Introduction to ST Codding

TUGraz

Institute of Signal Processing and Speech Communication

Seminar Presentation

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Outline of the Presentation

1. Introduction
2. Coding and interleaving architecture
3. ST coding for frequency flat channels
4. ST coding for frequency selective channels

1 - Introduction

-ST Coding is a means /technique to address broad goals in maximizing link performance, maximize link throughput and minimize error

- -signaling rate (in bps/Hz or bits per transmission)
- -diversity gain (or diversity order, which is the slope of the error vs SNR curve)
- -coding gain (from code design that increases effective SNR)
- -array gain (from antenna combining that also increases effective SNR)

-Improving error performance implies maximizing diversity (upper bound is $M_T M_R$)

-increasing coding gain depends on the minimum distance of the code

-increasing array gain is upper-bounded by M_R

2 - Coding and interleaving architecture

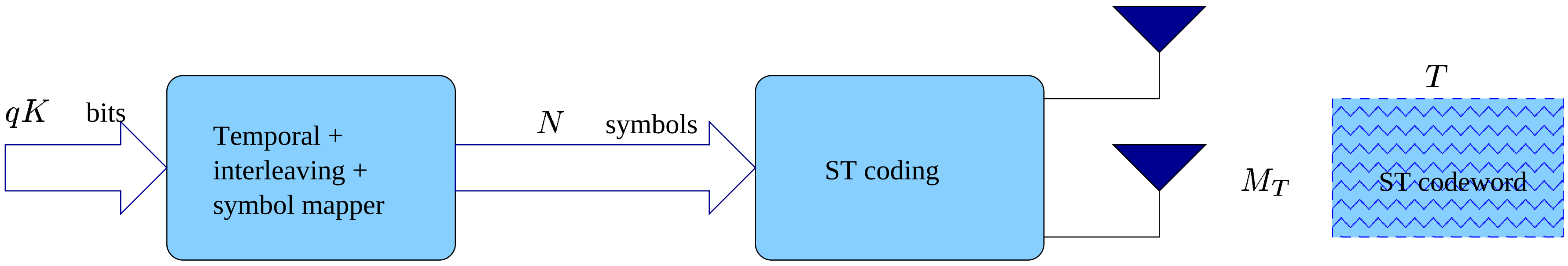


Figure 1: Coding archictecture

-A block of qK bits is input for temporal coding, interleaving and mapping and in the process $q(N-K)$ parity bits are added and N symbols are output.

-Nsymbols are added to the ST coder that adds an additional $M_T T - N$ parity symbols and packs the resulting $M_T T$ symbols into an $M_T \times T$ frame of length T

-The block/frame transmitted over T symbol period is noted as an ST codeword.

Coding and interleaving architecture continued

-signaling data rate is qK/T bits/transmission

(should be with in channel capacity)

$$\frac{qK}{T} = q\left(\frac{qK}{qN}\right)\left(\frac{N}{T}\right) = qr_tr_s \tag{1}$$

-r_t and r_s are temporal and spacial code rates respectively

-the spacial code rate r_s varies between 0 to M_T

-interleaving is used to spread burst errors

3 - ST coding for frequency flat channels

3.1 Signal model

-consider a MIMO system with M_T and M_R and a code word $M_T \times T$

-we denote ST code word by $\mathbf{S} = [\mathbf{s}[1]\mathbf{s}[2] \ldots \mathbf{s}[T]]$, where

$\mathbf{s}[k] = [s_1[k] \ldots s_{M_T}[k]]^T$, is the transmitted vector symbols

over kth symbols period.

$$\mathbf{y}[k] = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{s}[k] + \mathbf{n}[k], k = 1, 2, \ldots, T, \quad (2)$$

$$\mathbf{Y} = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{S} + \mathbf{N}, k = 1, 2, \ldots, T, \quad (3)$$

where $\mathbf{Y} = [\mathbf{y}[1]\mathbf{y}[2] \ldots \mathbf{y}[T]]$ and $\mathbf{N} = [\mathbf{n}[1]\mathbf{n}[2] \ldots \mathbf{n}[T]]$

are matrices of size $M_R \times T$

Decoding

-receiver is assumed to use a $\mathbf{ML}$ detection with perfect channel knowledge

$$\widehat{S} = \arg\min_s \|\mathbf{Y} - \mathbf{HS}\|_F^2$$
$$= \arg\min_s \|\mathbf{y}[k] - \sqrt{\frac{E_s}{M_T}}\mathbf{Hs}[k]\|_F^2 \quad (4)$$

