Multi User Detection I

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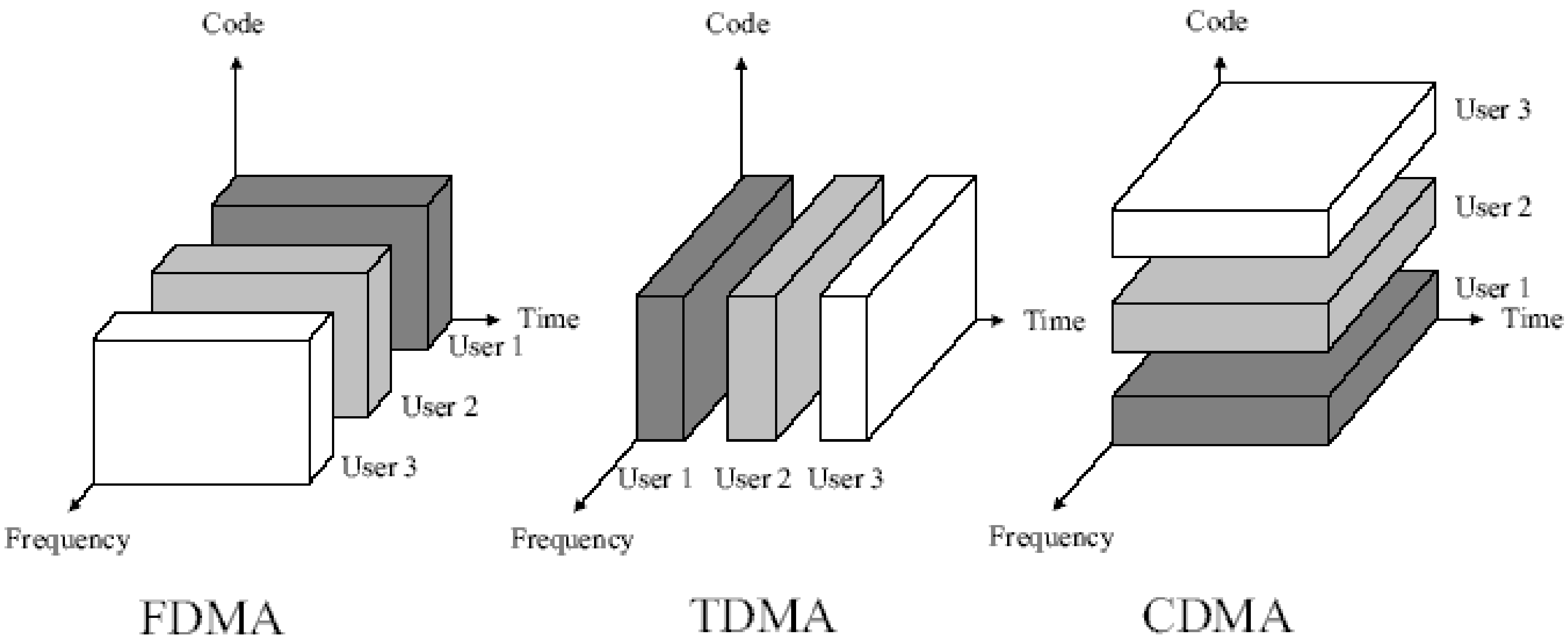
Overview Multiple Access Communication

- **Random Multi Access** (e.g. ALOHA) one user can work simultaneously, otherwise collision occur.
- **FDMA** many user can work simultaneously, but in non overlapping frequency bands.
- **TDMA** many user in one frequency band, but one user per time.
- **CDMA** many user simultaneously in the same frequency band. The user are separated by orthogonal or **non orthogonal signature waveforms**.

