Tutorial on Channel Equalization for Mobile Channels

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***Abstract*— Equalizer design for multi-path channels is closely linked to the distribution of zeros of mobile channels. While in a deterministic scenario, equalizers can be designed based on knowledge of the exact locations of zeros in the complex plane, mobile channels like in GSM/EDGE systems ask for a different approach. Various channel distortions make the derivation of a deterministic model a hard if not infeasible task. However, one can show that a statistical channel description can help in the development of suitable equalizers. We summarize some of the results obtained in [Schober01] on the importance of the distribution of zeros of the channel and finally explain some equalization respectively prefiltering strategies to overcome the burden of inter-symbol interference. Based on given power delay profiles, one can choose proper equalizer concepts for improving overall transmission performance.**

I. Introduction

WHILE this paper's title says '*Tutorial on Channel Equalization for Mobile Channels*', it mainly summarizes the work presented in [Schober01], titled '*On the Distribution of Zeros of Mobile Channels with Application to GSM/EDGE*'. We start with a brief introduction to multi-path channels (MPCs) in Section II, showing the richness of distortions that occurs within mobile transmission channels. In Section III we discuss the general problem of channel equalization and in particular explain some solution-strategies: Linear Equalization (LE), Decision-Feedback Equalization (DFE), Maximum Likelihood Sequence Estimation (MLSE), Reduced-State Sequence Estimation (RSSE) and Delayed Decision Feedback Sequence Estimation (DDFSE). Further in Section IV an introduction is given on impulse response truncation and allpass prefiltering as a means of improving the equalizer performance. After that, we give a statistical model of multi-path channels in Section V, leading to a description of the distribution of zeros in MPCs. Finally, we describe power delay profiles (PDPs) and give application examples in Section VI.

II. MPCs - Intuitive Characterization

Although not appropriate, we start with the introduction of multi-path channels as systems causing delays at symbol-intervals, leading to a first idea on the channel's properties.
Mobile transmission channels (model see Figure 1) can be seen as the combination of several propagation paths, each featuring an individual delay. With only a line-of sight (LOS) component, the channel's impulse response $h_C(t)$ features only one bin. With every additional delayed path added, it features more and more bins. This leads to inter-symbol interference (ISI), or, from a different point of view, to a certain distribution of zeros in the complex plane.
To reduce the ISI / place poles at the location of zeros, one can design equalizers acting at the receiver side, $h_R(t)$. The expected result is an overall impulse response featuring only one significant bin. There exist various approaches of which some are described below.

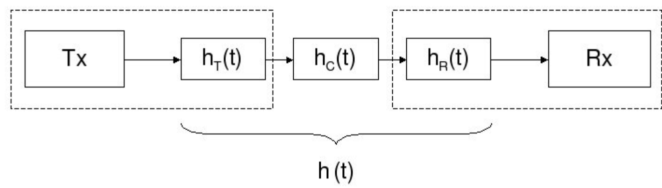

Fig. 1: Overall mobile channel model. During transmission from transmitter Tx to receiver Rx, the signal is distorted by the overall impulse response $h(t)$. It is composed out of the impulse responses of the transmitter ($h_T(t)$), the channel ($h_C(t)$) and the receiver ($h_R(t)$).

III. Channel Equalization

The goal of channel equalization is to remove the effects of the channel on the transmitted symbol sequence $[a_k]$, i.e. inter-symbol interference (ISI). This can be done either by inverse filtering (e.g. Linear- (LE) or Decision-Feedback-Equalization (DFE)) or by applying sequential detection (e.g. Viterbi algorithm).
An equalizer-filter can be optimized according to three different cost functions:

- Zero forcing criterion: invert the channel impulse response
- MMSE criterion: minimize the mean-squared-error
- Min. Bit-Error-Rate (BER) criterion

In the following discussions on different equalizers only the first two criteria are used.

The channel dynamics may not be known at startup. Further the channel may vary with time, so an adaptive implementation of the equalizer is necessary.The following different modes of adaptation can be distinguished:

- Adaptation using a training signal (cp. periodic training sequence in GSM)
- Decision directed adaptation: An error signal is generated by comparing in- and output of the decision device

- Blind adaptation: Exploiting signal properties instead of using an error signal for adaptation

For the discussions on LE and DFE the second strategy is assumed.

A. Linear Equalization

In Figure 2a the basic structure of a linear equalizer is shown. The received sequence r_k is obtained by filtering of the symbol sequence a_k by the linear channel $H(z)$ and adding noise n_k. Then the linear equalization filter $C(z)$ is applied to the input. The error signal e_k is defined as the difference between the output of the equalizer and the output of the decision device (slicer). The slicer simply quantizes the input to the nearest alphabet symbol.

