Frequency-Response Masking FIR Filters

Georg Holzmann

June 14, 2007

With the frequency-response masking technique it is possible to design sharp and linear phase FIR filters. Therefore a model filter and its complementary filter is generated and then each delay of these filters is replaced by M delays, which results in periodic, complementary model filters with much sharper transition bands. Finally two masking filters extract the desired bands to generate low-pass, high-pass, bandpass or bandstop filters with arbitrary bandwidth.

Contents

1 Introduction

Frequency-response masking filters are a technique to design sharp low-pass, high-pass, bandpass and bandstop filters with arbitrary passband bandwidth, as first proposed by Lim in [4]. Furthermore this technique generates linear phase FIR filters, which have advantages such as guaranteed stability and are free of phase distortion.

However, usually the problem with FIR filters is the high complexity for sharp filters, therefore the frequency-response masking technique results in filters with very sparse coefficients. Since only a very small fraction of its coefficient values are nonzero, its complexity is very much lower than the infinite wordlength minimax optimum filter (see [1]). With an additional multiplierless design method the complexity can be reduced to a minimum.

This report is structured in five sections. In the rest of this Introduction some general principles of filters preserving phase are shown. Section 2 presents the general idea behind the frequency-response masking technique and Section 3 shows some optimization methods for its parameters. Finally examples are presented in Section 4 and some final remarks are given in Section 5.

1.1 Filters Preserving Phase

Linear Phase FIR filters have the following properties (according to [6]):

- in linear phase FIR filters phase is a linear function of frequency
- they have a symmetric impulse response
- the phase delay $(-\frac{phase}{\omega})$ is constant $(\frac{N-1}{2})$ at every frequency
- also the group delay $(-\frac{d}{d\omega}phase)$ is constant $(\frac{N-1}{2})$

A special case of linear phase filters are zero phase filters, where the phase delay is zero. The impulse response of such a filter is even about time 0

$$h(n) = h(-n)$$

therefore a zero phase filter cannot be causal (see figure 1). A real, even impulse response also corresponds to a real, even frequency response.

Symmetric Linear Phase Filters are derived from a delayed zero-phase filter to be causal, therefore they are symmetric about the midpoint:

$$h(n) = h(N-1-n), n = 0, 1, ..., N-1$$

If H_{ZP} is a zero-phase filter and N is odd, the following relationships are valid for the symmetric linear phase filter $H(z)$:

$$h_{ZP}(n) = h(n - \frac{N-1}{2}), n = 0, 1, ..., N-1$$

$$H(z) = z^{-\frac{N-1}{2}} H_{ZP}(z)$$

$$H(e^{j\omega T}) = e^{-j\omega\frac{N-1}{2}T} H_{ZP}(e^{j\omega T})$$

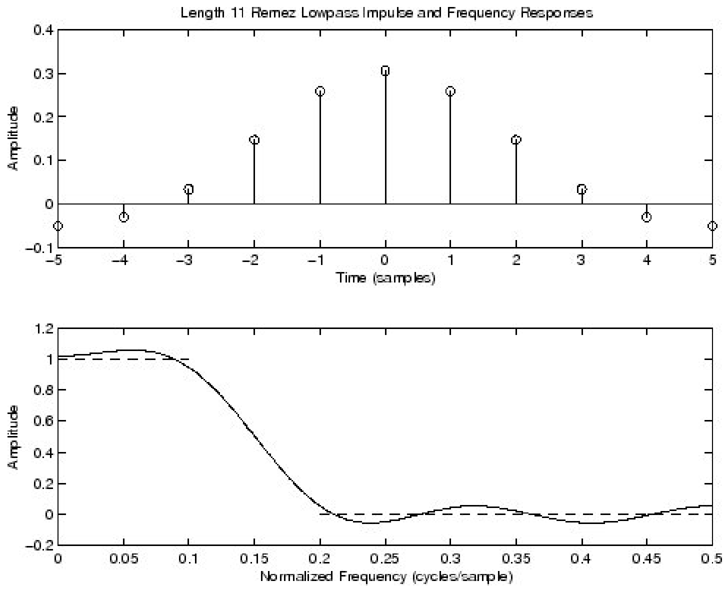

Figure 1: Impulse and frequency response of an non causal, length 11 zero phase FIR lowpass filter, both are real and even.