Access Techniques



Motivation: What is MU Detection?

$$y_k = \underbrace{A_k b_k}_{1\ \text{term}} + \underbrace{\sum_{j \neq k} A_j b_j \rho_{jk}}_{\text{MAI}} + \underbrace{n_k}_{n}$$

first term:	**desired information**
MAI:	**Multi Access Interference**
n:	**noise**

Cross correlation:

$$\rho_{ij} = \langle s_i, s_j \rangle = \int_0^T s_i(t) s_j(t)\, dt$$

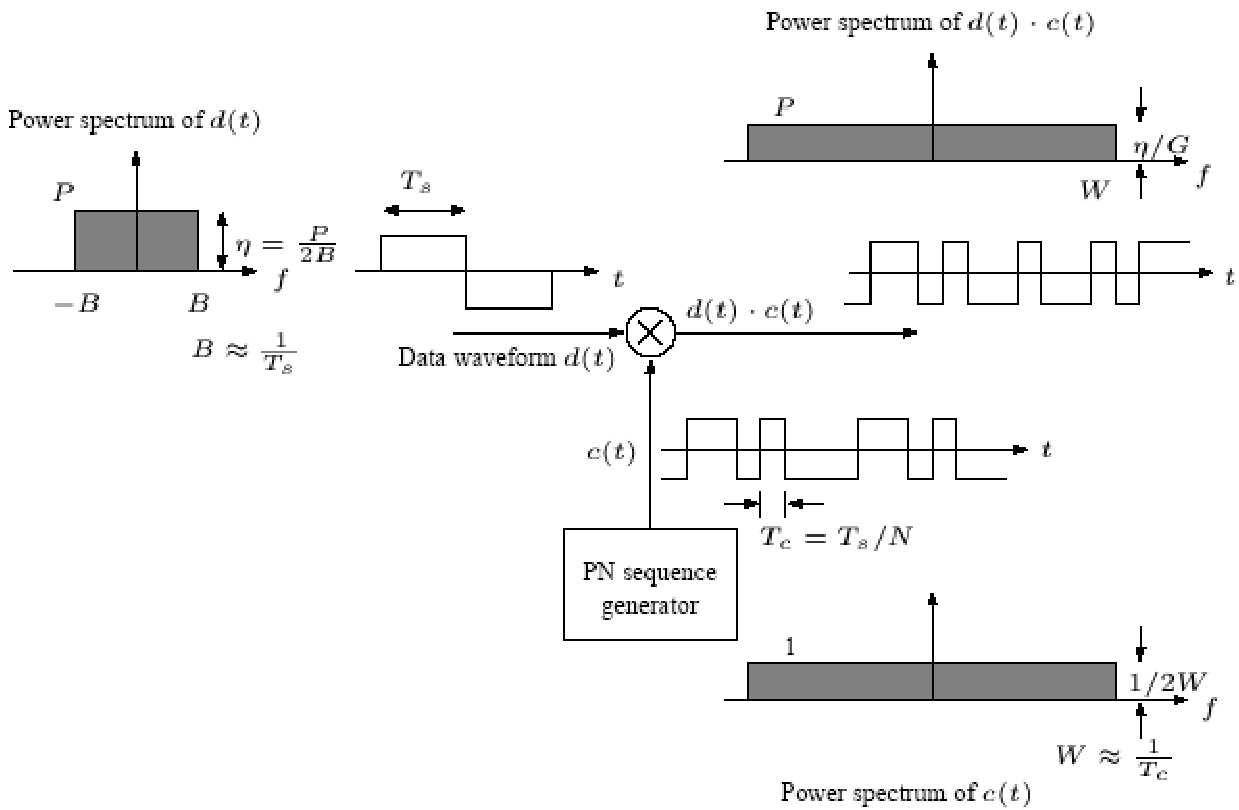
s_i . . . signature waveform of the i^{th} user

Where MU Detection can be used?

Answer: For nonorthogonal multiple access due to design or due to unwanted crosstalk!

