MIMO systems and Spatial Diversity.

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Content of the talk

- PART I.
  - Wireless communication and MIMO systems.
  - Wireless channels.
- Part II.
  - Spatial Diversity

Outline – PART I.

PART I.

- What is MIMO
  - Multiple antennas in communication
- Channel and channel models.
  - Channel and signal models.

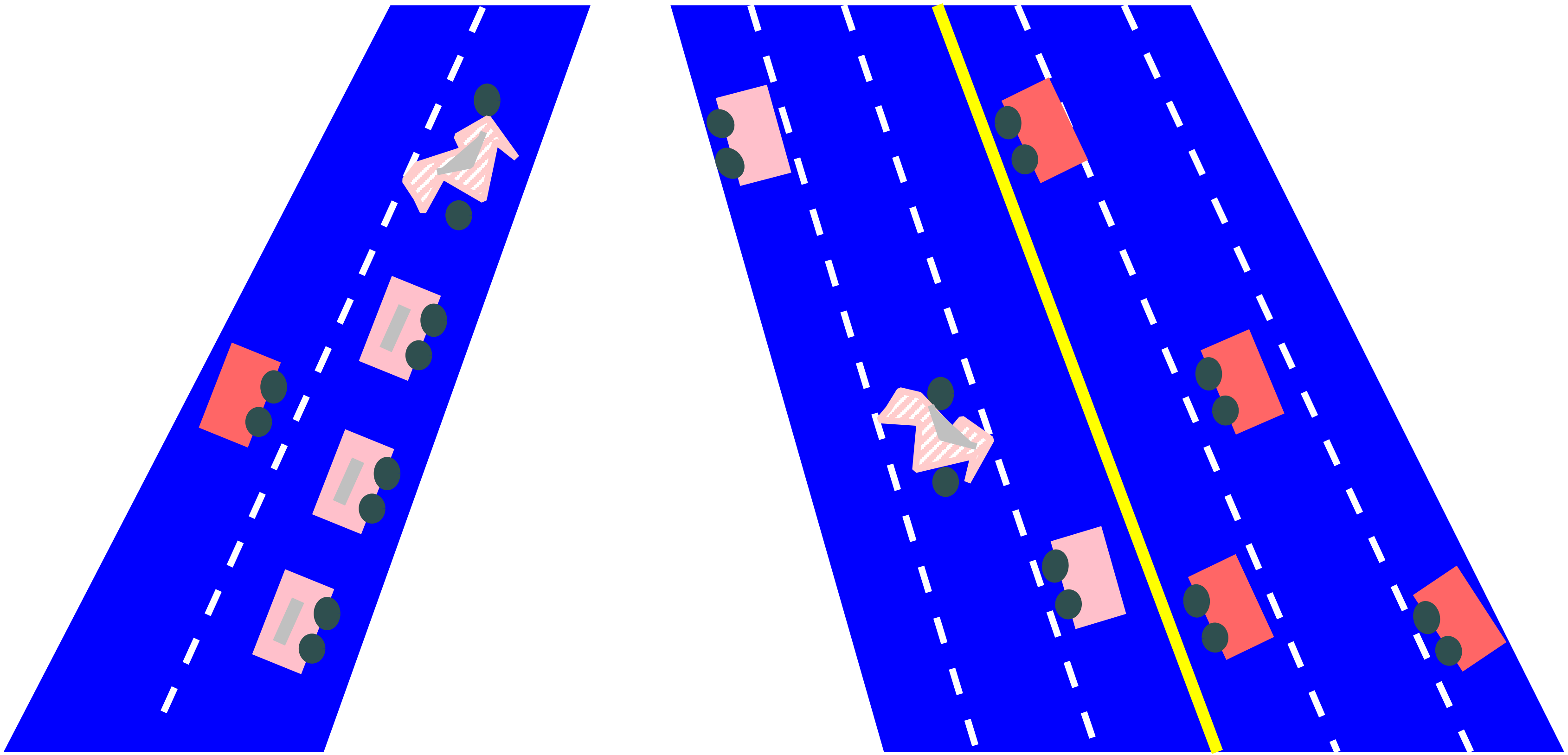
PART II.

Outline – PART II.

- What is diversity?
  - Diversity schemes.
  - What do we actually win.
- Implementing diversity – IID channels
  - RX diversity(SIMO).
  - Alamouti scheme and TX diversity (MISO).
  - RX-TX diversity (MIMO).
- Extended channels.