2 Frequency Response Masking

2.1 Narrow Band Filter Design

In the basic frequency masking principle a linear phase model filter is used and each delay of this filter is replaced by M delays (which corresponds to an up sampling). This results in a periodic filter, the model and its images, with much sharper transition bands. Finally a masking filter extracts the desired band.

Figure 2 illustrates this concept: A low-pass filter $H_a(z)$ with transition width Δ_a is used as a model filter. Then each delay of $H_a(z)$ is replaced by M delays to get a periodic filter with sharper transition bands: $H_b(z) = H_a(z^M)$.
If one uses the masking filter $H_c(z)$ the resulting frequency response $H_d(e^{j\omega}) = H_b(e^{j\omega})H_c(e^{j\omega})$ can be generated with transition width Δ_a/M.
If the masking filter $H_e(z)$ is used the resulting frequency response will be $H_f(e^{j\omega}) = H_b(e^{j\omega})H_e(e^{j\omega})$, which is a bandpass filter.

With this method it is possible to derive sharp filters (transition width Δ_a/M) from filters with much wider transition band (transition width Δ_a). The advantages are, that only a few coefficients in the model filter are nonzero, so the overall complexity is very low.
However, this is only suitable for narrow-band filters, because the passband bandwidth is reduced by the same factor.

2.2 Arbitrary Bandwidth Filter Design

To extend this idea to an arbitrary bandwidth design, we construct an additional complementary filter to the model filter F_a.

Consider a filter F_c complementary to the model filter F_a. The z-transform of the symmetric

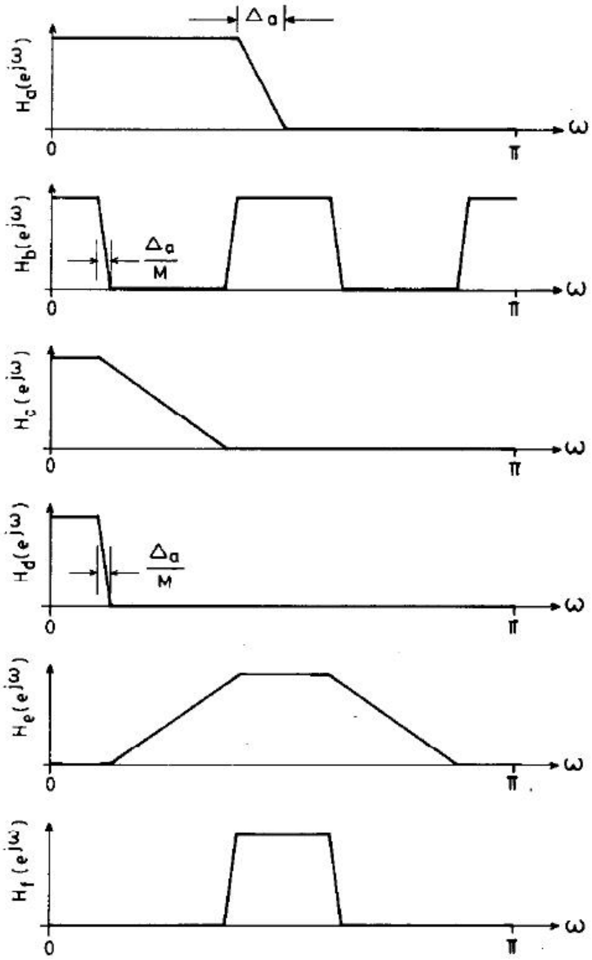

Figure 2: Frequency-response masking principle for narrow band filters. Description in the text.