-minimization is performed over all admissible codewords S

3.2 ST codeword design criteria:

-pairwise error probability

$$P(\mathbf{S}_{(i)} \rightarrow \mathbf{S}_{(j)}|\mathbf{H}) = Q\left(\sqrt{\frac{E_s\|\mathbf{H}(\mathbf{S}_{(i)} - \mathbf{S}_{(j)})\|_F^2}{2M_T N_o}}\right)$$
$$= Q\left(\sqrt{\frac{\rho\|\mathbf{HE}_{i,j}\|_F^2}{2M_T}}\right) \quad (5)$$

ST code word design criteria continued:

$\mathbf{E}_{i,j} = \mathbf{S}^{(i)} - \mathbf{S}^{(j)}$ is the $M_T \times T$ codeword difference matrix

$\rho = E_s/N_0$ is the SNR, applying Chernoff bound

$$P(\mathbf{s}_{(i)} \rightarrow \mathbf{S}_{(j)}|\mathbf{H}) \leq e^{-\frac{\rho\|\mathbf{H}\mathbf{E}_{i,j}\|_F^2}{4M_T}} \quad (6)$$

$$P(\mathbf{s}_{(i)} \rightarrow \mathbf{S}_{(j)}|\mathbf{H}) \leq \frac{1}{\left(\prod_{k=1}^{r(\mathbf{G}_{i,j})} \lambda_k(\mathbf{G}_{i,j})\right)^{M_R}} \left(\frac{\rho}{4M_T}\right)^{-r(\mathbf{G}_{i,j})\mathbf{M}_R} \quad (7)$$

where $\lambda_k(\mathbf{G}_{i,j})(k = 1, 2, 3, \ldots, r(\mathbf{G}_{i,j}))$ are non-zero eigenvalues of

$\mathbf{G}_{i,j} = \mathbf{E}_{i,j}\mathbf{E}_{i,j}^H$.

-equation (7) leads to the two-well known criteria for ST codeword construction ”rank criterion” and ”determinant criterion”

Rank criterion:

-the rank criterion optimizes the spatial diversity extracted by an ST code.

-to extract the full spatial diversity gain of $M_T M_R$, the code design should

be such that $\mathbf{E}_{i,j}$ between any pair of codewords is full-rank $(r(\mathbf{G}_{i,j}) = \mathbf{M_T})$.

Determinant criterion:

-the determinant criterion optimizes the coding gain. Referring eq (7), the coding

gain depends on the term:

$$\left(\prod_{k=1}^{r(\mathbf{G}_{i,j})} \lambda_k(\mathbf{G}_{i,j})\right)$$

-for high coding gain, this term should be maximum over all possible pairs of

codeword matrices $\mathbf{S}^{(i)}$ and $\mathbf{S}^{(j)}$

3.3 ST diversity coding $(r_s \leq 1)$:

We have two flavors of ST diversity codes for discussion, STTC and STBC, which extract full diversity order $(M_R M_T)$ with $r_s \leq 1$