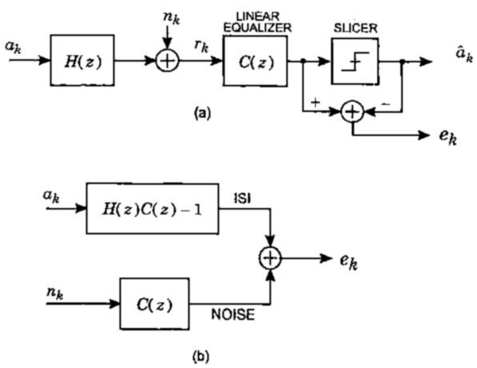

Fig. 2: Basic block diagram of a Linear Equalizer (from [BA])

Further in 2b an equivalent structure, emphasizing the contributions to the error signal, namely the rest-ISI and the filtered noise, is depicted.

Obviously the power spectrum of the error S_e can be written as

$$S_e = S_a \left| H(z) \cdot C(z) - 1 \right|^2 + S_n \left| C(z) \right|^2 \tag{1}$$

where S_a is the power spectrum of the data-symbols (assuming a modelation as a WSS random process), and S_n is the power spectrum of the noise process.

If we assume equalization according to the linear forcing criterion (i.e. $C(z) = H(z)^{-1}$) we see that the ISI-contribution to the error vanishes and an enhanced noise part remains.

If $H(z)$ is a non-minimum phase channel the equalizer has poles outside the unit circle and is thus not realizable. Further the problem of noise enhancement exacerbates when the zeros of the channel approach the unit circle.

If we consider also the impact of the additive noise on the error we end up in the idea to minimize the mean-squared error. In Eq. (2) the power spectrum of the received signal S_r is given and based on that and Eq. (1), Eq. (3) is obtained [BA]. The reflected channel transfer function $H^*(1/z^*)$ is called a matched filter.

$$S_r = S_a \left| H(z) \right|^2 + S_n \tag{2}$$

$$S_e = S_r \left| C(z) - S_a S_r^{-1} H^*(1/z^*) \right|^2 + S_a S_n S_r^{-1} \tag{3}$$

Clearly this term can be minimized by choosing

$$C(z) = S_a S_r^{-1} H^*(1/z^*) \tag{4}$$

An important difference to the Zero-forcing solution is that we have this matched filter term now which would not have been possible in the other case since the equalizer would have simply found the inverse of it. Note that if the channel has poles we get an anticausal IIR matched filter. From Eq. (4) we can conclude that the MMSE solution approaches the Zero-forcing solution for $S_n \to 0$.

The DFE results in less error-power at the slicer input since it avoids the noise enhancement by poles close to the unit circle.

A more detailed introduction to linear equalizers is given in [BA] and [MO].

B. Decision Feedback Equalization (DFE)

The DFE makes use of the regenerative effect of the non-linear decision device. In Figure 3 we see another interpretation of why the DFE is an improvement over the LE. Regarding Figure 3a we recognize an additional linear prediction block $E(z)$ filtering the correlated error e_k and producing an error e_k' which has, due to the properties of a linear predictor [BA], always a lower variance compared to e_k. In other words the linear predictor removes all predictable information resulting in white output noise.

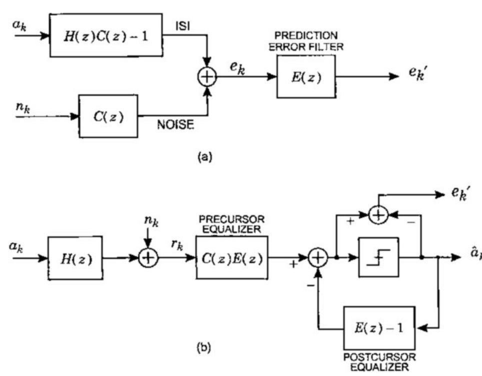

Fig. 3: Enhancement of the LE by linear prediction of the error (from [BA])

Figure 3b shows an equivalent structure. You see that the precursor equalizer consists of the Zero-forcing linear equalizer from above and the linear predictor given by Eq. (5) [BA] where $M_n(z)$ is the minimum phase component of the power spectrum of the noise denoted as $S_n(z)$ obtained by minimum-phase spectral factorization given in Eq. (6).

$$E(z) = \frac{H_{min}(z) \cdot H^*_{max}(z)}{M_n(z)} \quad (5)$$

$$S_n(z) = \gamma_n^2 \cdot M_n(z) \cdot M_n^*(1/z^*) \quad (6)$$

where γ_n^2 is the geometric mean of $S_n(e^{j\Theta})$

$$C(z) \cdot E(z) = \frac{1}{H_0(z)} \cdot \frac{H^*_{max}(1/z^*)}{H_{max}(z)} \cdot M_n^{-1}(z) \quad (7)$$

where H_0 contains all poles/zeros of $H(z)$ at $z = 0$

The overall precursor-equalizer is denoted in Eq. (7). It contains an Allpass part $\frac{H^*_{max}(1/z^*)}{H_{max}}$ which mirrors the maximum phase part of $H(z)$ into the unit circle. Thus the output of the precursor-equalizer is minimum phase with respect to the cursor. The post-cursor can then be equalized by the postcursor-equalizer $E(z) - 1$.
Note that the decision of the DFE solely relies on the cursor and completely ignores the signal energy embedded in the ISI-terms. The maximum-likelihood sequence detector (cp. Section III-C) on the other hand uses all the energy in the equivalent channel impulse response.