- nonorthogonal CDMA
- TDMA with $Multipath$
- bundle of twisted pair
- magnetic recording in adjacent tracks

DIRECT SEQUENCE (DS) SS SYSTEM



Properties of Random Binary Sequences (Codes in CDMA)

Autocorrelation only one single peak

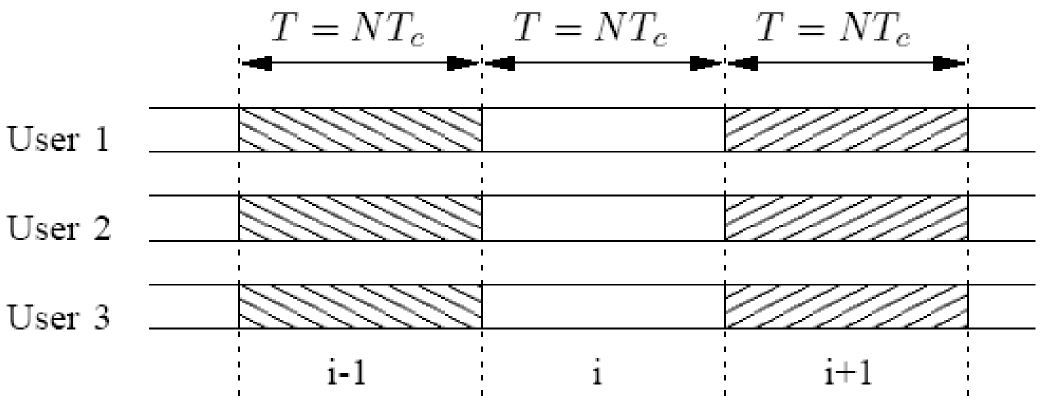
Cross-correlation should be low

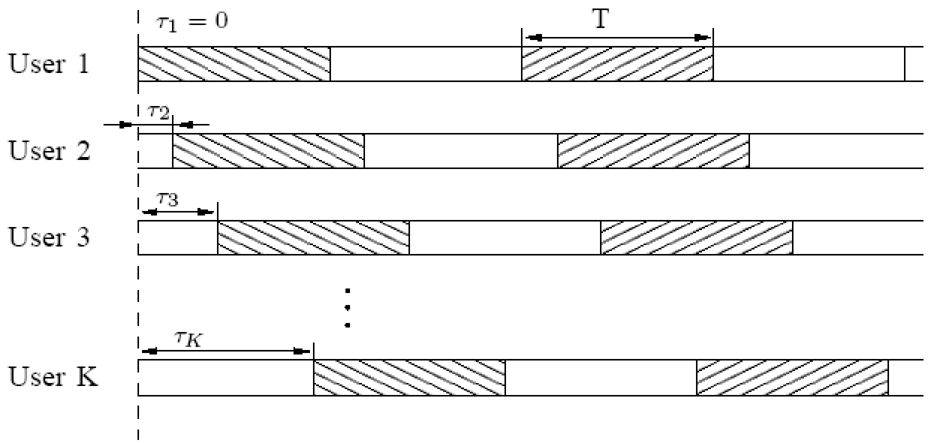
Frequency Spectrum the spread frequency should be flat

Orthogonal / Non Orthogonal

- Orthogonal code e.g. WALSH
  - bad spreading properties
  - more peaks in the autocorrelation
  - Orthogonality hold only for perfect synchronized and no multi-path
- Non Orthogonal code e.g. Gold, Kasami
  - good spreading properties
  - single peak in the autocorrelation
  - limited (low) cross-correlation