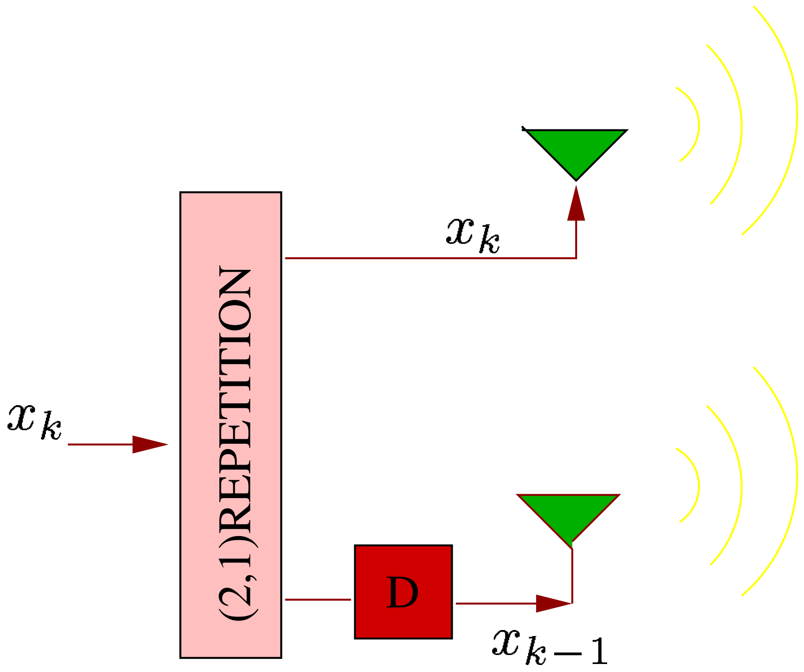
STTC

-they map an arbitrary number of information symbols to antenna outputs according to a finite-state machine.

-they are extensions of convolutional codes and trellis-coded modulation

-these codes can be designed to extract diversity gain and coding gain using the mentioned criteria.

-the simplest example of a STTC is the delay-diversity scheme



-this transmitter can be modeled as a finite state machine,

-previous and current sequence of input symbols uniquely defines a trellis path

-the number of trellis states is equal to the size of the input alphabet.

-the receiver can implement ML sequence detection using Viterbi algorithm

-fixing the number of input symbols to k

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_{K-1} & x_K & 0 \\ 0 & x_1 & x_2 & x_3 & \dots & x_{K-1} & x_K \end{bmatrix} \quad (8)$$

-the rank criterion ensures for the above code full diversity

-improved code is a case of 8-PSK alphabet $(A = e^{jl\pi/4})$

$$\mathbf{A}(\mathbf{l}) = \begin{bmatrix} l_1 & l_2 & l_3 & \dots & \\ 0 & 5l_1 & 5l_2 & 5l_3 & \dots \end{bmatrix} \quad (9)$$

