What is MIMO?

- Application of Multiple Antennas

  (at the Transmitter **and**/or Receiver)

  to improve the link performance:

- Coverage (range)
- Quality
- Interference Reduction
- Spectral Efficiency

Multiple Antennas in Wireless - History

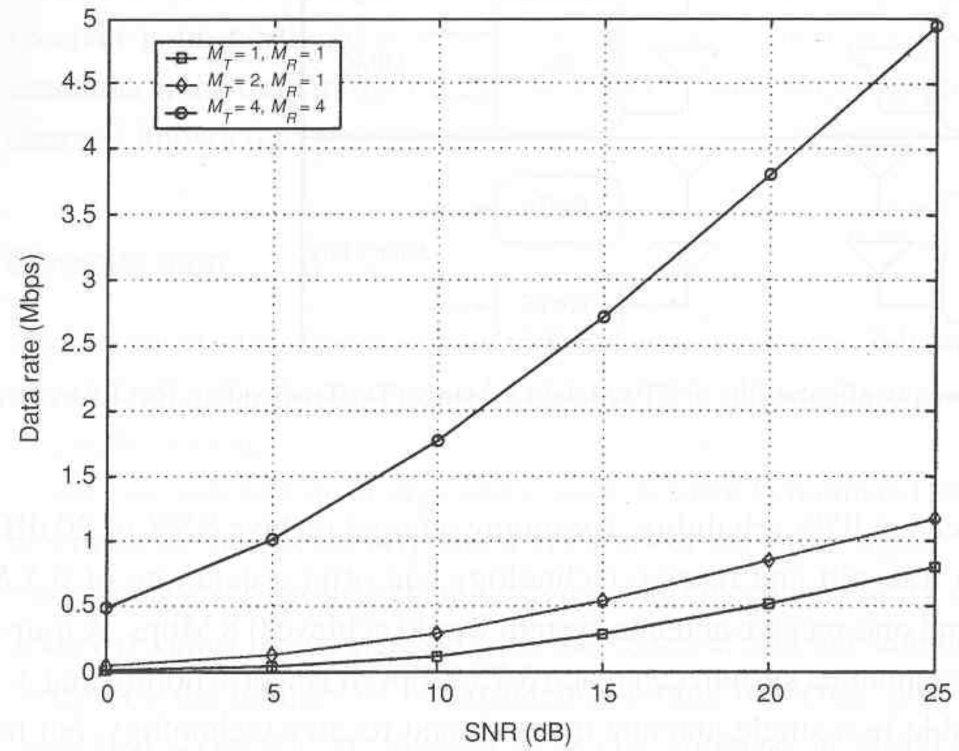
- Non-adaptive:
  - Diversity (Marconi 1900)

- Adaptive:
  - Interference suppression by beam steering (military: 70’s, 80’s)
  - Receiver ST-Techniques to support co-channel signals: 90’s
  - TX-RX ST-Techniques: 2000

