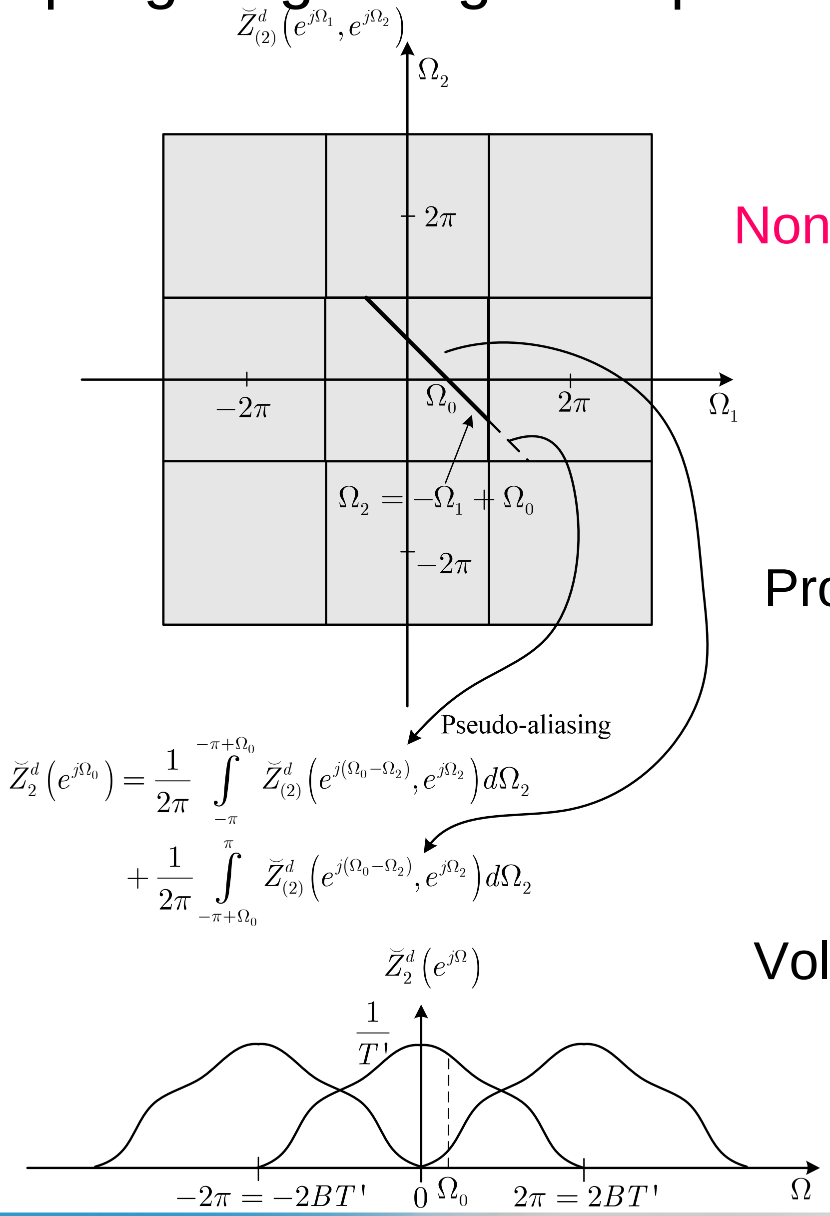
Nyquist Sampling Regarding the Input Signal