-in the above representation the parity symbol index is 5l (modulo8),

$C = \{00, 15, 22, 37, 44, 51, 66, 73\}$, optimal, maximized d_{min}

-using this code in the delay-diversity transmitter structure, two antenna

8-state 8-PSK STTC is achieved

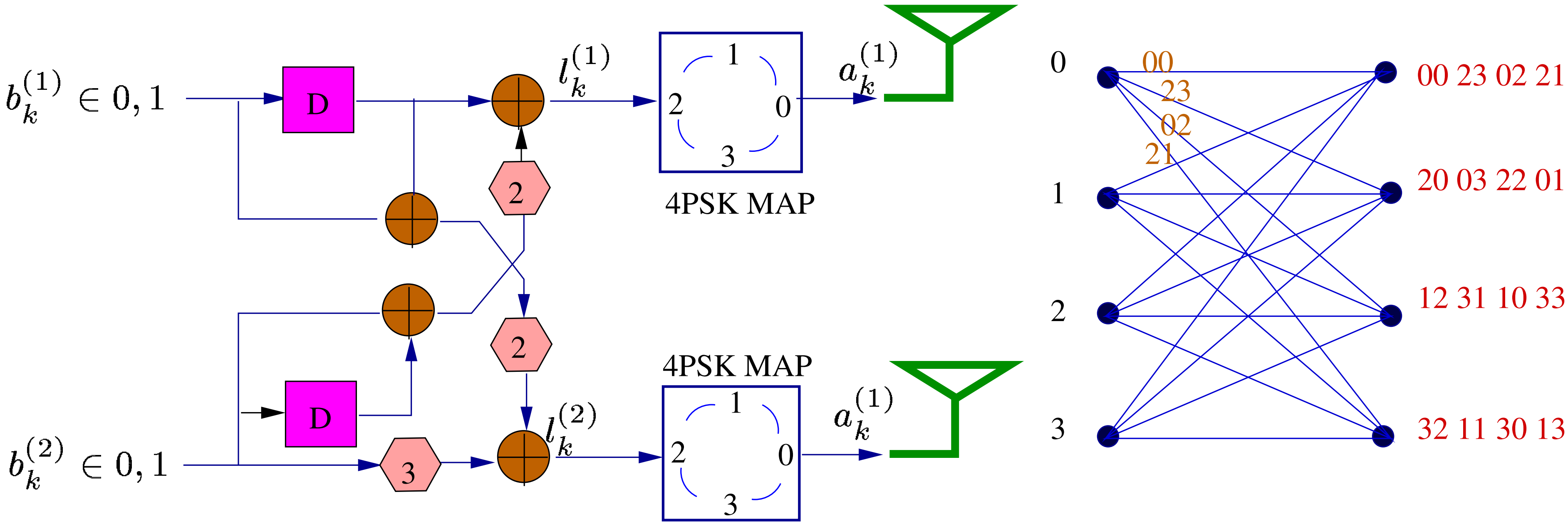


Fig 2 Trellis diagram for a 4-QAM, four state trellis code for M_T=2 with a rate of 2bps/Hz

ST diversity coding $(r_s \leq 1)$ continued

-number of nodes in the trellis diagram corresponds to number of states

-M_T entries in each constellation correspond to the symbols to be transmitted

from the M_T antennas

<u>for the specific example in fig 2:</u>

-the trellis has four nodes and four states.

-there are four group of symbols for the four possible inputs (4-QAM constellation)

-each group has two entries corresponding to the symbols to be output through

the two transmit antennas

-the outputs 0 ,1, 2 and 3 are mapped to data symbols 1, j, -1 and -j respectively.

-decoding will be done using ML sequence estimation using Viterbi algorithm

ST diversity coding $(r_s \leq 1)$ continued

Important points to note

-increasing the number of states increases the coding gain.

-the computational load for decoding an STTC increases exponentially with the number of states

STBC

-generalizations of Alamouti's scheme (two-antenna transmit diversity scheme)

-it is a scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas at the opposite end

