

# Frequency-Response Masking FIR Filters

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## Introduction

Introduction to FRM Filters  
Filters Preserving Phase

## Frequency Response Masking

Narrow Band Filter Design  
Arbitrary Bandwidth Filter Design

## Parameter Optimization

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Optimizing F and M  
Further Optimization

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## Conclusion

# Frequency-Response Masking Filters

- ▶ frequency-response masking filters are a technique to design sharp low-pass, high-pass, bandpass and bandstop filters with arbitrary passband bandwidth
- ▶ furthermore linear phase FIR filters are generated, which have advantages such as guaranteed stability and are free of phase distortion
- ▶ however, the problem with FIR filters is the high complexity for sharp filters

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# Advantages

- ▶ with the frequency-response masking technique the resulting filter has very sparse coefficients
- ▶ since only a very small fraction of its coefficient values are nonzero, its complexity is very much lower than the infinite wordlength minimax optimum filter
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# Linear Phase FIR Filters

- ▶ in linear phase FIR filters phase is a linear function of frequency
- ▶ they have a symmetric impulse response
- ▶ the phase delay ( $-\frac{phase}{\omega}$ ) is  $\frac{N-1}{2}$  at every frequency
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# Zero Phase Filters

- ▶ are a special case of linear phase filters, where the phase delay is zero
- ▶ impulse response of a zero phase filter is even about time 0:

$$h(n) = h(-n)$$

therefore this filter cannot be causal

- ▶ a real, even impulse response corresponds to a real, even frequency response
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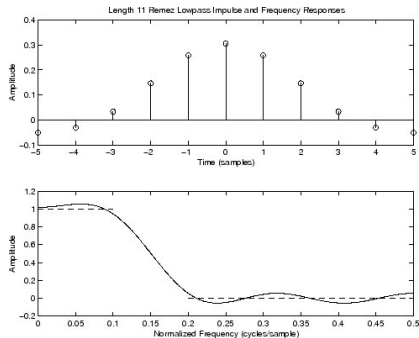
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# Zero Phase Filters

Impulse and frequency response of a length 11 zero-phase FIR lowpass filter:



# Symmetric Linear Phase Filters

- ▶ are derived from a delayed zero-phase filter
- ▶ are causal and symmetric about the midpoint:

$$h(n) = h(N - 1 - n), n = 0, 1, \dots, N - 1$$

- ▶  $H_{ZP}$  is a zero-phase filter,  $N$  is odd:

$$h_{ZP}(n) = h\left(n - \frac{N-1}{2}\right), n = 0, 1, \dots, N - 1$$

$$H(z) = z^{-\frac{N-1}{2}} H_{ZP}(z)$$

$$H(e^{j\omega T}) = e^{-j\omega \frac{N-1}{2} T} H_{ZP}(e^{j\omega T})$$

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## Basic Principle

The basic principle of frequency masking is the following:

- ▶ in a linear phase model filter each delay is replaced by  $M$  delays
- ▶ this results in a periodic filter with much sharper transition bands
- ▶ finally a masking filter extracts the desired band

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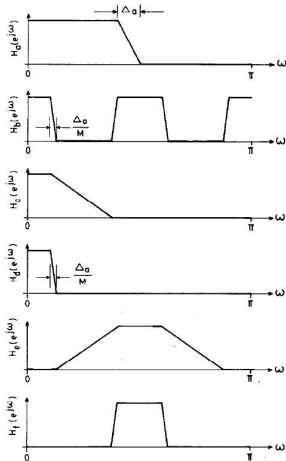
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# Narrow Band Filter Design

## Frequency Masking Principle:



- ▶ low-pass filter  $H_a(z)$  with transition width  $\Delta_a$  (model filter)
- ▶ replacing each delay by  $M$  delays:  $H_b(z) = H_a(z^M)$
- ▶ masking filter  $H_c(z)$
- ▶ resulting frequency response:  $H_d(e^{j\omega}) = H_b(e^{j\omega})H_c(e^{j\omega})$  with transition width  $\Delta_a/M$
- ▶ masking filter  $H_e(z)$
- ▶ resulting frequency response:  $H_f(e^{j\omega}) = H_b(e^{j\omega})H_e(e^{j\omega})$



# Narrow Band Filter Design

- ▶ This describes a method of deriving sharp filters ( $\Delta_a/M$ ) from filters with much wider transition band ( $\Delta_a$ )
- ▶ Advantages: only a few coefficients in the model filter are nonzero, so the complexity is very low
- ▶ Problem: only suitable for narrow-band filters, because the passband bandwidth is reduced by the same factor

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## Arbitrary Bandwidth Design

Consider a filter  $F_c$  complementary to the masking filter  $F_a$ :