Now we want to use the MMSE-criterion to derive a solution for the optimal equalizer. The slicer-error before linear prediction is the same as in the Zero-forcing case, so the optimal transfer function $C(z)$ doesn't change compared to the linear equalizer. The combination of Eq. (3) and (4) shows that the power spectrum of the residual error is

$$S_e = \frac{S_a S_n}{S_r} = \frac{\gamma_a \gamma_n}{\gamma_r} \cdot \frac{M_a(z) M_n(z) \cdot M_a^*(1/z^*) M_n^*(1/z^*)}{M_r(z) \cdot M_r^*(1/z^*)} \quad (8)$$
$$= \gamma_e \cdot M_e(z) \cdot M_e^*(1/z^*)$$

We are looking for the filter $E(z)$ that whitens the slicer error. On the other hand we know from theory that the prediction filter is a minimum-phase filter [BA]. Therefore Eq. (9) and (10) follow [BA].

$$E(z) = \frac{1}{M_e(z)} = \frac{M_r(z)}{M_a(z) \cdot M_n(z)} \quad (9)$$

$$C(z) \cdot E(z) = \frac{\gamma_a^2}{\gamma_r^2} \cdot H^*(1/z^*) \cdot \frac{M_a^*(1/z^*)}{M_r^*(1/z^*)} \cdot M_r^{-1}(z) \quad (10)$$

As in the LE-MMSE case there is a matched filter part and a noise whitening part M_n^{-1} which were absent in the solution for the DFE-Zero-forcing.
Generally speaking, DFEs offer ISI cancellation with reduced noise enhancement and may thus provide a significantly lower BER compared to linear equalizers.

C. Maximum Likelihood Sequence Estimation (MLSE)

Maximum likelihood sequence estimation is the optimal minimum probability of error detector on ISI channels [BA]. The strategy is to look for the most likely sequence out of all possible sequences $X[z] = \{x_0 + x_1 z^{-1} + x_2 z^{-2} + \cdots + x_{N-1} z^{N-1} | x_k \in X\}$ using the minimum distance rule from Eq. (11) where $y(z)$ is the observation sequence and $H(z)$ is the channel transfer function.

$$\hat{x}(z) = argmin_{x(z) \in X[z]} \left\| y(z) - \underbrace{(H(z) - 1)x(z)}_{ISI} - x(z) \right\|^2 \quad (11)$$

Please note that the number of states, i.e. the number of different symbol sequences, is growing exponentially depending on the alphabet size $|X|$ and the length of the channel impulse response K with $|X|^K$.
A popular algorithm implementing the MLSE is the Viterbi algorithm which searches a state sequence through the trellis that minimizes the distance to the observation sequence.

D. Reduced State Sequence Estimation (RSSE)

Coming from the MLSE the idea that gives rise to RSSE [[RSSE88]] (and as a special case of RSSE to DDFSE [[DDFSE00]]) is to narrow down the number of states M by combining states to sub-states. For a certain delay k ($1 < k \leq K$) a 2-dim. set partitioning $\Omega(k)$ is defined where the signal set is partitioned into J_k subsets ($1 < J_k < M$). For this set partitioning two conditions are defined:

- $J_1 \geq J_2 \geq \ldots \geq J_K$
- $\Omega(k)$ is a further partition of the subsets of $\Omega(k+1)$

Equation (III-D) means that when we look further into the past we never see an increase in sub-states.
An example of a set partitioning is presented in Figure 4 where you can see that Eq. (III-D) is fulfilled since in the next deeper level of the tree the states from the previous level are each divided into two sub-states.

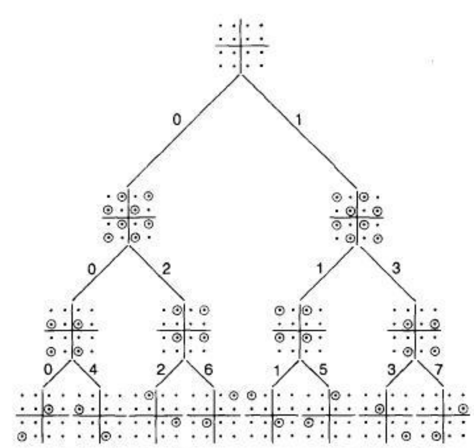

Fig. 4: Ungerboeck partition tree for the rectangular 16-QAM signal set

But also in the other direction, i.e. from regular state to substate, a connection has to be defined: $a_i(k)$ is the index

$(1 \leq a_i \leq J_k)$ of the sub-state a certain state belongs to.
Finally by this merging into sub-states we reduced the number of states to $\prod_{k=1}^{K} J_k$. Note that from one state to the next there are always J_1 transitions, i.e. parallel transitions occur when $J_1 < M$.