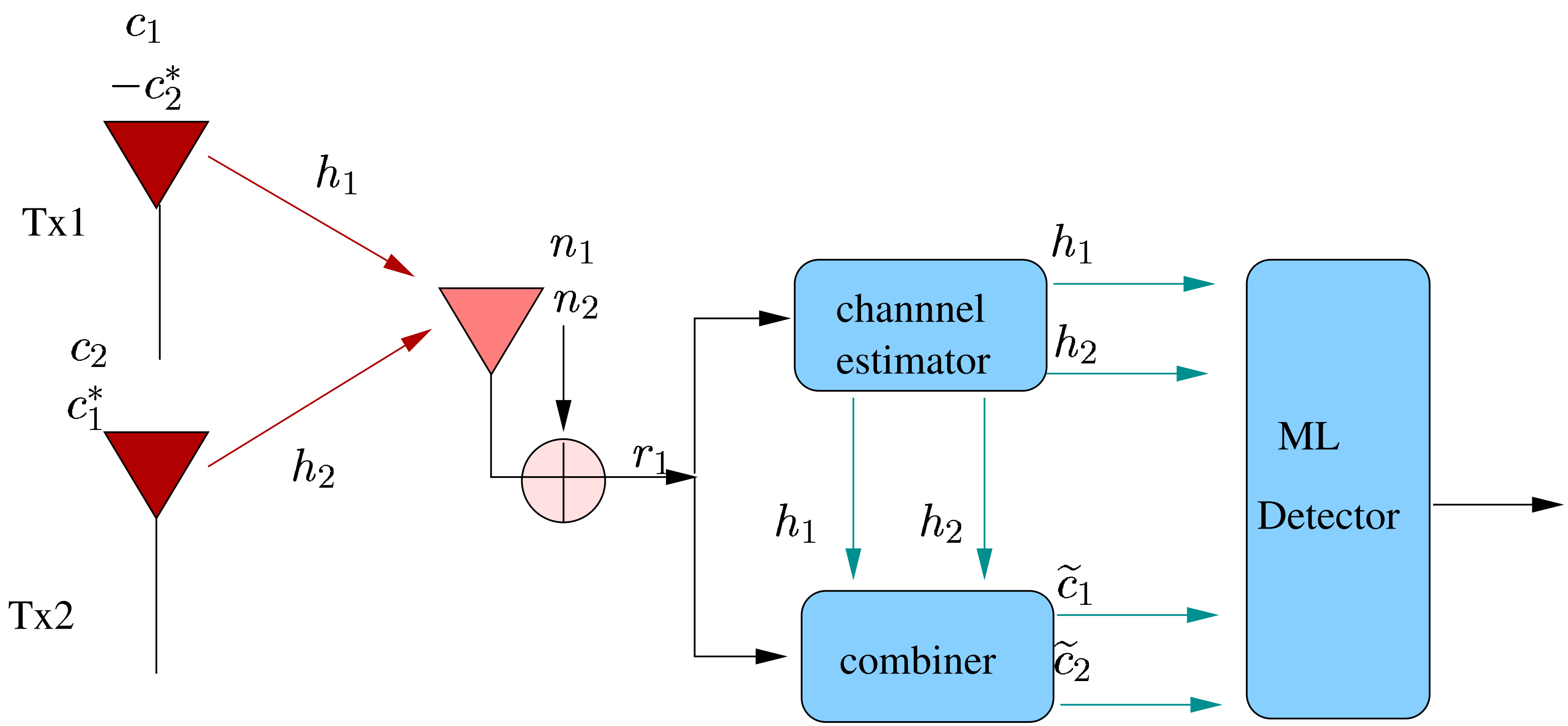


Fig3 Alamouti's Two Antenna Transmit Diversity Scheme

STBC continued

-the transmitted code word can be expressed as:

$$\mathbf{S} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \tag{10}$$

-the codeword difference matrix between any pair of codewords($S^{(i)}$ and $S^{(j)}$)

$$\mathbf{E}_{i,j} = \begin{bmatrix} e_1 & -e_2^* \\ e_2 & e_1^* \end{bmatrix} \tag{11}$$

$E_{i,j}$ is an orthogonal matrix with two non-zero eigenvalues (rank 2) of

equal magnitude

-the Alamouti scheme therefore delivers full $2M_R$ order diversity

STBC continued

Alamouti's receiver output can be stated us:

$$y_i = \sqrt{\frac{E_s}{2}} \|\mathbf{H}\|_F^2 s_i + n, \qquad i = 1, 2, \tag{12}$$

$y_i$is the scalar processed received signal corresponding to transmitted

symbol s_i and n_i is ZMCSCG noise with variance $\|H\|_F^2 N_0$

-ST code construction for Alamouti type scheme to an arbitrary number

of transmit antennas is possible

-An example of an orthogonal design for $M_T = 4$

$$\mathbf{S} = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \\ s_4 & s_3 & -s_2 & s_1 \end{bmatrix} \tag{13}$$

STBC continued

-symbols s_1, s_2, s_3, s_4 are all drawn from a real constellation

-the difference matrix between such type codewords $E_{i,j}$ is orthogonal matrix

-the average PEP in the high SNR regime for an orthogonal STBC(OSTBC)is

$$P(\mathbf{s}_{(i)} \rightarrow \mathbf{S}_{(j)}|\mathbf{H}) \leq \left(\frac{M_T}{\|\mathbf{E}_{i,j}\|_F^2}\right)^{M_T M_R} \left(\frac{\rho}{4M_T}\right)^{-M_T M_R} \quad (14)$$

-from the above equation we learn that OSTBC extract the full diversity gain

of $M_T M_R$

STBC continued

-in the case of complex constellations, an orthogonal design with spatial

rate 1 doesn't exist for systems more than two transmit antennas

-Orthogonal designs for $r_s = \frac{1}{2}$ and $r_s \geq \frac{1}{2}$ are known to exist

-OSTBC are attractive due to their low complexity implementation;

-their main feature is the provision of full diversity with very simple decoding

-they have to be concatenated with an outer code to provide coding gain

3.4 Spatial multiplexing as a ST code $(\mathbf{r_s} = \mathbf{M_T})$:

-in SM we transmit M_T independent symbols per symbol period

-in an uncoded SM scheme $r_t = 1$ and $r_s = M_T$ the signaling rate will be qM_T bits/transmission

-the receiver treats each received signal vector as a codeword, and performs ML decoding on every vector symbol

-the code word difference matrix$E_{i,j}$ is now an $M_T \times 1$ vector and the $\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H$ is matrix with one rank and thus the average PEP is written as

$$P(\mathbf{s}^{(i)} \to \mathbf{S}^{(j)}) \leq \frac{1}{\lambda(\mathbf{G}_{i,j})^{M_R}} \left(\frac{\rho}{4M_T} \right)^{-M_R} \qquad (15)$$