Multiple Antennas in Wireless – Potential

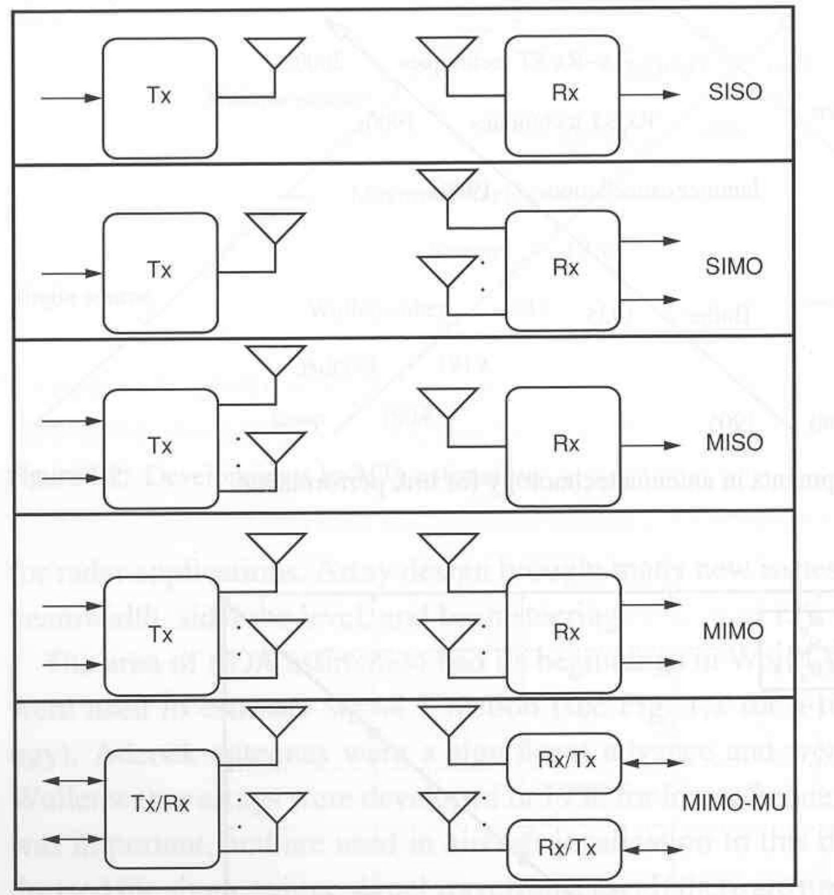
- Data rate at 95% reliability in a 200 kHz fading channel

- At SNR = 20 dB:
  - SISO: 0.5 MBit/s
  - 2 TX, 1 RX: 0.8 MBit/s
  - 4 TX, 4 RX: 3.75 MBit/s



Antenna Configurations

- Number of TX-antennas: M_T
- Number of RX-antennas: M_R
- MIMO channels can be exploited in several ways



Exploiting Multiple Antennas – Array Gain

- Array Gain:
  - Average increase in SNR due to coherent combining (at TX / RX or both) → beamforming
  - Average increase in SNR at RX is prop. M_R
  - MISO/MIMO (if $M_T > 1$): Channel knowledge required at TX to obtain array gain

Exploiting Multiple Antennas – Diversity Gain (1)

- Fading channel: variations of signal power
  - Diversity is used to combat fading

- Receive antenna diversity (SIMO)

- **Diversity order**:
  - number of **independently** fading branches

  - In SIMO: number of RX antennas

Fading (Small-Scale, Microscopic)

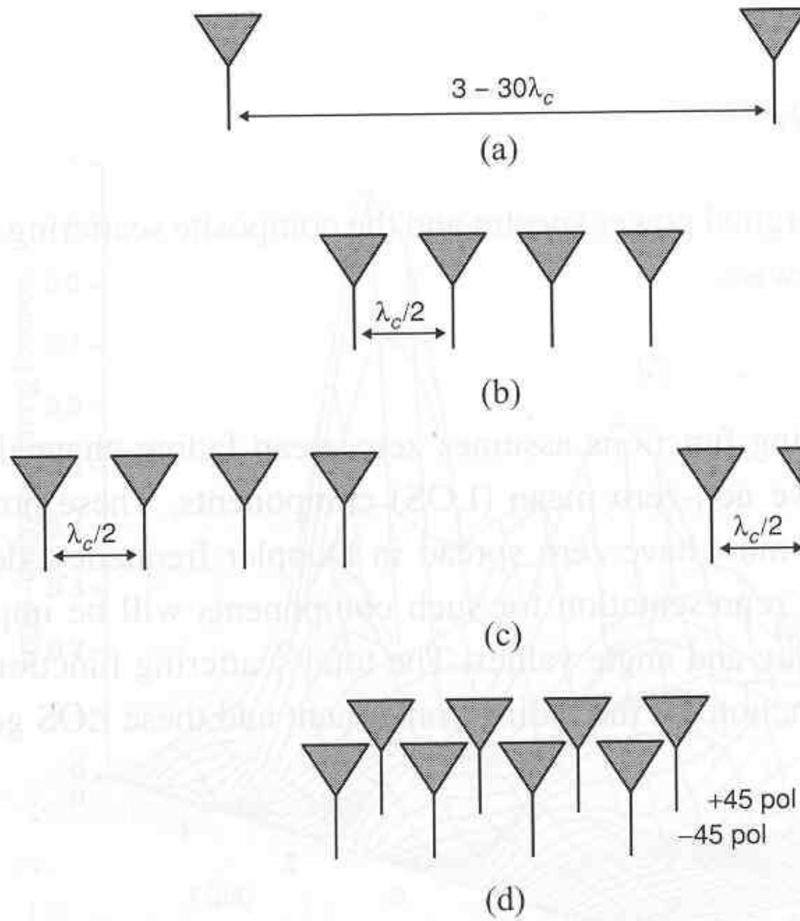
- Multipath:
  - Superposition of large number of scattered waves
    - Various magnitudes and phases
  - I- and Q-components add up to complex Gaussian (CLT)