Nyquist Sampling Regarding the Input Signal



Nyquist Sampling Regarding the Input Signal

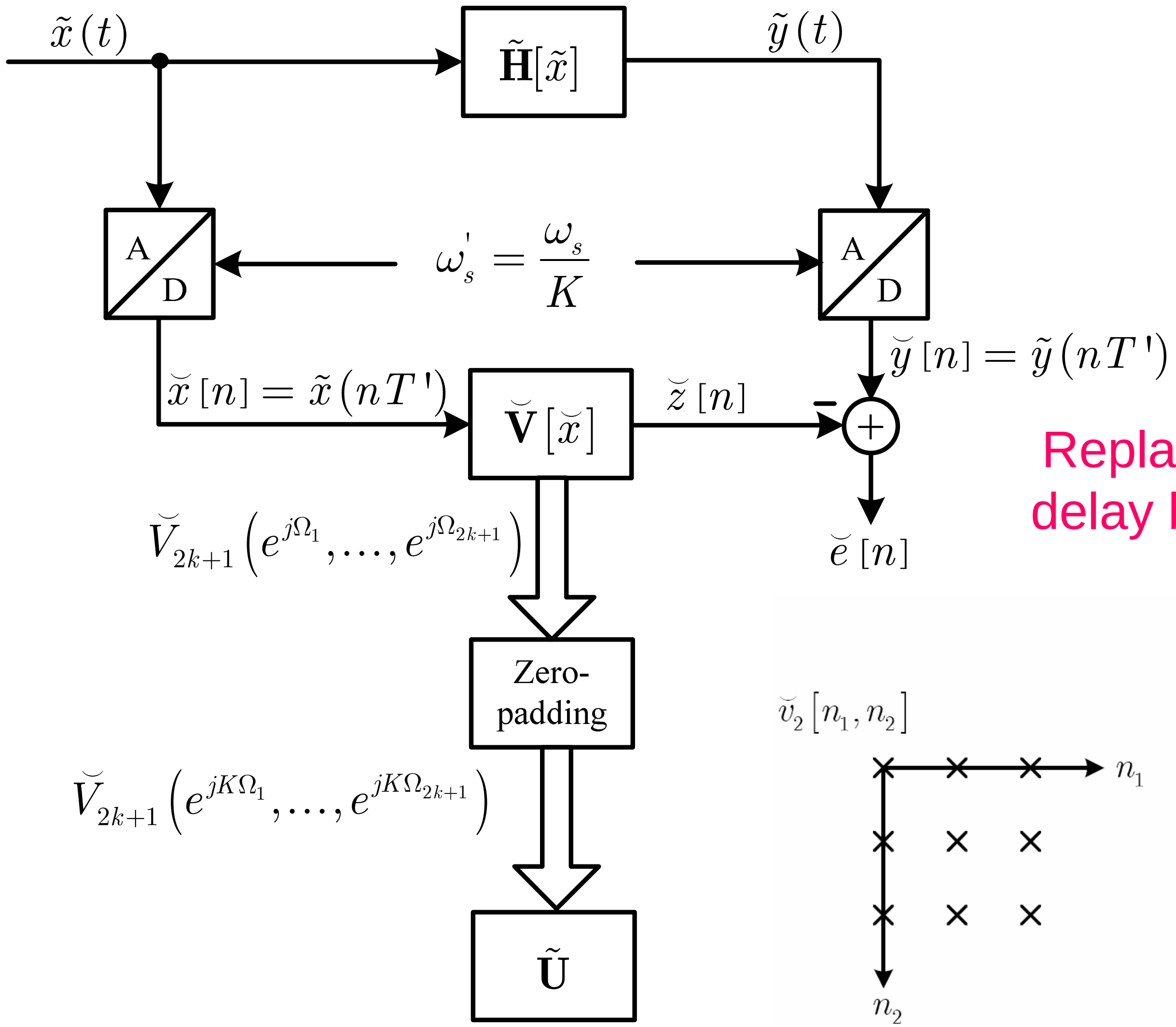


Nonlinear system identification is possible!