Synchronous/Asynchronous CDMA Model



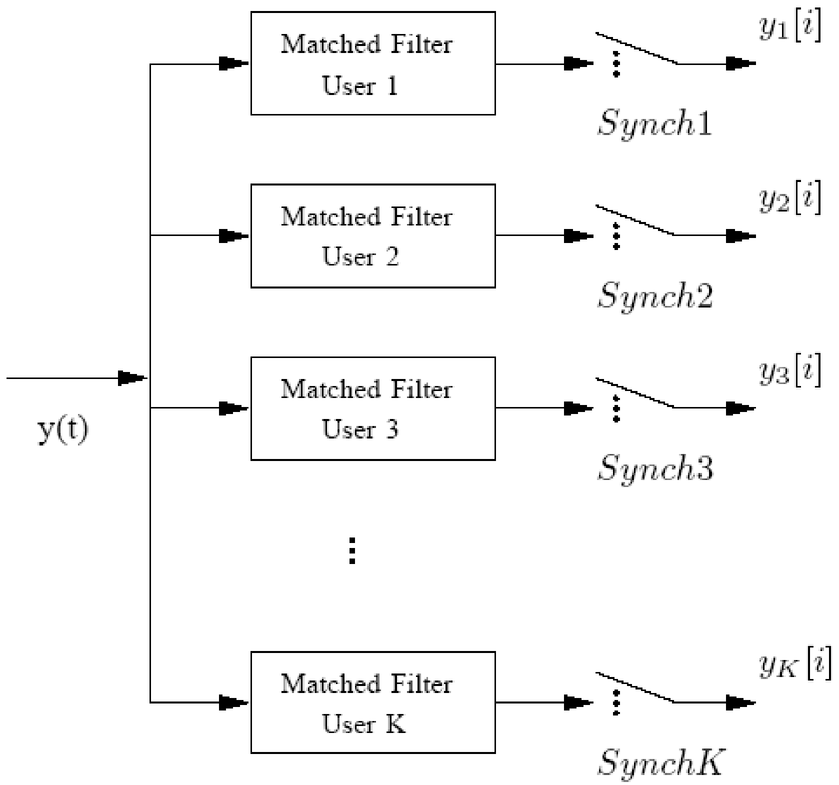


Basic Synchronous CDMA Model (K-user)

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t),\ \ t \in [0, T]$$

- $s_k(t)$ deterministic signature waveform assigned to the k^{th} user, with normalized energy
- A_k amplitude of the k^{th} user
- $b_k \in \{-1, +1\}$ is the bit transmitted by the k^{th} user
- $n(t)$ is white Gaussian noise with unit power

Discrete Time Synchronous CDMA Model (K-user)



Matched Filter Output

$$y_1 = \int_0^T y(t) s_1(t)\, d\, t$$

$$\vdots$$

$$y_k = \int_0^T y(t) s_k(t)\, d\, t$$

Cross Correlation

$$\rho_{ij} = \langle s_i, s_j \rangle = \int_0^T s_i(t) s_j(t)\, dt$$

Cross correlation matrix for example in the 3-user case:

$$R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}$$

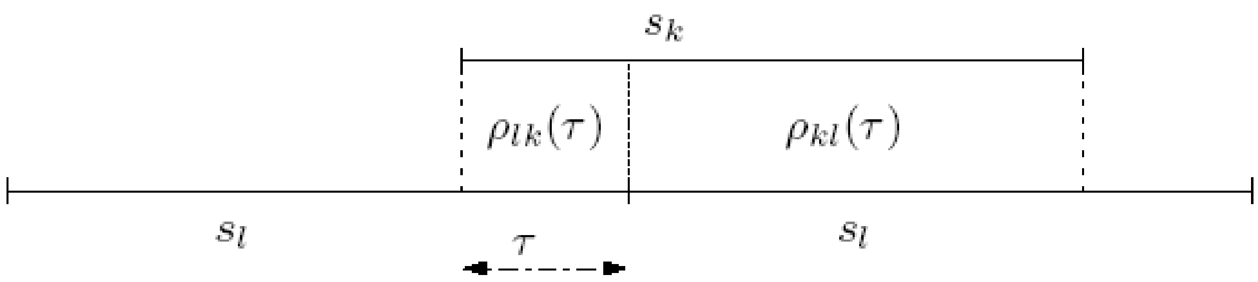
$$\boldsymbol{y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k}$$