linear phase filter F_a is

$$F_a(z) = z^{-\frac{N-1}{2}} F_{a,ZP}(z)$$

where $F_{a,ZP}(z)$ is a zero-phase filter and N is odd.
The complementary filter F_c can be written as

$$F_c(z) = z^{-\frac{N-1}{2}} (1 - F_{a,ZP}(z))$$

which results in

$$F_c(z) = z^{-\frac{N-1}{2}} - F_a(z)$$

Therefore we can implement the complementary filter F_c by subtracting the output of F_a from a delayed version of the input (see figure 3).

This can be realized without using extra delays, when we reuse the delays in F_a as illustrated in figure 4.

If now two masking filters, F_{Ma} and F_{Mc} for F_a and F_c, are used, it's possible to design wide-band sharp filters as shown in figure 5. Then we get the following filter $F(z)$:

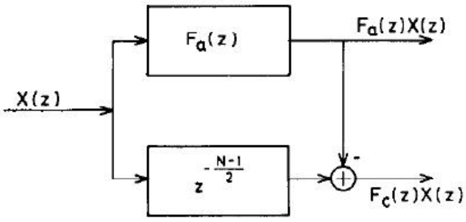

Figure 3: Realization of the complementary filter F_c.

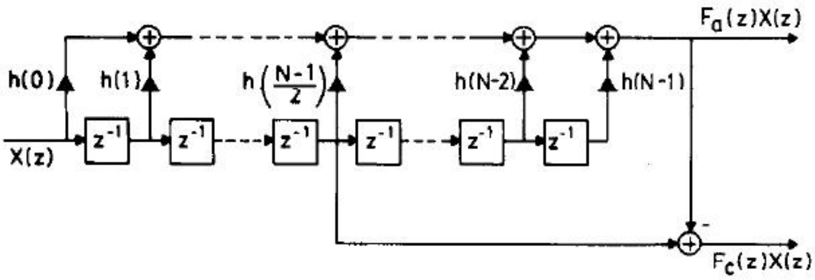

Figure 4: Realization of the complementary filter F_c reusing the delays of F_a.

$$F(z) = F_a(z^M)F_{Ma}(z) + (z^{-\frac{N-1}{2}} - F_a(z^M))F_{Mc}(z)$$

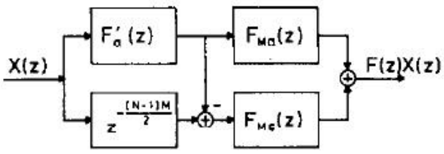

Figure 5: General structure of frequency-response masking filters.

The whole concept is explained again according to figure 6: A model filter F_a with cutoff frequencies θ and ϕ and its complementary filter F_c is used. Replacing each delay of F_a and F_c by M delays results in periodic, complementary model filters.
In the first example the masking filters F_{Ma} and F_{Mc} are used and one gets a resulting frequency response $F(e^{j\omega})$ with band edges ω_P and ω_S.
In the second example the masking filter results in a different frequency response $F(e^{j\omega})$.

Here it is possible to distinguish between two cases:

- Case1: the frequency response of F near the transition band is determined mainly by F_a (as in the first example of figure 6), then pass- and stopband is defined by

