MIMO systems and Spatial Diversity.

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Content of the talk

- PART I.
  - Wireless communication and MIMO systems.
  - Wireless channels.
- Part II.
  - Spatial Diversity
Outline – PART I.

PART I.

- What is MIMO
  - Multiple antennas in communication
- Channel and channel models.
  - Channel and signal models.
PART II.
Outline – PART II.

- What is diversity?
  - Diversity schemes.
  - What do we actually win.
- Implementing diversity – IID channels
  - RX diversity (SIMO).
  - Alamouti scheme and TX diversity (MISO).
  - RX-TX diversity (MIMO).
- Extended channels.
What is diversity?

Diversity – Multiple independent look at the same transmitted signal.
What is diversity?

- Fading impairs wireless link.
- Use multiple antennas to create alternative transmission branches.
What is diversity?

- If \( p = P(\text{signal is in fade}) \)
- Having \( M \) independent branches results in
  \[
P(\text{all } M \text{ branches are in fade}) = p^M \ll p
  \]
- There are
  - Frequency diversity
  - Time diversity
  - Space diversity
Diversity schemes.

- **Space diversity**: RX/TX has multiple antennas, spaced $d$ meters apart such that
  
  \[d > D_c\]

- **Frequency diversity**: signal is transmitted over two carrier frequencies, spaced $\Delta f$, such that
  
  \[\Delta f > B_c \sim \frac{1}{\text{RMS delay spread}}\]

- **Time diversity**: the same signal is re-transmitted with a delay $\Delta t$ such that
  
  \[\Delta t > T_c = \frac{1}{B_D}\]
Diversity Gain

To exploit diversity we need:

- Combine multiple branches in some way.
- Make sure branches are independent. Separation
  \[ B_c, T_c, D_c \]

Signal model for each branch:

\[
y_i = \sqrt{\frac{E_s}{M}} h_i s + n_i, \quad n = 1, \ldots, M
\]

- \( h_i \) – flat-fading channel coefficient;
- \( E_s \) – symbol energy;
- \( n_i \) – additive noise.
Diversity Gain, cont’d

- To combine branches – Maximum Ratio Combining:

\[ z = \sum_{i=1}^{M} h_i^* y_i \]

- Each branch increases SNR

\[ \eta = \frac{1}{M} \sum_{i=1}^{M} \left( |h_i|^2 \cdot \frac{E_s}{N_0} \right) = \alpha \cdot \rho \]

\[ \alpha = \frac{1}{M} \sum_{i=1}^{M} \left( |h_i|^2 \right) \text{— branches gain,} \quad \rho = \frac{E_s}{N_0} \text{— Single-branch SNR} \]
Diversity Gain, cont’d

Assuming ML detection, averaged probability of the symbol error can be upper-bounded as

\[ P_e \leq N_e \left( \frac{\rho d_{\text{min}}^2}{4M} \right)^{-M} \]

- \( N_e \) - number of nearest neighbors
- \( d_{\text{min}}^2 \) – minimum separation of the symbol constellation.
- \( \rho = \frac{E_s}{N_0} \) – averaged SNR at the receiver antenna.
Diversity Gain, cont’d

Effect of diversity on the SER performance in fading channels.

Diversity affects the slope of the SER-SNR curve.
Approximate $P_e$ can be expressed

$$\bar{P}_e \approx \frac{c}{(\gamma_c \rho)^M}$$

$c$ – modulation/channel constant
$\gamma_c \geq 1$ – coding gain
$M$ – diversity order
**Spatial vs Time/Frequency diversity**

- **Spatial diversity**
  - No additional bandwidth required
  - Increase of average SNR is possible
  - Additional array gain is possible

- **Time/Frequency diversity**
  - Time/frequency is sacrificed
  - No array gain.
  - Averaged receive SNR remains as that for AWGN channel.
Implementing Spacial diversity

- Receiver diversity – SIMO
- Transmit diversity – MISO
- RX/TX diversity – MIMO
Receive antenna diversity

Let us assume flat fading

\[ y = \sqrt{E_s} h S + n, \quad h = [h_1, \ldots, h_{MR}]^T \]