As a Matrix:

$$\text{y} = \text{RAb} + \text{n}$$

$$\begin{aligned} R &= \text{normalized cross correlation} \\ y &= [y_1, y_2, \ldots, y_k]^T \\ b &= [b_1, b_2, \ldots, b_k]^T \\ A &= diag\,(A_1, A_2, \ldots, A_k) \\ n &= \text{n is zero mean Gaussian random vector} \end{aligned}$$

Asynchronous Model
Cross correlation



$$\rho_{kl}(\tau) = \int_{\tau}^{T} s_k(t)s_l(t-\tau)\,d\,t$$

$$\rho_{lk}(\tau) = \int_{0}^{\tau} s_k(t)s_l(t+T-\tau)\,d\,t$$

$$\rho_{kl} := \rho_{kl}(\tau_l-\tau_k)$$

Basic Asynchronous CDMA Model

Assuming (without losing generality) that a user sends data packets with $2M+1$ bits.

continuous time CDMA Model for K user:

$$\boldsymbol{y(t) = \sum_{k=1}^{K} \sum_{i=-M}^{M} A_k b_k[i] s_k(t - iT - \tau_k) + \sigma n(t)}$$

$$\boldsymbol{t \in [-MT, MT + 2T]}$$

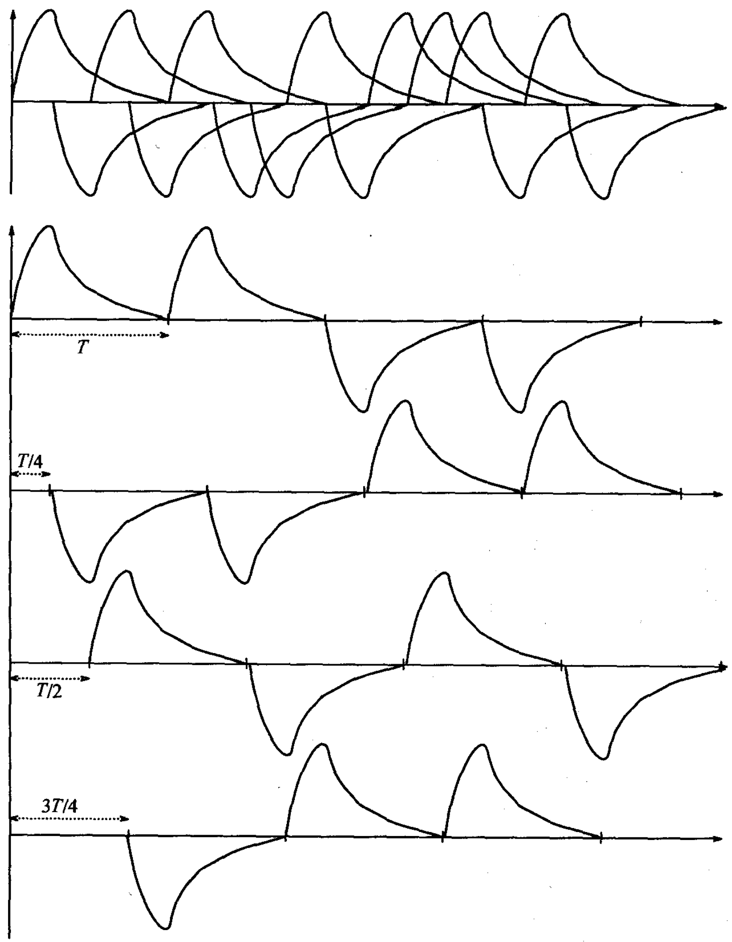
special case:

$$\begin{aligned} A_1 &= \ldots = A_K = A \\ s_1 &= \ldots = s_K = s \\ \tau_k &= \frac{(k-1)T}{K} \end{aligned}$$