$$\omega_P = \frac{2m\pi + \theta}{M}, \omega_S = \frac{2m\pi + \phi}{M}$$

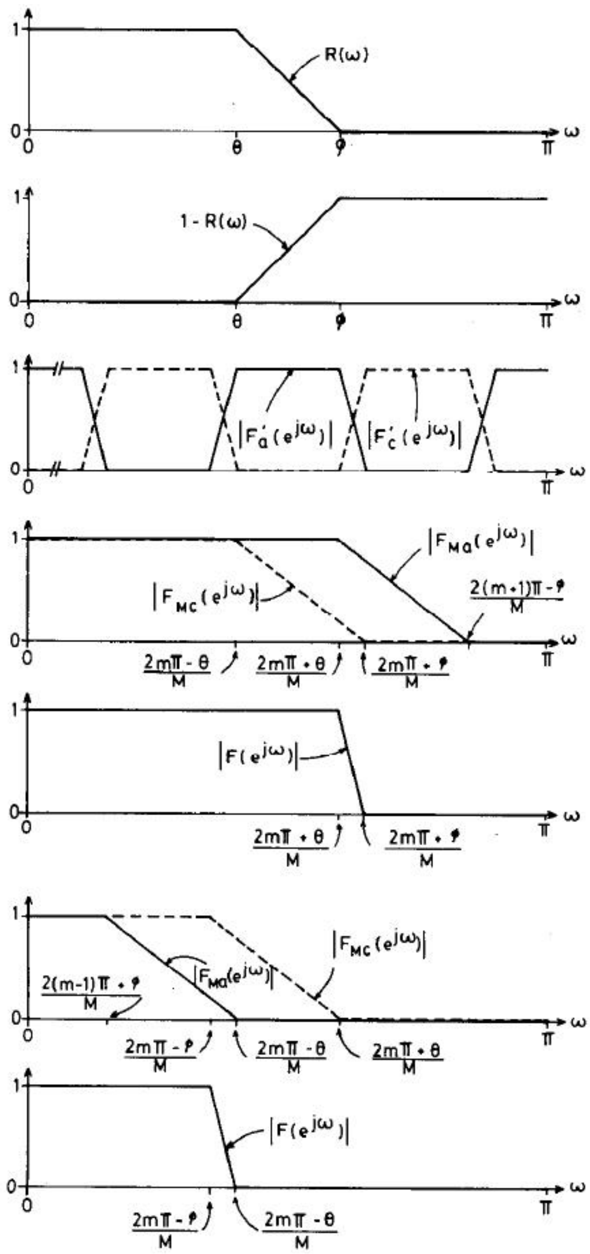

Figure 6: Frequency-response masking principle for arbitrary bandwidth filters. Description in the text.

where m is an integer, indicating the images of the model filter.

- Case2: the frequency response of F near the transition band is mainly determined by F_c (as in the second example of figure 6), then pass- and stopband is defined by

$$\omega_P = \frac{2m\pi - \phi}{M}, \omega_S = \frac{2m\pi - \theta}{M}$$

If one wants to synthesize a frequency-response masking filter, the parameters ω_P and ω_S are given and m,M,θ,ϕ must be determined. M should be chosen that the overall complexity of the filter is minimized: for increasing M the masking filters must be sharper (higher complexity) and the model filters can be broader (lower complexity). Therefore this leads to an optimization problem, which is the topic of the next section.

3 Parameter Optimization

3.1 Ripples of F

An other important issue are the ripples of the overall filter $F(e^{j\omega})$. In order to choose the right parameters we have to analyze the influence of the model and masking filters on the ripples.

Let $G(\omega)$ be the desired value and $\delta(\omega)$ the deviation from this value for each filter. Then we can express the filter $F(e^{j\omega})$ with G and δ of the individual filters:

$$G(\omega) + \delta(\omega) = (G_{Ma}(\omega) + \delta_{Ma}(\omega))(G_a(\omega) + \delta_a(\omega))$$

$$+(G_{Mc}(\omega) + \delta_{Mc}(\omega))(1 - G_a(\omega) - \delta_a(\omega))$$

We will now examine the effects of the parts of this formula for three different frequency ranges.