- ▶ z-transform of the symmetric linear phase filter  $F_a$ :

$$F_a(z) = z^{-\frac{N-1}{2}} F_{a,ZP}(z)$$

where  $F_{a,ZP}(z)$  is a zero-phase filter and  $N$  is odd

- ▶ the complementary filter  $F_c$ :

$$F_c(z) = z^{-\frac{N-1}{2}} (1 - F_{a,ZP}(z))$$

- ▶ this results in

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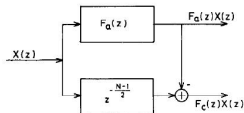
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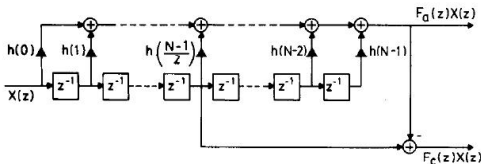
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## Complementary Filter Pair

$F_c$  can be implemented by subtracting the output of  $F_a$  from a delayed version of the input:

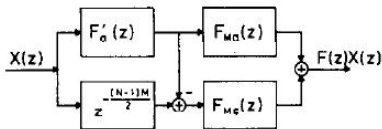


without extra delays:



# Masking Filters

If two masking filters,  $F_{Ma}$  and  $F_{Mc}$  for  $F_a$  and  $F_c$ , are used, it's possible to design wide-band sharp filters:

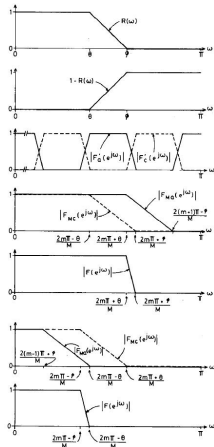


$$F(z) = F_a(z^M)F_{Ma}(z) + (z^{-\frac{N-1}{2}} - F_a(z^M))F_{Mc}(z)$$



# Arbitrary Bandwidth Filter Design

## Arbitrary Bandwidth Masking Principle:



- ▶ model filter  $F_a$ , cutoff frequencies  $\theta$  and  $\phi$
- ▶ complementary filter  $F_c$
- ▶ replacing each delay of  $F_a$  and  $F_c$  by  $M$  delays to get periodic, complementary model filters
- ▶ masking filters  $F_{Ma}$  and  $F_{Mc}$
- ▶ resulting frequency response  $F(e^{j\omega})$  with band edges  $\omega_P$  and  $\omega_S$
- ▶ other masking filters  $F_{Ma}$  and  $F_{Mc}$
- ▶ resulting frequency response  $F(e^{j\omega})$

# Arbitrary Bandwidth Filter Design

One can distinguish two cases:

- ▶ Case1: the frequency response of  $F$  near the transition band is determined mainly by  $F_a$ , pass- and stopband is defined by

$$\omega_P = \frac{2m\pi + \theta}{M}, \omega_S = \frac{2m\pi + \phi}{M}$$

- ▶ Case2: mainly determined by  $F_c$ , then pass- and stopband is defined by

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# Synthesis Problem

In a synthesis problem the following has to be considered:

- ▶  $\omega_P$  and  $\omega_S$  are given and  $m, M, \theta, \phi$  must be determined
- ▶  $M$  should be chosen that the overall complexity of the filter is minimized
- ▶ this leads to an optimization problem:  
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# Ripples of F

In this section the ripples of the overall filter  $F(e^{j\omega})$  are analyzed:

- ▶ let  $G(\omega)$  be the desired value and  $\delta(\omega)$  the deviation from this value for each filter
- ▶ for  $F(e^{j\omega})$  this leads to:

$$G(\omega) + \delta(\omega) = (G_{Ma}(\omega) + \delta_{Ma}(\omega))(G_a(\omega) + \delta_a(\omega)) \\ + (G_{Mc}(\omega) + \delta_{Mc}(\omega))(1 - G_a(\omega) - \delta_a(\omega))$$

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## Ripples, Fr. Range 1 + 2

- ▶ *Frequency Range 1:  $G_{Ma}(\omega) = G_{Mc}(\omega) = 1$  (passband)*

$$G(\omega) = 1$$

$$G_a(\omega) = 1, \delta(\omega) \approx \delta_{Ma}(\omega)$$

$$G_c(\omega) = 0, \delta(\omega) \approx \delta_{Mc}(\omega)$$

- ▶ *Frequency Range 2:  $G_{Ma}(\omega) = G_{Mc}(\omega) = 0$  (stopband)*

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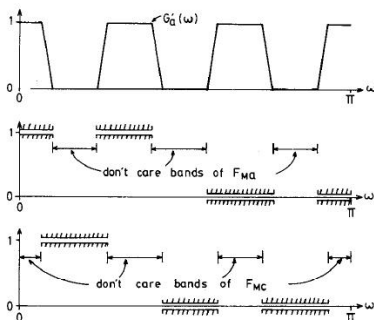
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## Don't Care Bands

Therefore  $F_{Ma}$  and  $F_{Mc}$  could be interpreted as low-pass filters with don't care bands within their pass- and stopbands:



These don't care bands help to reduce the complexity of the masking filters.