Maximum Ratio Combining at the receiver

\[ z = h^H y = \sqrt{E_s} h^H h S + h^H n \]

MRC assumes perfect channel knowledge at RX.
Averaged probability of symbol error is given as (at high SNR regime and rich scattering)

$$\overline{P_e} \geq \overline{N_e} \left( \frac{\rho d_{\text{min}}^2}{4} \right)^{-M_R}$$

Averaged SNR at the receiver

$$\overline{\eta} = M_R \cdot \rho$$

Array gain is $$M_R \ (10 \log_{10} M_R \ [\text{dB}])$$
Receive antenna diversity - performance

- Can be better than AWGN due to array gain
- For BER $> 10^{-5}$ outperforms AWGN due to array gain.

RX diversity extracts full diversity and array gain!
Transmit antenna diversity

Straightforward approach is useless:

- Signal model (for $M_T = 2$); $h_1, h_2 \sim \mathcal{N}(0, 1)$

$$y = \sqrt{\frac{E_s}{2}}(h_1 + h_2)s + n$$

- Equivalently, $h \equiv (h_1 + h_2)/\sqrt{2}$, and $h \sim \mathcal{N}(0, 1)$

$$y = \sqrt{E_s}hs + n$$

Unlike RX diversity, pre-processing is needed.
TX diversity – Alamouti (MISO)

Preprocessing without channel knowledge.
Alamouti scheme, cont’d

- Channel is constant over two symbols intervals and flat-fading
  \[ h = [h_1, h_2] \]
- Two received symbols are:
  \[ y_1 = \sqrt{\frac{E_s}{2}} h_1 s_1 + \sqrt{\frac{E_s}{2}} h_2 s_2 + n_1 \]
  \[ y_2 = -\sqrt{\frac{E_s}{2}} h_1 s_2^* + \sqrt{\frac{E_s}{2}} h_2 s_1^* + n_2 \]
Now, received vector is formed as $y = [y_1, y_2]^T$

$$y = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \mathbf{H}_{\text{eff}} \mathbf{s} + \mathbf{n}$$

MRC gives (using $\mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} = \| \mathbf{h} \|^2 \mathbf{I}$)

$$z = \mathbf{H}_{\text{eff}}^H y = \sqrt{\frac{E_s}{2}} \| \mathbf{h} \|^2 \mathbf{I} \mathbf{s} + \tilde{\mathbf{n}}$$

which simplifies to $z_i = \sqrt{\frac{E_s}{2}} \| \mathbf{h} \|^2 s_i + \tilde{n}_i$. 
Alamouti scheme – Performance

- High SNR regime gives

\[ P_e \leq Ne \left( \frac{\rho d_{\text{min}}^2}{4 \cdot 2} \right)^{-2} \]

- No array gain: \( \bar{\eta} = \rho \)

- Full transmit diversity for \( M_R = 2 \)

How to implement array gain?
MISO with known channel

- Flat-fading channel is given as \( h = [h_1, \ldots, h_{MT}]^T \)
- Signal at the receiver

\[
y = \sqrt{\frac{E_S}{MT}} h^H \mathbf{w} s + n
\]

where \( \mathbf{w} = \sqrt{MT} \frac{h}{\|h\|} \)
- This scheme is known as transmit MRC.
Transmit MRC – Performance

- High SNR regime gives

\[ P_e \leq N_e \left( \frac{\rho d_{\text{min}}^2}{4} \right)^{-M_T} \]

- Array gain in rich scattering \( \bar{\eta} = M_T \cdot \rho \)

- Equivalent to RX diversity.