- Amplitude distribution:
  - Rayleigh fading:
    - Means of I/Q are zero
  - Ricean fading:
    - Dominant component
    - K-factor: power in dominant /power in scattered rays

Channel Variability

- Time Variability – Doppler Spread
  - Coherence time and Doppler spread: $T_C = 1/\nu_{rms}$
- Frequency Selectivity – Delay Spread
  - Coherence BW and RMS Delay spread: $B_C = 1/\tau_{rms}$
- Space Selective Fading – Angle Spread
  - Coherence distance and Angle spread: $D_C = 1/\theta_{rms}$
  - Doppler/Delay/Angle power spectra: average power as a function of …

Array Topologies



Signal Models

- Input output relation
- Classifications:
  - SISO, SIMO, MISO, MIMO
  - Continuous time – discrete time (sampled)
  - Frequency flat channel (τ_{rms} ~ 0) – frequency selective channel ($B\tau_{rms}$ > 0.1)

- For sampled signal model (single carrier), normalizations are introduced:
  - Bandwidth = 1 Hz, symbol period = 1 s

Sampled Signal Model - Normalizations

- Channel
  - Channel in frequency flat channels: $E\{|h_{i,j}|^2\} = 1$
  - Varying part of $h_{i,j}$ is ZMCSCG (zero-mean circular symmetric complex Gaussian)
  - Multipath channels: total average energy of all taps = 1
- Signal
  - Signal energy: average transmit symbol energy (= power, since T_s = 1 s) E_s
  - MIMO, MISO: energy per symbol per antenna E_s / M_T
  - data are IID with zero mean, unit average energy symbol constellations
- Noise
  - noise power = noise PSD N_0 due to B = 1 Hz

Sampled Signal Model – SISO (1)

- $h[l]$ … T_s spaced sampled channel
  - $l = 0, 1, \ldots, L-1$; L … channel length in samples
  - incorporates:
    - physical channel, pulse-shaping at TX, matched filter on RX, sampling delay
- $s[l]$ … symbols to be transmitted
  - scalar linear modulation: PAM, QAM
- $n[k]$ … noise samples
  - assumed white ZMCSCG noise; $\mathrm{var}\{n[k]\} = N_0$
- $y[k]$ … received signal
  - Multiple samples per symbol period should be used (2)

Sampled Signal Models (2)