It is clear that now paths merge earlier than in the original trellis diagram. Therefore it is important that you can reliably distinguish between states at the point of merging into one sub-state. Looking at Figure 4 it becomes reasonable that for that reason, for every Ω_k you have to maximize the minimum intra-subset Euclidean distance, i.e. the distance between two states of the same sub-state.
Everytime you merge two states you have to decide which one of them was more likely and store this symbol in a path history associated with a certain state t_k.
An important fact to notice is that in contrast to the MLSE the performance of the RSSE is affected by the phase response since the more you look into the past, the less different states are possible because you already merged (i.e. decided for) some of them. In other words the algorithm doesn't make the decision about the most likely symbol immediately, but gradually dismisses certain possibilities. Therefore the earlier samples of the impulse response have a higher impact on the decision than later ones. This explains why the RSSE performs better for minimum phase responses, where the energy in the first K' samples is maximized for every K' [[RSSE88]].
Based on how fast you merge you can construct other equalizers like the DFE ($J_k = 1, \forall k$) or the DDFSE ($J_k = \begin{cases} M & 1 \leq k \leq K' \\ 1 & K' \leq k \leq K \end{cases}$) which are thus just special cases of the RSSE.

E. Delayed Decision Feedback Sequence Estimation (DDFSE)

The DDFSE can be regarded as a hybrid between MLSE and Zero-Forcing DFE or, as described in Section III-D, as a special case of RSSE. It's like a Viterbi algorithm working on a truncated (at a length K') channel impulse response and using a Zero-forcing DFE on each branch of the trellis subtracting the postcursor ISI caused by samples $x_{k-K'-1}, x_{k-K'-2}, ..., x_{k-K}$.
Equation (12) finally shows the calculation of the branch metric, where y_k is the observation at time k, x_k is the symbol at time k, $\hat{x}_k$ is the symbol estimate at time k and h is the channel impulse response. The result is shown in Equation (13).

branch metric calculation:

$$L = \left| y_k - \underbrace{\underbrace{\sum_{i=1}^{K'} h_i x_{k-i}}_{\text{state contribution}} - \underbrace{\sum_{i=K'}^{K} h_i \hat{x}_{k-i}}_{\text{delayed decision contribution}}}_{\text{ISI}} - x_k \right|^2 \tag{12}$$

$$L = \left| y_k - \sum_{i=0}^{K'} h_i x_{k-i} - \hat{w}_{k-K'-1} \right|^2 \tag{13}$$

where $\hat{w}$ is the postcursor ISI using decisions made

IV. Prefiltering

A. Impulse Response Truncation

We know that for channels with large delay spread, i.e. long impulse responses **h**, or transmissions using large alphabets, a very high complexity may result for the MLSE. The aim of impulse response truncation is to minimize the power of all impulse response samples $h_e(k)$ for $k > N_g$ by applying a prefilter with impulse response **c** at the input to the equalizer and thus restricting the complexity of the equalizer drastically (cp. Figure 5).

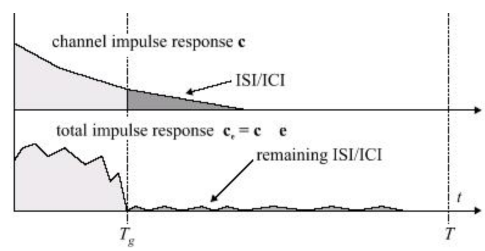

Fig. 5: Goal of Impulse Response Truncation (from [[Schmidt00]])

In Section II we have heard that the extent to which we can shorten the impulse response depends on the number of zeros of the channel transfer function close to the unit circle. Otherwise we get lower equalizer performance [Schober01].
The reason why in the following we concentrate on FIR-prefilters is that the performance of the IIR-prefiltered system deteriorates in case of noise and non-ideal channel knowledge [[Schmidt00]].
In the following we also assume ideal knowledge of the channel impulse response.
Convolution of the channel- with the prefilter impulse response yields Eq. (14), where K and P are the lengths of the impulse responses of **h** resp. **c** and **H** is the convolution matrix of **h**. Eq. (15) formulates our demands: the convolved impulse response is truncated to a length N_g, the rest of the impulse response can take arbitrary values and the first sample shall be 1 (to avoid a zero solution). Excluding all lines of the convolution matrix we don't have any restrictions on, results in the reduced convolution matrix H_r and our problem formulation can be written as denoted in Eq. (16).