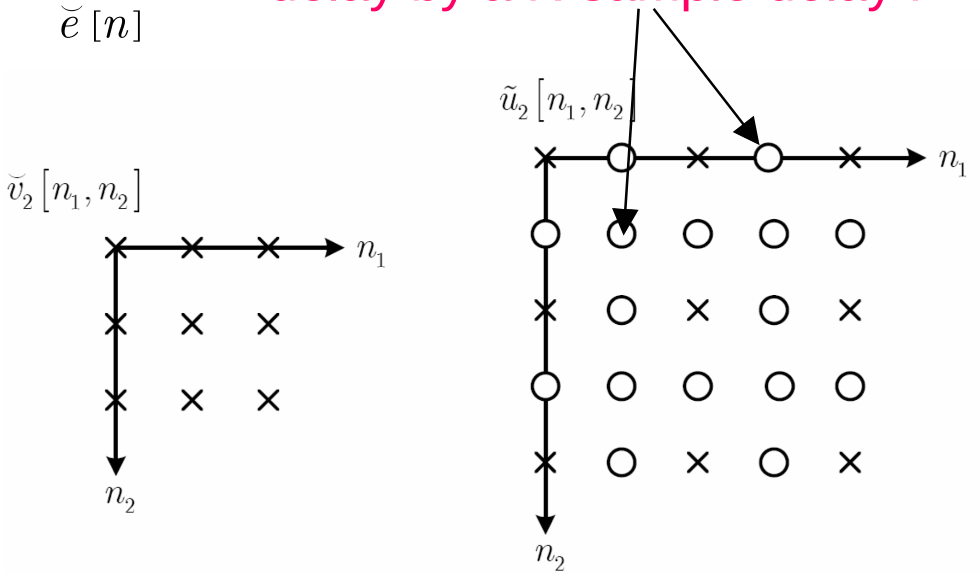
Problem: aliased output signal !