## Ripples, Fr. Range 3

*Frequency Range 3:  $G_{Ma}(\omega) \neq G_{Mc}(\omega)$ , transition band*

- ▶ here  $\delta(\omega)$  is a function of  $\delta_a(\omega)$ ,  $\delta_{Ma}(\omega)$  and  $\delta_{Mc}(\omega)$
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## Optimization of F

$F_a$  has to be designed to compensate for  $\delta_{Ma}(\omega)$  and  $\delta_{Mc}(\omega)$

- ▶ a linear equation relating  $\delta(\omega)$  and  $F_a$  must be obtained:

$$\delta(\omega) = F_{a,ZP}(M\omega)(G_{Ma}(\omega) + \delta_{Ma}(\omega) - G_{Mc}(\omega) - \delta_{Mc}(\omega)) \\ + G_{Mc}(\omega) + \delta_{Mc}(\omega) - G(\omega)$$

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# Optimization of M

- ▶ there is no closed-form analytic expression for finding the optimum  $M$  (in the paper from 1986)
- ▶ a good choice of  $M$  can be obtained by estimating the filter complexity for each  $M$  (nonzero multipliers) and then selecting the  $M$  which corresponds to the lowest estimate
- ▶ however, many more recent papers address the right selection of the parameter  $M$  and suggest optimized designs

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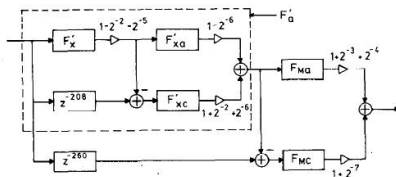
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# Multistage Frequency Response Masking Design

The model and masking filters may again be synthesized using the frequency response masking technique, producing a multistage frequency response masking design:



Optimizing this technique is again subject of many more recent papers.

# Powers-of-Two Design Technique

- ▶ the complexity of the filter may be further reduced by constraining all the coefficient values to be a sum or difference of two powers-of-two using the powers-of-two design technique
- ▶ in this case, the multiplication can be performed just by using shifts and adds

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## Single-Stage Design

- ▶ The single-stage FRM low-pass filter, using the powers-of-two design technique, should meet the following specifications:
- ▶ bandedges at 0.3 and 0.305 sampling frequencies, maximum passband deviation is 0.1 dB and minimum stopband attenuation is -40 dB
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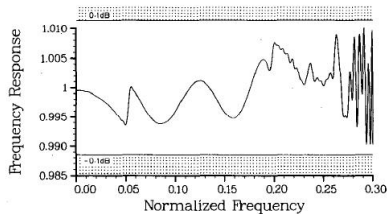
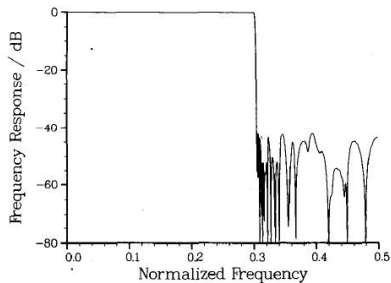


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# Single-Stage Design Frequency Response

Frequency Response of the single-stage FRM low-pass filter:



## Multi-Stage Design

- ▶ Now a multi-stage FRM low-pass filter with the following specifications should be designed:
- ▶ bandedges at 0.2 and 0.2001 sampling frequencies, maximum passband deviation is 0.05 dB and minimum stopband attenuation is -50 dB
- ▶ a five stage design was used with  $M_1 = M_2 = M_3 = M_4 = 4$  and  $M_5 = 3$
- ▶ the total number of multipliers is 125, whereas the infinite precision minimax optimum design requires 12055 multiplications (!)

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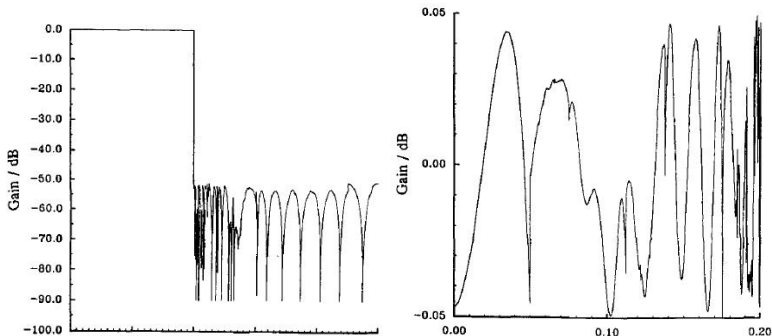
- ▶ Now a multi-stage FRM low-pass filter with the following specifications should be designed:
- ▶ bandedges at 0.2 and 0.2001 sampling frequencies, maximum passband deviation is 0.05 dB and minimum stopband attenuation is -50 dB
- ▶ a five stage design was used with  $M1 = M2 = M3 = M4 = 4$  and  $M5 = 3$
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# Multi-Stage Design Frequency Response

Frequency Response of the multi-stage FRM low-pass filter:



# Conclusion

- ▶ in the frequency-masking technique a model filter and its complementary filter is generated
- ▶ then each delay of these filters is replaced by  $M$  delays
- ▶ this results in periodic, complementary model filters with much sharper transition bands
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# Questions

Questions ... ?