$$h_e(k) = \mathbf{h} * \mathbf{c} = [h_e(0), h_e(1), \ldots, h_e(K+P-1)]^T$$
$$= \mathbf{H} \cdot \mathbf{c} \quad (14)$$

$$h_e(k) = \begin{cases} 1 & k = 0 \\ * & 0 < k \leq N_g \\ 0 & k > N_g \end{cases} \quad (15)$$

$$\mathbf{H}_r \cdot \mathbf{c} = \mathbf{d} + \delta \quad (16)$$

where $\mathbf{d} = [1, 0, 0, \ldots, 0]^T$

error vector $\delta = [\delta(0), \delta(1), \ldots, \delta(K+P-N_g)]^T$

The optimal solution, as given in Eq. (17), of this problem in a MMSE sense leads to the pseudo-inverse of H_r (since you can only invert square matrices), and changes to Eq. (18) if you assume additive white Gaussian noise [[Schmidt00]]. The term γ_{pe}^2 equals $\frac{1}{Signal-To-Noise\ Ratio}$.

$$\mathbf{c} = (\mathbf{H}_r^T \mathbf{H}_r)^{-1} \mathbf{H}_r^T \cdot \mathbf{d} \quad (17)$$
$$\mathbf{c} = (\mathbf{H}_r^T \mathbf{H}_r + \gamma_{pe}^2)^{-1} \mathbf{H}_r^T \cdot \mathbf{d} \quad (18)$$

Finally we'd like to repeat that for practical systems the impulse truncation technique must be combined with time-domain channel estimation in order to get an estimate for **h**.

B. Allpass Prefilter Computation

In Section II we have seen that suboptimum sequence estimators clearly rely on a discrete-time minimum phase impulse response. This section is denoted to a way of finding a closed form solution, necessary if the training sequence is too short, for an Allpass prefilter that transforms our transfer function of the channel H into it's minimum phase part. A more precise description of the following is given in [[Gerstacker02]].

Equation (19) defines our optimal prefilter for the task mentioned. So the most direct approach would be to calculate $H_{min}(z)$ (e.g. by spectral factorization, root finding, prediction-error filter etc.).

$$A(z) = \frac{H_{min}(z)}{H(z)} \quad (19)$$

A problem quite obvious is that $A(z)$ is non-stable since we assumed $H(z)$ has maximum phase components. Possible solutions to that problem are [[Gerstacker02]]:

- Viewing $A(z)$ as a transfer function corresponding to a noncausal and stable impulse response and using a FIR-filter approximation
- Time reverse a block of received data, filter the block with A(1/z) and time-reverse the output sequence again
- Filter the data with $\tilde{A}(z) = \frac{H_{max}(z)}{H(z)}$, and since the resulting impulse response is maximum-phase then you simply apply reduced-state equalization in negative time direction then ("backward decoding")

Another possibility instead of computing H_{min} directly is to apply the FIR feedforward filter of a MMSE-DFE though this solution doesn't seem to be robust to a mismatch of design parameters in certain cases [[Gerstacker02]].

In [[Gerstacker02]] a computationally less expensive approach based on linear prediction is presented.
First we see in Eq. (20) that our previous demand can be rewritten. The "new" allpass prefilter consists of a matched filter and a second part $\frac{1}{H_{min}^*(1/z^*)}$ (cp. Eq. (21)). We will approximate this second part by a FIR filter $F_2(z) \approx C \cdot \frac{1}{H_{min}^*(1/z^*)} = G^*(1/z^*)$.

$$H_{min}(z) \cdot H_{min}^*(1/z^*) = H(z) \cdot H^*(1/z^*)$$
$$\frac{H_{min}(z)}{H(z)} = \frac{H^*(1/z^*)}{H_{min}^*(1/z^*)} \quad (20)$$
$$A(z) = A_1(z) \cdot A_2(z)$$
$$= H^*(1/z^*) \cdot \frac{1}{H_{min}^*(1/z^*)} \quad (21)$$

The striking idea now is that we say this filter $G(z)$ is a prediction error filter as denoted in Eq. (22) where $P(z)$ is a prediction filter.

$$G(z) = 1 - P(z) \quad (22)$$

... prediction-error filter of order q_p

The optimum coefficients for minimization of the output power of the error filter are given by the solution of the Yule-Walker equations [[Gerstacker02]], where Φ_{hh} is the autocorrelation matrix of h, φ_{hh} is the autocorrelation vector and p is the coefficient vector of the prediction filter:

$$\Phi_{hh}\mathbf{p} = \varphi_{hh} \quad (23)$$

These equations can be solved recursively via the Levinson Derbin algorithm. Further in [[Gerstacker02]] it is shown that for infinite order ($q_p \to \infty$) $G(z) = 1 - P(z) = \frac{C^*}{H_{min}(z)}$.
So finally the allpass prefilter looks like the following:

$$A(z) = z^{-(q_h+q_p)} \cdot H^*(1/z^*) \cdot (1 - P^*(1/z^*) \quad (24)$$

where q_h is the length of the channel impulse response

V. MPCs - Statistical Model

As opposed to Section II, we give a more realistic description of multi-path channels in this section, incorporating information on Equalizers out of Section III as well.