Frequency Range 1: $G_{Ma}(\omega) = G_{Mc}(\omega) = 1$, so we are in the passband of F. In this range $G(\omega) = 1$ and δ only depends on the masking filters:

$$G(\omega) = 1$$

$$G_a(\omega) = 1, \delta(\omega) \approx \delta_{Ma}(\omega)$$

$$G_a(\omega) = 0, \delta(\omega) \approx \delta_{Mc}(\omega)$$

Frequency Range 2: $G_{Ma}(\omega) = G_{Mc}(\omega) = 0$, which is the stopband of F. Here $G(\omega) = 0$ and δ is the same as in Range 1:

$$G(\omega) = 0$$

$$G_a(\omega) = 1, \delta(\omega) \approx \delta_{Ma}(\omega)$$

$$G_a(\omega) = 0, \delta(\omega) \approx \delta_{Mc}(\omega)$$

Therefore, as a result from analyzing Range 1 and 2, F_{Ma} and F_{Mc} could be interpreted as low-pass filters with don't care bands within their pass- and stopbands (see figure 7), because F only depends on $\delta_{Ma}(\omega)$ if $G_a(\omega) = 1$ or on $\delta_{Mc}(\omega)$ if $G_a(\omega) = 0$. These don't care bands help to reduce the complexity of the masking filters.

Frequency Range 3: $G_{Ma}(\omega) \neq G_{Mc}(\omega)$, this is the transition band of F. Here $\delta(\omega)$ is a more complex function of $\delta_a(\omega)$, $\delta_{Ma}(\omega)$ and $\delta_{Mc}(\omega)$. However, it is possible to design the filter F_a such that $\delta_a(\omega)$ partially compensates $\delta_{Ma}(\omega)$ and $\delta_{Mc}(\omega)$ in the transition band, as described in 3.2.

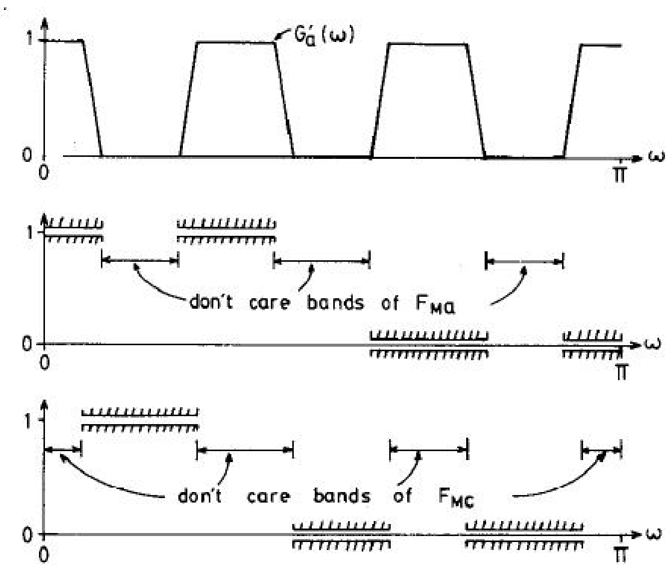

Figure 7: Don't Care bands of the masking filters.

3.2 Optimizing F

As already said, F_a has to be designed to compensate for $\delta_{Ma}(\omega)$ and $\delta_{Mc}(\omega)$, therefore a linear equation relating $\delta(\omega)$ and F_a must be obtained.
According to [4] the following relation can be used:

$$\delta(\omega) = F_{a,ZP}(M\omega)(G_{Ma}(\omega) + \delta_{Ma}(\omega) - G_{Mc}(\omega) - \delta_{Mc}(\omega)) + G_{Mc}(\omega) + \delta_{Mc}(\omega) - G(\omega)$$

Now the minimization of $|\delta(\omega)|$ in the transition band is a linear programming filter design problem and can be solved by a standard mathematical programming package as described in [3].

3.3 Optimizing M

There is no closed-form analytic expression for finding the optimum value of M (in [4]).
Therefore a good choice of M can be obtained by estimating the filter complexity for each M (the number of nonzero multipliers) and then selecting the M which corresponds to the lowest estimate.