Spatial multiplexing as a ST code $(\mathbf{r_s = M_T})$ continued

-SM with no coding may be considered as a ST code with spatial rate M_T with M_R order diversity

Horizontal Encoding (HE)

-the bit stream is first demultiplexed into M_T separate streams to undergo temporal coding, interleaving and symbol mapping before transmission

-the spatial rate is $r_s = M_T$ and the signaling rate is qr_tM_Tbits/transmission

-like the uncoded SM it can at most achieve M_R order diversity

-coding gain depends on the strength of the temporal code and array gain of M_R is achievable

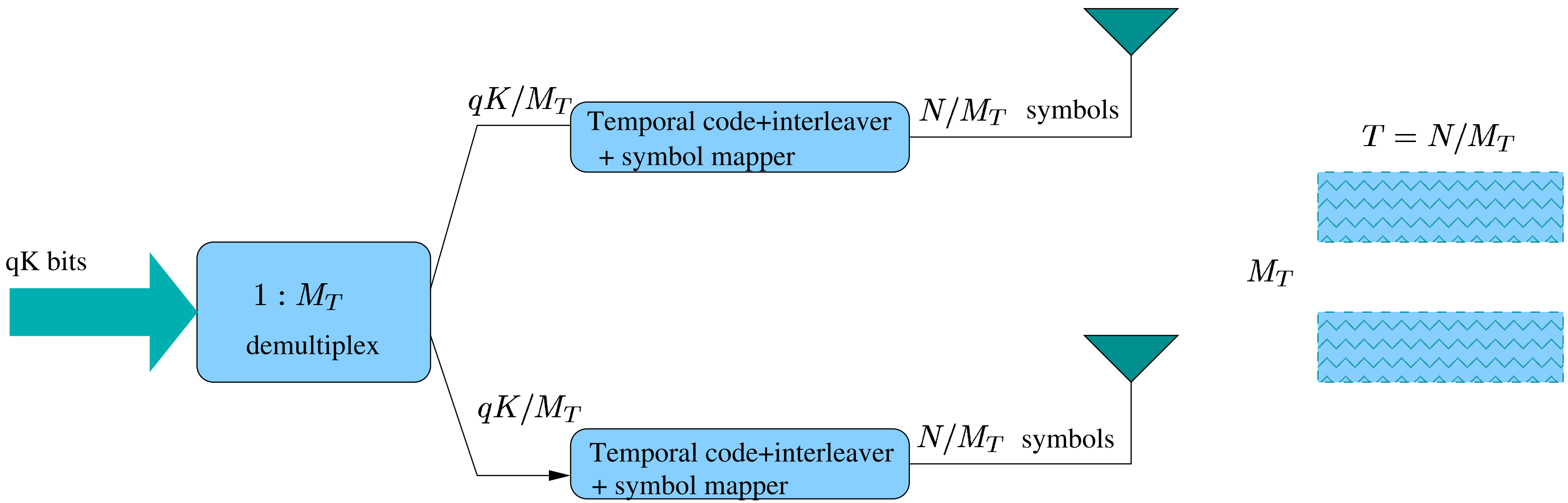


Figure 4 Horizontal Encoding (sub-optimal encoding technique that captures at most M_R order diversity

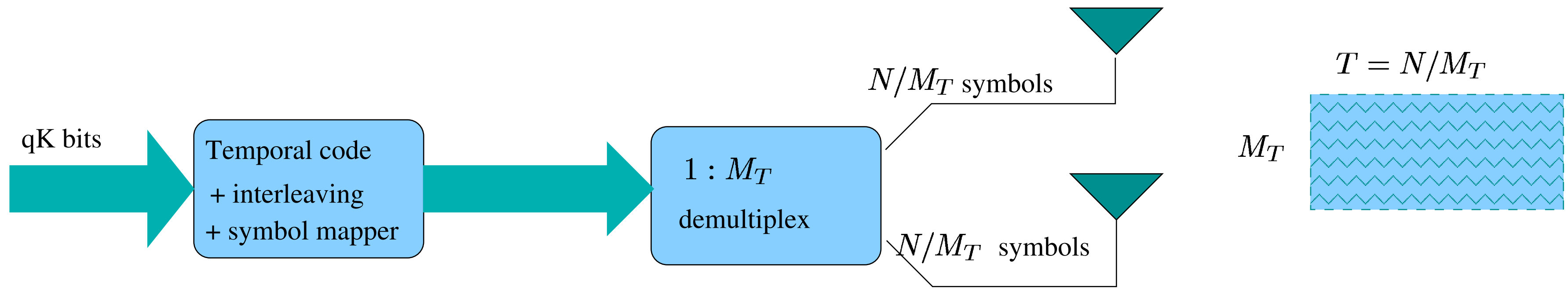


Figure 5 Vertical Encoding (allows spreading of information bits across all antennas, needs complex decoding tech)