$$\boldsymbol{y(t) = \sum_{k=1}^{K} \sum_{i=-M}^{M} Ab_k[i]s(t - iT - (k-1)T/K) + \sigma n(t)}$$

single user channel with ISI:

$$\boldsymbol{y(t) = \sum_{j} Ab[j]s(t - jT/K) + \sigma n(t)}$$



ISI as an asynchronous CDMA channel with 4 users

Discrete Time Asynchronous CDMA Model (K-user)

$$\tau_1 \leq \tau_2 \ldots \leq \tau_k$$

Output of the matched Filter:

$$y_k = A_k b_k[i] + \sum_{j<k} A_j b_j[i+1]\rho_{kj} + \sum_{j<k} A_j b_j[i]\rho_{jk} + \sum_{j>k} A_j b_j[i]\rho_{kj} + \sum_{j>k} A_j b_j[i-1]\rho_{jk} + n_k[i]$$

Asynchronous Cross correlation Matrix

Cross correlation matrix for example in the 3-user case:

$$\boldsymbol{R}[0] = \begin{bmatrix} \mathbf{1} & \boldsymbol{\rho_{12}} & \boldsymbol{\rho_{13}} \\ \boldsymbol{\rho_{12}} & \mathbf{1} & \boldsymbol{\rho_{23}} \\ \boldsymbol{\rho_{13}} & \boldsymbol{\rho_{23}} & \mathbf{1} \end{bmatrix} \quad \boldsymbol{R}[1] = \begin{bmatrix} \mathbf{0} & \boldsymbol{\rho_{21}} & \boldsymbol{\rho_{31}} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\rho_{32}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Time discret asynchronous model in matrix form:

$$\boldsymbol{y}[\boldsymbol{i}] = \boldsymbol{R}^{\boldsymbol{T}}[\mathbf{1}]\,\boldsymbol{A}\boldsymbol{b}[\boldsymbol{i}+\mathbf{1}] + \boldsymbol{R}[\mathbf{0}]\,\boldsymbol{A}\boldsymbol{b}[\boldsymbol{i}] + \boldsymbol{R}[\mathbf{1}]\,\boldsymbol{A}\boldsymbol{b}[\boldsymbol{i}-\mathbf{1}] + \boldsymbol{n}[\boldsymbol{i}]$$

Matched Filter for Single User

$$y(t) = A \cdot b \cdot s(t) + \sigma \cdot n(t),\ t \in [0, T]$$

We want to find a **linear** Filter, which minimize the probability of error.

$$\widehat{b} = sgn(\langle y, h \rangle) = sgn\left(\int_0^T y(t)h(t)d\,t \right)$$

due to linearity of decision statistic

$$Y = \langle y, h \rangle = \underbrace{Ab \langle s, h \rangle}_{signal} + \underbrace{\sigma \langle n, h \rangle}_{noise}$$

Some Properties:

If h is finite-energy deterministic signal and $n(t)$ is withe noise with unit spectral density, then:

1. $E\left[\langle n,h\rangle\right]=0$

2. $E\left[\langle n,h\rangle^{2}\right]=\|h\|^{2}$

3. If n(t) is a Gaussian process, then $\langle n,h\rangle$ is a Gaussian random variable.

Maximize SNR of Y

$$\max_h \frac{A^2 \left(\langle s, h \rangle\right)^2}{\sigma^2 \left\|h\right\|^2}$$

Cauchy-Schwarz

$$\left(\langle s, h \rangle\right)^2 \leq \left\|h\right\|^2 \left\|s\right\|^2$$