However, many more recent papers address the right selection of the parameter M and suggest optimized designs (e.g. in [5]).

3.4 Multistage Frequency Response Masking

The model and masking filters may again be synthesized using the frequency response masking technique, producing a multistage frequency response masking design. In figure 8 a two-stage design is shown, replacing the model filter with another frequency-response masking filter.

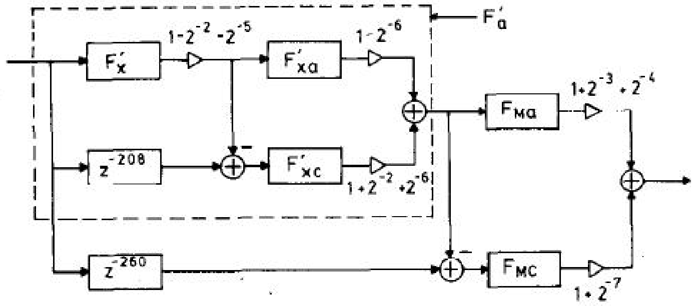

Figure 8: Multistage Frequency Response Masking: the model filter is replaced with another frequency-response masking filter.

More details to find the right parameters for multistage frequency-response masking filters can be found in [5].

3.5 Powers-of-Two Design Technique

The complexity of the filter may be further reduced by constraining all the coefficient values to be a sum or difference of two powers-of-two using the powers-of-two design technique as described in [2].
In this case, all the multiplications can be performed just by using shifts and adds.

4 Examples

4.1 Single-Stage Design

In this example a single-stage frequency-response masking filter, using the power-of-two design technique is designed (taken from [4]).
The synthesized filter should meet the following specifications:

- bandedges at 0.3 and 0.305 sampling frequencies
- a maximum passband deviation of 0.1 dB
- a minimum stopband attenuation of -40 dB

The frequency response of the synthesized filter is shown in figure 9.
This filter requires 202 shift-add operations per sampling interval, whereas the infinite precision minimax optimum design requires 383 multiply and 382 add operations.

4.2 Multi-Stage Design

The next example uses a multi-stage design technique (taken from [5]).
The synthesized filter should meet the following specifications:

- bandedges at 0.2 and 0.2001 sampling frequencies
- a maximum passband deviation of 0.05 dB

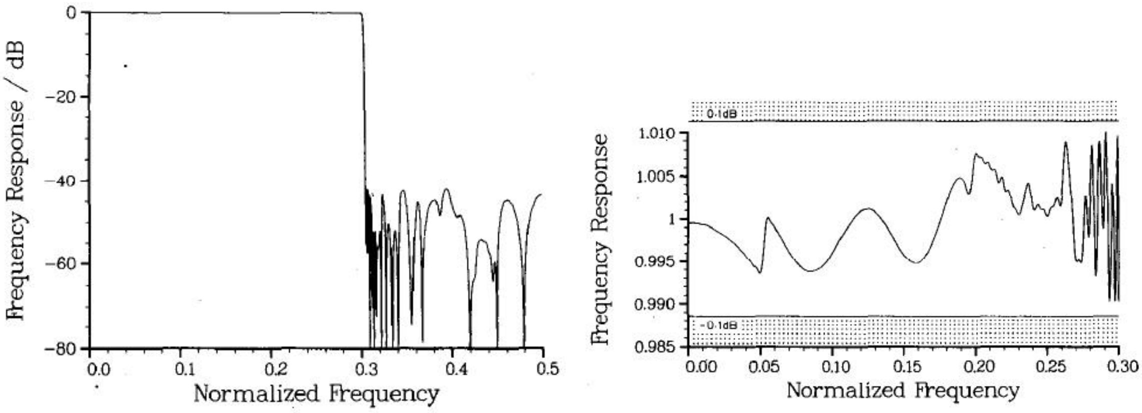

Figure 9: Synthesized, single-stage frequency-response masking low-pass filter as described in the text. Left the whole frequency response, right zoomed into the passband.