Spatial multiplexing as a ST code $(\mathbf{r_s} = \mathbf{M_T})$ continued

Vertical Encoding (VE):

-the bit stream undergoes temporal coding, interleaving and symbol mapping

before demultiplexing and transmission

-optimality is achievable since potentially each bit can be spread across all antennas

-it requires joint decoding at the receiver and hence may be complex

-the spatial rate and the signaling rate are the same as that of HE scheme

-can achieve a diversity order greater than M_R (bits are possibly spread

over all antenna)

-coding gain depends on the temporal code design, and array gain of M_R

is achievable

Spatial multiplexing as a ST code $(\mathbf{r_s = M_T})$ continued

Diagonal Encoding (DE):

-DE is a variation of the HE and VE schemes

-incoming data streams first undergoes HE encoding and split into frames/slots

-the frames pass through a stream rotator so the that the bit stream-antenna association is periodically cycled

-for large codeword, the codeword from any one of the demultiplexed stream could be transmitted over all M_T antennas (D-BLAST transmission technique)

-in D-BLAST an initial wastage (no transmission) is required for optimal encoding

-similarly the spatial rate is M_T and the signaling rate is qr_tM_T bits/transmission

-M_TM_R diversity is possible if the stream rotation is optimal

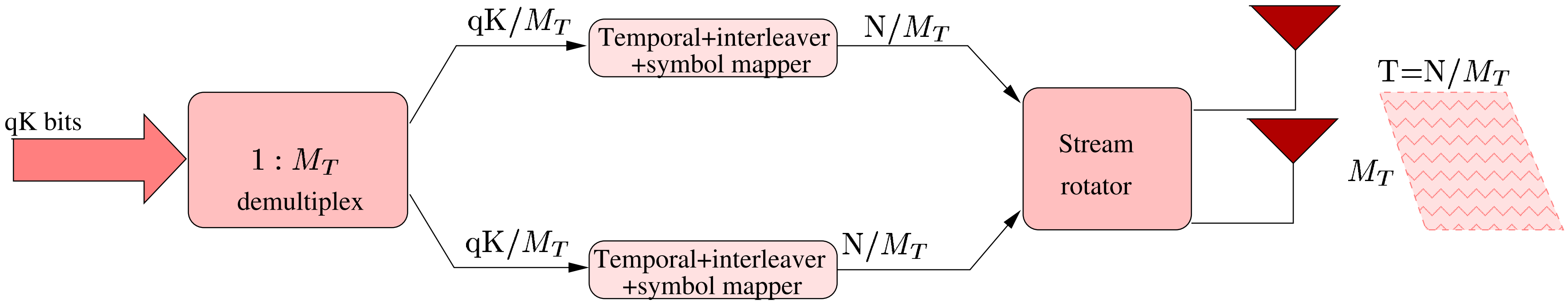


Fig6 Diagonal Encoding (HE) with stream rotation

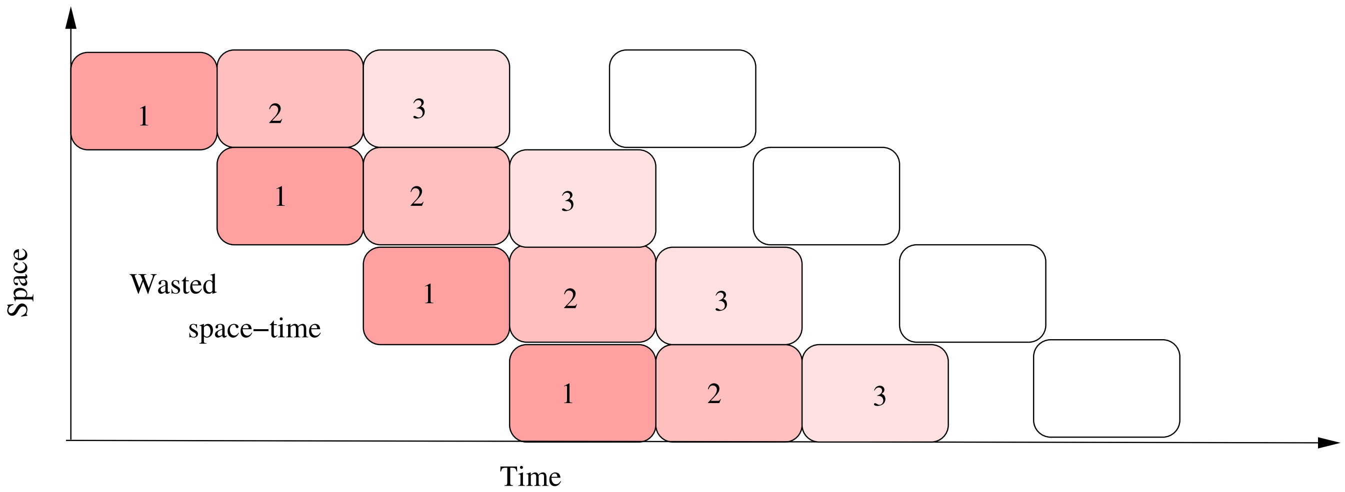


Fig7 D–Blast encoding–numerals represent layers belonging to the same codeword

3.5 ST coding for intermediate rates $(1 \leq r_s \leq M_T)$

-is it possible to trade-off rate and reliability?

-the design metrics (rank and determinant) are not directly related to the

capacity of encoding scheme

-the encoding scheme can be viewed as an operator on the channel,

to yield a new effective channel whose capacity can be computed

-linear dispersion framework proposed by [Hassib ad Hochwald]

-symbols are spread across time and space through matrix modulation and