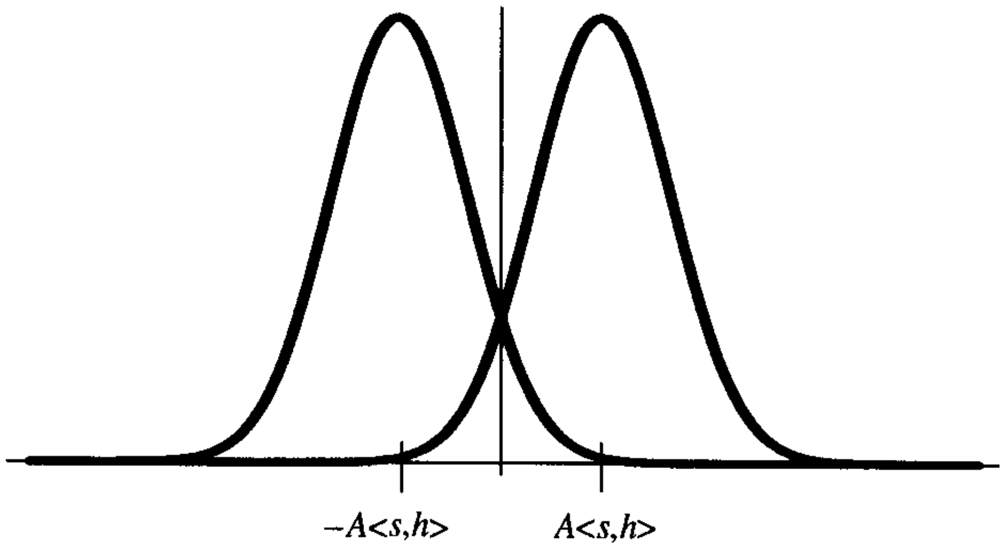
$$\frac{A^2 \left(\langle s, h \rangle\right)^2}{\sigma^2 \left\|h\right\|^2} \leq \frac{A^2}{\sigma^2} \left\|s\right\|^2$$

with equality if and only if h is a multiple of s!

The matched filter can be implemented either as a correlator (mult. with s(t) and integration) or by a linear filter with an impulse response s(T-t) sampled at time T.

Proof: Hypothesis Testing Problem:

$$H_1 : Y \sim f_{Y|1} \quad = \quad N(A\langle s,h\rangle, \sigma^2 \|h\|^2)$$
$$H_{-1} : Y \sim f_{Y|-1} \quad = \quad N(-A\langle s,h\rangle, \sigma^2 \|h\|^2)$$



Probability of Error

$$
\begin{aligned}
P &= \frac{1}{2}\int_{0}^{\infty} f_{Y|-1}(v)\,d\,v + \frac{1}{2}\int_{-\infty}^{0} f_{Y|1}(v)\,d\,v \\
&= \frac{1}{2}\int_{A\langle s,h\rangle}^{\infty} \frac{1}{\sqrt{2\pi}\sigma\,\|h\|}\exp\left(-\frac{v^2}{2\sigma^2\,\|h\|^2}\right)\,d\,v + \\
&\quad \frac{1}{2}\int_{-\infty}^{-A\langle s,h\rangle} \frac{1}{\sqrt{2\pi}\sigma\,\|h\|}\exp\left(-\frac{v^2}{2\sigma^2\,\|h\|^2}\right)\,d\,v \\
&= \int_{\frac{A\langle s,h\rangle}{\sigma\|h\|}}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{v^2}{2}\right)\,d\,v = Q\left(\frac{A\,\langle s,h\rangle}{\sigma\,\|h\|}\right)
\end{aligned}
$$

Probability of Error(2)

$$\boldsymbol{P = Q\left(\frac{A\,\langle s, h\rangle}{\sigma\,\|h\|}\right)}$$

The Q function is monotonic decreasing. We assume that argument of Q-function is non negative and so we can maximize the square of the argument to minimize the error probability. And that we already did before.

$$\boldsymbol{P^C = Q\left(\frac{A}{\sigma}\right)}$$

References

- Multiuser Detection, Serio Verdu
- Spread Spectrum Technique and its Application to DS/CDMA, Bernard H. Fleury and Alexander Kocian
  http://kom.aau.dk/project/sipcom/sites/sipcom9/courses/CDMA/SS_all_2.pdf

THE END