A. Channel Physics

There exist following propagation phenomena:

- Reflection, which occurs on smooth surfaces
- Transmission through buildings, walls, etc.
- Diffraction at solid edges
- Scattering on rough surfaces

These phenomena cause waves with different phase shifts to arrive at the receiver, resulting in destructive interference. It is also termed *small-scale fading*, which varies in the range of up to $10 \cdot \lambda$ as the receiver moves (see Figure 6). In the following, only small-scale fading is considered, as opposed to *large-scale fading*, e.g. shadowing by hills.

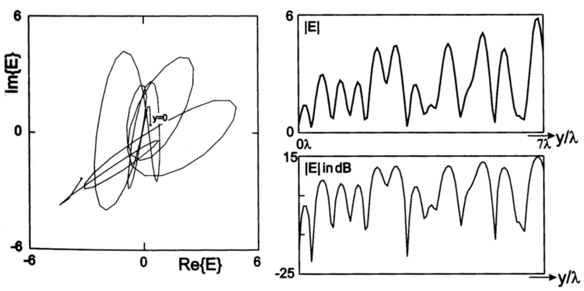

Fig. 6: Phasor (left) and amplitude (right) of received field E. In the amplitude plots, one can clearly observe fading, characterized by low amplitudes [MO].

B. Channel Impulse Response

Due to the varying configuration of transmitter, receiver and interacting objects between them, $\{h(t)\}$ varies with time. In fact, we have a time-variant impulse response $\{h_\tau(t)\}$. When considering its sampled version, $\{h_m[n]\}$, we have to keep in mind that infinitely many multi-path components contribute to its individual bins. This means further on that we can view $\{h_m[n]\}$ as a random process. With assuming the multi-path components of $\{h_\tau(t)\}$ to be identically, independently distributed in absence of a dominant component, we get a circular symmetric zero-mean Gaussian distribution of the bins $h_m[n]$ via the central limit theorem. For that, the absolute values of $\{h_m[n]\}$ are Rayleigh-distributed.
With the presence of a dominant component, we get a Rice distribution as shown in Figure 7.

As shown in Section III, equalizer performance heavily depends on how close zeros lie next to the unit circle $|z| = 1$ in the complex plane. For that, we are interested in statistical measures describing the radial distribution of zeros. At first, we have the marginal density given as [PA] (25), then the expected number of zeros inside the disc $|z| = r \leq R$ (26), and the expected number of zeros inside $\rho \leq |z| \leq 1/\rho, 0 < \rho < 1$ (27). Of special interest for equalizer design is the number of zeros "close" to the unit circle, given by the disc for $\rho = 0.9$ ($0.9 \leq |z| \leq 1.11$).

$$f_r(r) \triangleq r \cdot \int_0^{2\pi} f_z(r \cdot cos(\varphi) + j \cdot r \cdot sin(\varphi)) \cdot d\varphi \qquad (25)$$

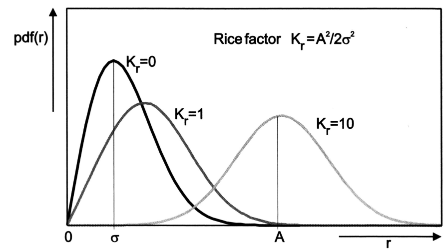

Fig. 7: Rice distribution. For the Rice factor $K_r \rightarrow 0$ (no dominant component), it approaches a Rayleigh distribution. The higher the impact of the dominant component ($K_r \uparrow$), the more it approaches a biased Gaussian distribution [MO].

$$n(R) = \int_0^R f_r(r) \cdot dr \qquad (26)$$

$$d(\rho) = n(1/\rho) - n(\rho) \qquad (27)$$

Assuming uncorrelated impulse response coefficients, one can provide a closed-form solution for $f_r(r)$ as specified in Equation 28. Important to note here is the dependence on the *variances* of impulse response bins, σ_h^2 and radius r. The corresponding $n(r)$ is given in Equation 29. For an exponential decay of channel impulse response bin variances as shown in Figure 8, the distribution of zeros looks like depicted in Figure 9.

$$f_r(r) = \frac{2}{r}\left(\frac{\sum\limits_{n=0}^{L-1} n^2\sigma_h^2[L-1-n]r^{2n}}{\sum\limits_{n=0}^{L-1}\sigma_h^2[L-1-n]r^{2n}} - \left(\frac{\sum\limits_{n=0}^{L-1} n^2\sigma_h^2[L-1-n]r^{2n}}{\sum\limits_{n=0}^{L-1}\sigma_h^2[L-1-n]r^{2n}}\right)^2\right) \qquad (28)$$

$$n(R) = \frac{\sum\limits_{n=0}^{L-1} n \cdot \sigma_h^2[L-1-n] \cdot R^{2 \cdot n}}{\sum\limits_{n=0}^{L-1} \sigma_h^2[L-1-n] \cdot R^{2 \cdot n}} \qquad (29)$$

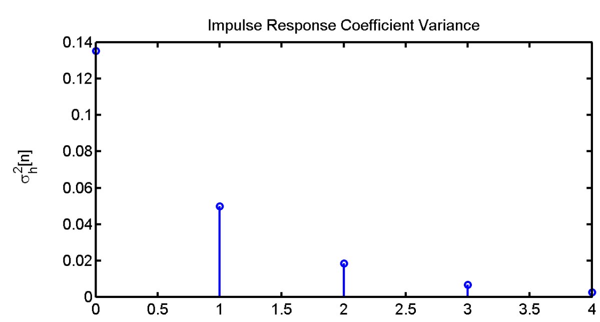

Fig. 8: Baseband channel impulse response variances with exponential decay.