superposition (ergodic capacity maximization)

$$\bar{C} = \max_{Tr(\mathcal{X}^H X)=M_T T} \frac{1}{T}\mathcal{E}\{log_2 det(I_{M_R T} + \frac{\rho}{M_T}\mathcal{H}\mathcal{X}\mathcal{X}^H\mathcal{H}^H)\} \quad (16)$$

Signal Model

$N \times 1$ vector s of N complex data symbols is modulated by $M_T \times N$ code

matrix, and transmitted over the $M_R \times M_T$ channel H for each symbol period

-assume there are T distinct code matrices and at time $1 \leq k \leq T$ signal $\mathbf{X}[k]\mathbf{s}$

(kth code matrix) is transmitted

-the received symbol vector at time instant k will be

$$\mathbf{y}[k] = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{X}[k]\mathbf{s} + \mathbf{n}[k] \tag{17}$$

-the code design involves identifying the matrices $\mathbf{X}[k]$ that constitute the code

-matrices are computed to maximize both diversity and ergodic capacity

4 - ST coding for frequency selective channels

4.1 Signal model

-the symbol-sampled baseband impulse response:

$$h_{i,j}[l], (l = 0, \ldots, L-1)$$

is assumed to be (CCGRZ) and correlations depending on the base band

pulse, the RF channel time response and the sampling frequency

-assumption is UnCh at the transmitter and KnCh with ML decoding at the receiver

$$\mathbf{y}[k] = \sqrt{\frac{E_s}{M_T}} \begin{bmatrix} \mathbf{h}_{1,1} & \ldots & \mathbf{h}_{1,M_T} \\ \vdots & \vdots & \vdots \\ \mathbf{h}_{M_R,1} & \ldots & \mathbf{h}_{M_R,M_T} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1[k] \\ \vdots \\ \mathbf{s}_{M_T}[k] \end{bmatrix} + \mathbf{n}[k] \qquad (18)$$

4.2 ST codeword design criteria

-the codeword construction criterion to obtain full diversity is

similar to the flat fading case

-the codeword has in effect $\mu = \mathbf{M_T L}$ virtual antennas

-although there are $M_T L$ virtual antennas, the additional structure imposed

may prevent these codes from exploiting full spatio-temporal diversity equal to

$M_T M_R L_{eff}$

References

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