Volterra kernel interpolation

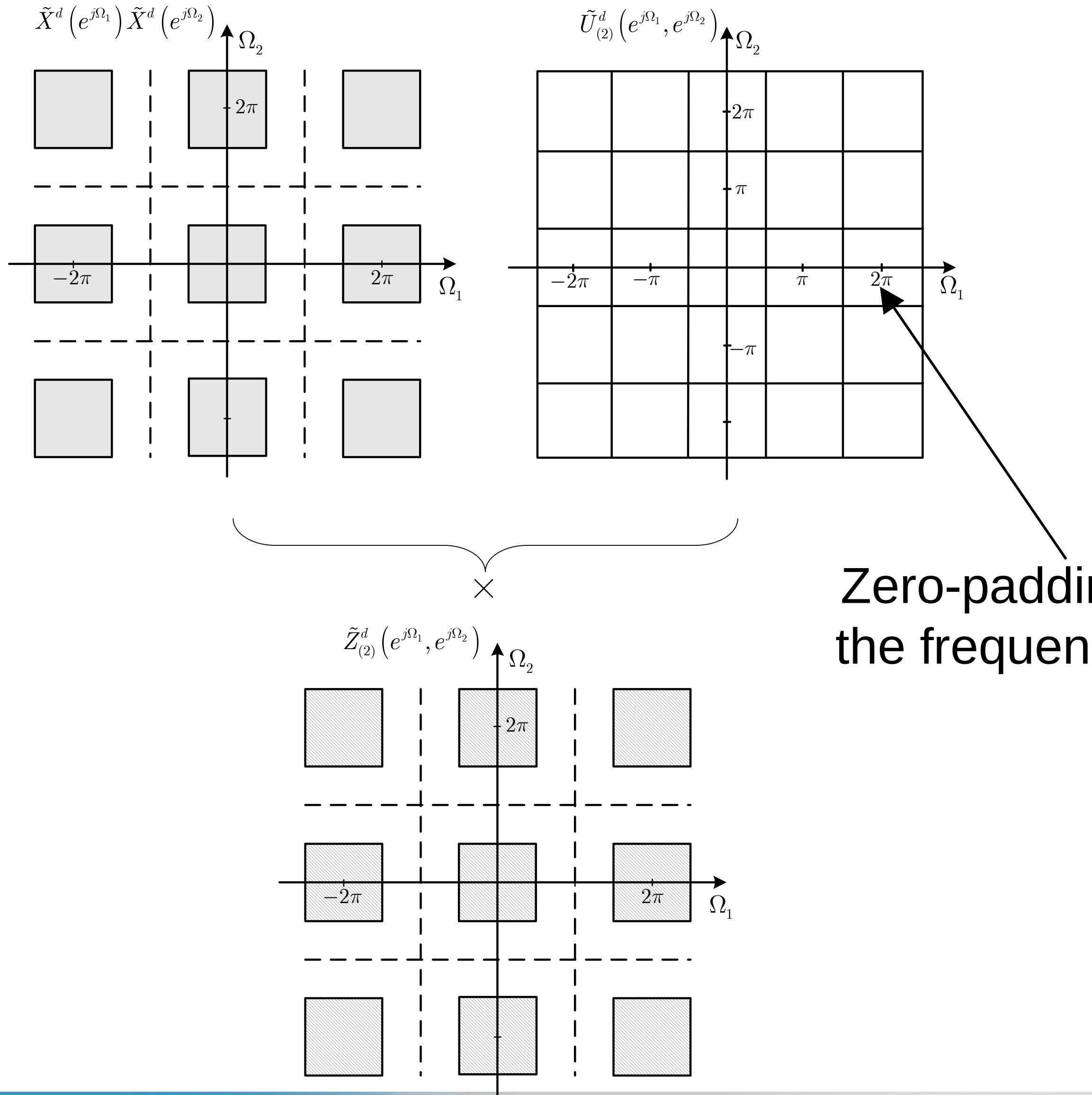
Volterra Kernel Interpolation



Replace each unit-sample delay by a K-sample delay !

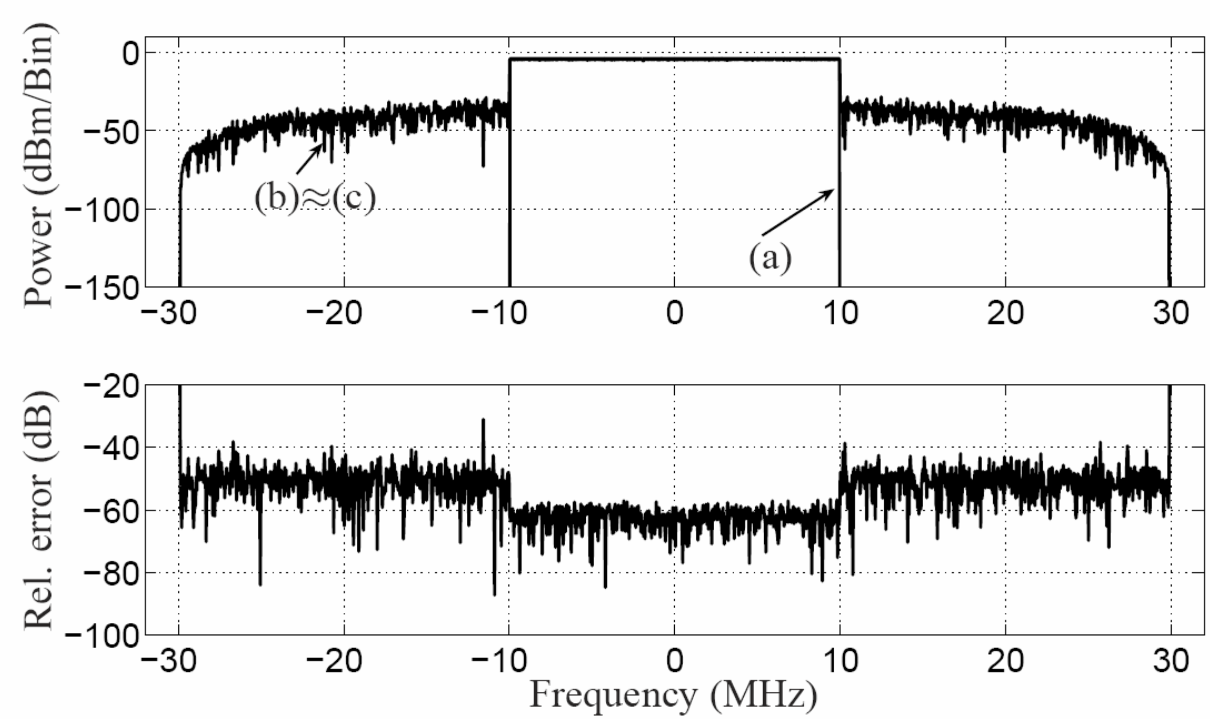


Volterra Kernel Interpolation



Zero-padding change
the frequency scaling!

Volterra kernel interpolation



Error signal is caused by the imperfect system identification (model uncertainties and noise) !

Conclusion

System identification on the Nyqist rate regarding the output signal

- Numerical unstable, low convergence speed
- high sampling rate ADC's (expensive, high power consuming)

System identification on the Nyquist rate regarding the input signal

- can be accomplished with Volterra kernel interpolation
- No additional computational complexity is needed!