Transmitter should be aware of the channel.
Alamouti scheme & MIMO

- Transmit symbols like MISO Alamouti
- Flat-fading channel matrix

\[ H = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \]

- Receive symbols \((M_R = 2)\)

\[ y_1 = \sqrt{\frac{E_s}{2}} H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + n_1, \quad y_2 = \sqrt{\frac{E_s}{2}} H \begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix} + n_2, \]

equivalently...
Alamouti scheme & MIMO, cont’d

By defining \( y = [y_1, y_2^*]^T \)

\[
y = \sqrt{\frac{E_s}{2}} \begin{bmatrix}
  h_{1,1} & h_{1,2} \\
  h_{2,1} & h_{2,2} \\
  h_{1,2}^* & -h_{1,1}^* \\
  h_{2,2}^* & -h_{2,1}^*
\end{bmatrix} \begin{bmatrix}
  s_1 \\
  s_2
\end{bmatrix} + \begin{bmatrix}
  n_1 \\
  n_2
\end{bmatrix}
\]

MRC is formed (using \( H_{\text{eff}}^H H_{\text{eff}} = \|H\|^2_F I \))

\[
z = \sqrt{\frac{E_s}{2}} \|H\|^2_F I s + \tilde{n}, \text{ or,}
\]

\[
z_i = \sqrt{\frac{E_s}{2}} \|H\|^2_F s_i + \tilde{n}_i
\]
Alamouti scheme & MIMO, cont’d

- High SNR regime gives and rich scattering

\[
\overline{P_e} \leq \overline{N_e} \left( \frac{\rho d^2_{\text{min}}}{4M_T} \right)^{-M_R M_T}
\]

- Diversity order \(M_R M_T\)

- Array gain \(\overline{\eta} = M_R \rho\)

Thus, without channel knowledge only array gain is obtained.
MIMO with channel knowledge

- If channel known – transmit MRC is used
  \[ y = \sqrt{\frac{E_s}{M_T}} H^H w_s + n \]

- Receiver forms weighted sum \( z = g^H y \)

- Vectors \( w \) and \( g \) are singular vectors from SVD decomposition of \( H \)
  \[ H = U \Sigma V^H; \quad w \in U, g \in V \]
  \( w \) and \( g \) correspond to the max singular value \( \sigma_{max}^2 \) of \( H \)

This is known as **dominant eigenmode transmission** – DET
**MIMO with DET – Performance**

- Diversity order $M_R M_T$
- SNR $\bar{\eta} = E\{\sigma_{max}^2\} \rho$

\[
\max(M_T, M_R) \leq E\{\sigma_{max}^2\} \leq M_T M_R
\]

Channel knowledge increases array gain.
# Summary

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Array gain</th>
<th>Diversity gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMO (Ch.Un.)</td>
<td>$M_R$</td>
<td>$M_R$</td>
</tr>
<tr>
<td>SIMO (Ch.Kn.)</td>
<td>$M_R$</td>
<td>$M_R$</td>
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<tr>
<td>MISO (Ch.Un.)</td>
<td>1</td>
<td>$M_T$</td>
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<tr>
<td>MISO (Ch.Kn.)</td>
<td>$M_T$</td>
<td>$M_T$</td>
</tr>
<tr>
<td>MIMO (Ch.Un.)</td>
<td>$M_R$</td>
<td>$M_R M_T$</td>
</tr>
<tr>
<td>MIMO (Ch.Kn.)</td>
<td>$\max(M_T, M_R) \leq E{\sigma^2_{\text{max}}}$, $E{\sigma^2_{\text{max}}} \leq M_T M_R$</td>
<td>$M_R M_T$</td>
</tr>
</tbody>
</table>
Diversity order and channel variability

- Can be quantified as

\[ \mu_{\text{var}} = \frac{1}{\sqrt{M_R M_T}} \]

- Link stabilizes as

\[ \mu_{\text{var}} \to \infty \]
Diversity order for extended channels

- Elements in channel matrix $H$ are
  - Correlated
  - Have gain imbalances
  - Fade with Ricean statistics

- Highest possible diversity $\Rightarrow$ rich scattering (IID elements in $H$)

- Let us consider Alamouti scheme with $M_T = M_R = 2$
Correlations in $H$

- Diversity order decreases to $r(R)$, where $R$ is $(4 \times 4)$ covariance matrix:
  \[ R = E\{\text{vec}(H)\text{vec}(H)^H\} \]
- Rank $r(R) = 1$ → full correlation.
- No spatial diversity.
- Array gain is present.
Ricean fading of $H$

- Ricean fading occurs when LOS is present.
- As $K \to \infty$ link stabilizes. $K$-factor is given as

$$K = \frac{A_{LOS}^2}{\sigma_{diffuse}^2}$$