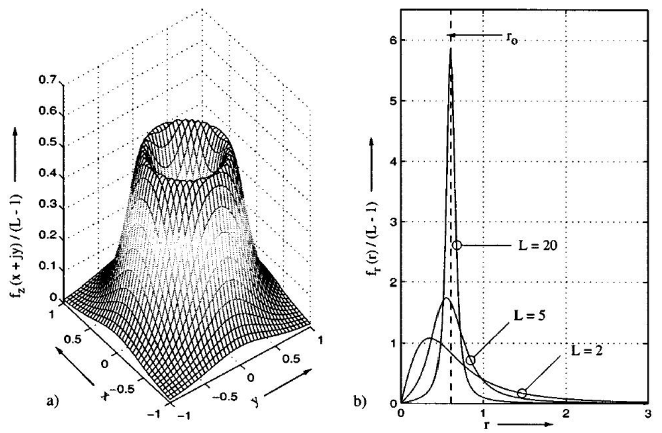

Fig. 9: Normalized density of zeros for impulse response length $L = 5$ (left), marginal density of zeros for exponential decay of channel impulse response variances and different impulse response lengths L [Schober01].

VI. Power Delay Profiles

In Section V we showed the distribution of zeros for theoretical impulse responses. Here, we give some practical examples, considering adequate equalizer design. The European Telecommunications Standards Institute (ETSI) recommends standardized measures to characterize channel impulse responses, called "Delay Power Spectral Densities" or "Power Delay Profiles" (PDPs). These PDPs are based on measurements and represent a simplified form of the channel's autocorrelation function. Although the ACF would be an extensive description of the linear, time-varying impulse responses, it is too cumbersome. Additionally, the PDPs suffice for describing the channel.

There are four PDPs recommended by the ETSI, as shown in Figure 10. For the RA, TU and HT profiles, two alternatives are given. Furthermore, there are six-tap equivalents for the twelve-tap PDPs shown here.

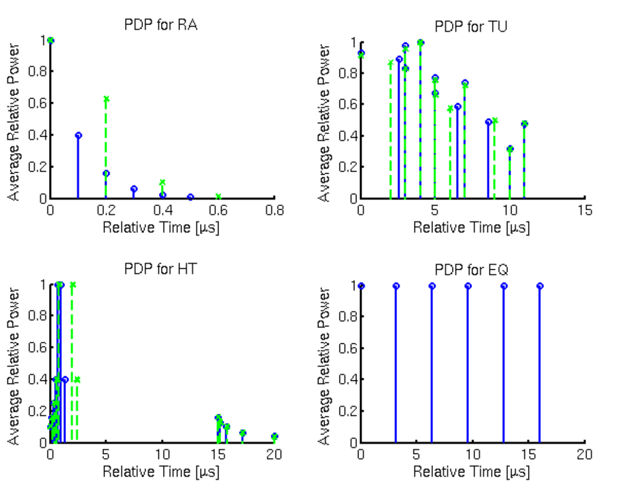

Fig. 10: Recommended PDPs for Rural Area (RA), Typical Urban (TU), Hilly Terrain (HT) and Equalizer Test (EQ). Optional alternative PDPs are plotted using dashed lines.

Below we will show the distributions of zeros for these four PDPs and mention equalizer design considerations for each of them.

In the RA PDP, we have an essentially flat channel (∼"one tap only"). For that, the zeros of $h(t)$ are mainly influenced by the transmit- and receiver input filters. According to [Schober01], $h_T(t)$ is chosen to be an Gaussian-minimum phase shift keying (GMSK) pulse, as standardized for EDGE. The shape of $h_R(t)$ is the receiver designer's choice. In [Schober01], a squared-root raised cosine (SRC) filter with $\alpha = 0.3$ was chosen.

For HT, the distribution of zeros is shown in Figure VI. As can be observed, it is not rotational symmetric. That comes from the impulse response coefficients being correlated. Most of the zeros lie inside $|z| < 1$ - some in the area $|z| \geq 2$, however ⇒ DFE resp. DDFSE/RSSE performance increasable using an allpass prefilter that transforms the zeros inside the unit circle. Only one zero lies inside $0.9 \leq |z| \leq 1.11 \Rightarrow$ truncation of $h[n]$ to $L = 3$ possible.