- Frequency selective case (SISO)

$$y[k] = \sum_{l} \sqrt{E_s}\, s[l]h[k-l] + n[k]$$

- Frequency flat case (SISO) – channel is complex factor

$$y[k] = \sqrt{E_s}\, hs[k] + n[k]$$

- Frequency flat case (MIMO) – vector notation

$$\mathbf{y}[k] = \sqrt{\frac{E_s}{M_T}}\mathbf{Hs}[k] + \mathbf{n}[k]$$

TUG

Definitions

Drop time-index k

$$\mathbf{r} = \sqrt{\frac{E_s}{M_T}}\mathbf{H}\mathbf{s} + \mathbf{n}$$

with

$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \dots & r_{M_R} \end{bmatrix}^T$ ... $M_R x1$ receive signal vector

$\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \dots & s_{M_T} \end{bmatrix}^T$ ... $M_T x1$ transmit signal vector

$\mathbf{n} = \begin{bmatrix} n_1 & n_2 & \dots & n_{M_R} \end{bmatrix}^T$ ... $M_R x1$ noise vector

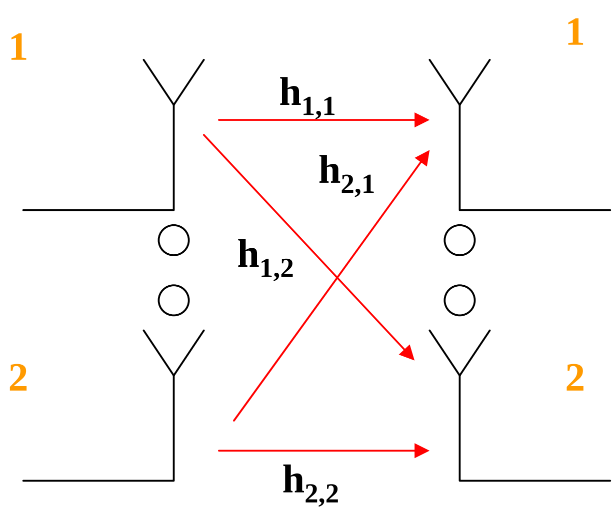
$\mathbf{H}$ ... M_R x M_T channel transfer matrix

Noise: $n_i \sim \left(0, \sigma_R^2\right)$ with $\mathrm{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \sigma_R^2 \mathbf{I}_{M_R}$

TUG

MIMO Sampled Signal Model

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$



- Frequency-flat channel
  - Channel impact expressed by complex factors: channel transfer matrix
- $\mathbf{H} = \mathbf{H}_w$ is often assumed IID (spatially white channel)
  - in rich scattering

Properties of H

- Singular values of **H**
  - **H** has rank r
  - SVD: $\mathbf{H} = \mathbf{U\Sigma V}^H$
  - **U**: M_R x r
  - **V**: M_T x r
  - $\mathbf{\Sigma} = \text{diag}\{\sigma_1\ \sigma_2 \ldots\ \sigma_r\}$ (singular values)

- EV Decomposition of $\mathbf{HH}^H = \mathbf{Q\Lambda Q}^{\mathrm{H}}$
  - $\mathbf{\Lambda} = \text{diag}\{\lambda_1\ \lambda_2 \ldots\ \lambda_r\}$

$$\lambda_i = \begin{cases} \sigma_i^2 & i = 1, 2, \ldots, r \\ 0 & i > r \end{cases}$$

TUG

Squared Frobenius Norm of H

- Definition

$$\|\mathbf{H}\|_F^2 = \mathrm{Tr}(\mathbf{H}\mathbf{H}^H) = \sum_{i=1}^{M_R}\sum_{j=1}^{M_T}\left|h_{i,j}\right|^2$$

- Interpretation: total power gain of channel

$$\|\mathbf{H}\|_F^2 = \sum_{i=1}^{M_R}\lambda_i$$

- PDF of power gain, when $\mathbf{H} = \mathbf{H}_w$ (IID channel)
  - chi-square distribution with $2M_TM_R$ degrees of freedom

$$f(x) = \frac{x^{M_T M_R - 1}}{(M_R M_T - 1)!} e^{-x} \sigma(x)$$