- a minimum stopband attenuation of -50 dB

The frequency response of the synthesized filter is shown in figure 10.

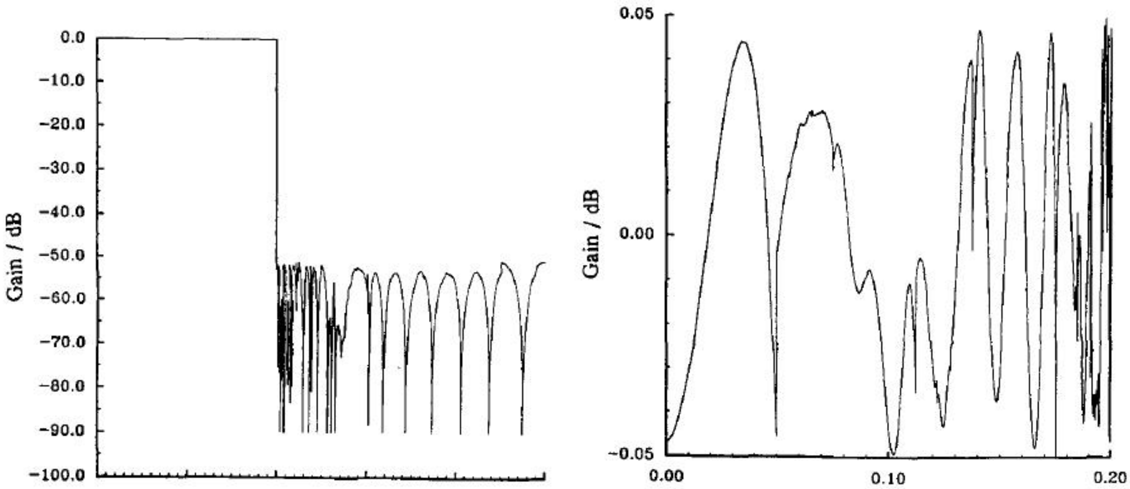

Figure 10: Synthesized, multi-stage frequency-response masking low-pass filter as described in the text. Left the whole frequency response, right zoomed into the passband.

In this example a five stage design was used with $M_1 = M_2 = M_3 = M_4 = 4$ and $M_5 = 3$. The total number of multipliers is 125, whereas the infinite precision minimax optimum design requires 12055 multiplications (!).

5 Conclusion

In the frequency-response masking technique a model filter and its complementary filter is generated, as first proposed in [4]. Then each delay of these filters is replaced by M delays, which results in periodic, complementary model filters with much sharper transition bands. Finally two masking filters extract the desired band.

So it is possible to design sharp low-pass, high-pass, bandpass and bandstop linear phase filters with arbitrary passband bandwidth and the complexity of the resulting filter is very low, because only a small fraction of its coefficients are nonzero. With additional optimization methods (like multiplierless or multi-stage design), which is subject of many recent papers, it is possible to reduce the complexity to a minimum.

References

[1] O. Herrmann. Practical design rules for optimum finite impulse response lowpass digital filters. 1973. Bell Syst. Tech. J., vol. 52.

[2] Y. C. Lim and S. R. Parker. Fir filter design over a dicrete powers-of-two coefficient space. 1983. IEEE transactions on circuits and systems.

[3] Yong Ching Lim. Efficient special purpose linear programming for fir filter design. 1983. IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-31.

[4] Yong Ching Lim. Frequency-response masking approach for the synthesis of sharp linear phase digital filters. 1986. IEEE transactions on circuits and systems.

[5] Yong Ching Lim. The optimum design of one- and two-dimensional fir filters using the frequency response masking technique. 1993. IEEE transactions on circuits and systems.

[6] Julius O. Smith. Introduction to digital filters. 2006. Center for Computer Research in Music and Acoustics (CCRMA), Stanford University.