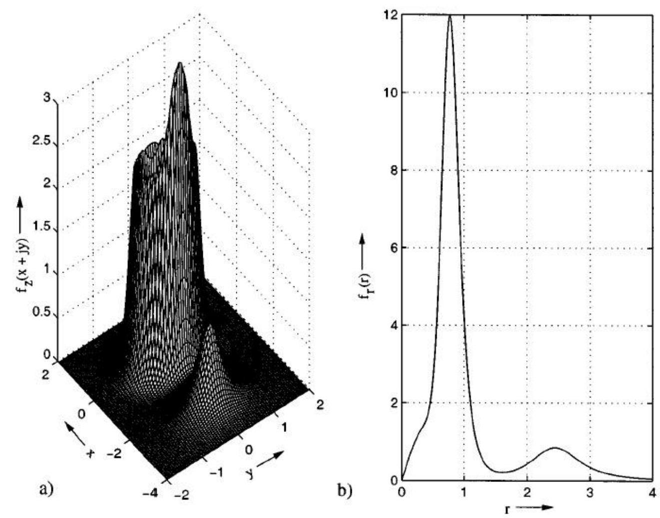

Fig. 11: HT: $f_z(z)$ and $f_r(r)$ for $L = 7$ [Schober01]

With the TU profile, we have a distribution of zeros that is not rotational symmetric as well - see Figure VI. As there are approximately only 0.07 zeros inside $0.9 \leq |z| \leq 1.11$ ⇒ truncation to $L = 2$ possible. We have 1.1 zeros lying outside $|z| = 1$ ⇒ prefiltering with an allpass prefilter to get a minimum phase system response will improve equalizer performance like in the HT profile.

In the EQ profile's distribution of zeros one can observe a peak at $z = 1$, see Figure VI. This indicates a strong correlation between neighboring bins of the channel impulse response. Like the distributions mentioned above, $f_z(z)$ is not circularly symmetric. Due to the correlated bins, there are many zeros near the unit circle with the number of zeros inside the unit circle being equal to the number of zeros outside the unit circle on average. Prefiltering can improve equalizer performance here as well. With 1.2 zeros inside

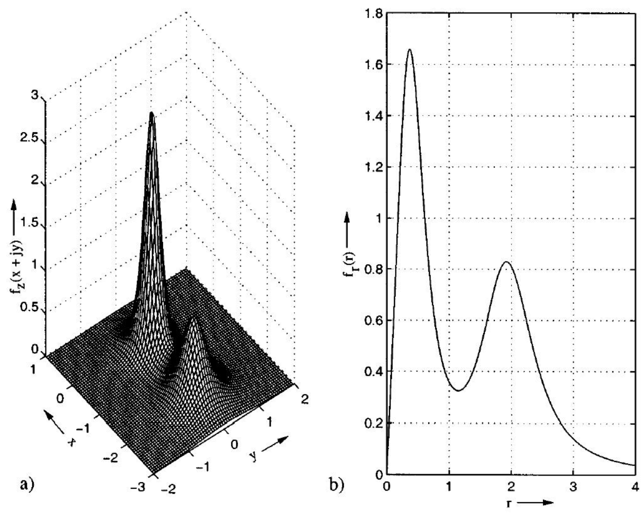

Fig. 12: TU: $f_z(z)$ and $f_r(r)$ for $L = 3$ [Schober01]

$0.9 \leq |z| \leq 1.11$, a truncation of $h[n]$ to $L = 3$ is possible without lowering equalizer performance.

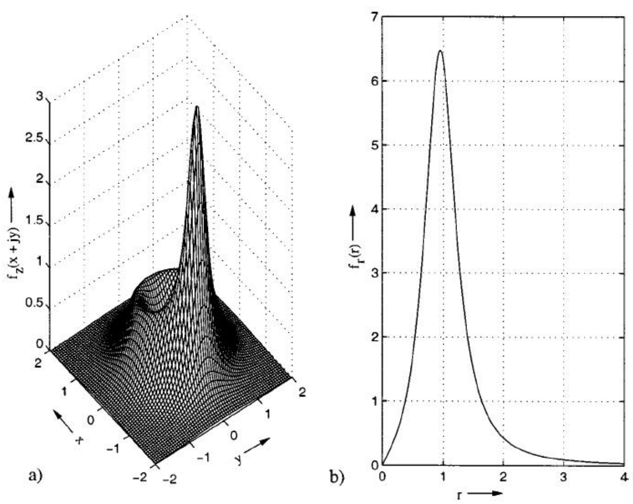

Fig. 13: EQ: $f_z(z)$ and $f_r(r)$ for $L = 6$ [Schober01]

VII. Conclusion

Given the knowledge of the distribution of zeros in a mobile channel, one can design suitable equalizers to improve the overall transmission performance in terms of inter-symbol interference / number of zeros in the complex plane. Considering the application of the statistical equalizer design concepts presented above, it is inevitable to have a statistical model of the channel, however, which is left open at this point.

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