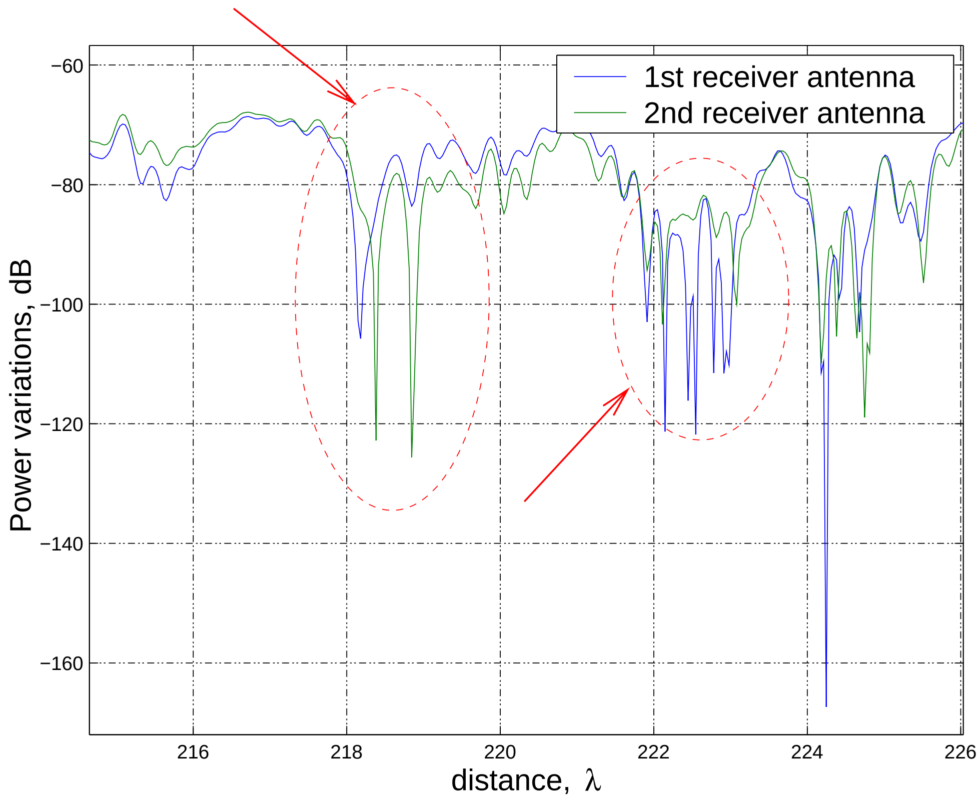
What is diversity?

Diversity – Multiple independent look at the same transmitted signal.



What is diversity?

- Fading impairs wireless link.
- Use multiple antennas to create alternative transmission branches.



What is diversity?

- If $p = P(\text{signal is in fade})$
- Having M independent branches results

$$P(\text{all } M \text{ branches are in fade}) = p^M \ll p$$

- There are
  - Frequency diversity
  - Time diversity
  - Space diversity

Diversity schemes.

- Space diversity: RX/TX has multiple antennas, spaced d meters apart such that

$$d > D_c$$

- Frequency diversity: signal is transmitted over two carrier frequencies, spaced Δf, such that

$$\Delta f > B_c \sim \frac{1}{\text{RMS delay spread}}$$

- Time diversity: the same signal is re-transmitted with a delay Δt such that

$$\Delta t > T_c = \frac{1}{B_D}$$

Diversity Gain

To exploit diversity we need:

- Combine multiple branches in some way.
- Make sure branches are independent. Separation $> B_c, T_c, D_c$

Signal model for each branch:

$$y_i = \sqrt{\frac{E_s}{M}} h_i s + n_i, \quad n = 1, \ldots, M$$

h_i – flat-fading channel coefficient;
E_s – symbol energy;

n_i – additive noise.

Diversity Gain, cont'd

- To combine branches – Maximum Ratio Combining:

$$z = \sum_{i=1}^{M} h_i^* y_i$$

- Each branch increases SNR

$$\eta = \frac{1}{M} \sum_{i=1}^{M} \left(|h_i|^2 \cdot \frac{E_s}{N_0} \right) = \alpha \cdot \rho$$

$\alpha = \frac{1}{M} \sum_{i=1}^{M} \left(|h_i|^2 \right)$ – branches gain, $\rho = \frac{E_s}{N_0}$ – Single-branch S

Diversity Gain, cont'd

Assuming ML detection, averaged probability of the symbol error can be upper-bounded as

$$\overline{P}_e \leq \overline{N}_e \left(\frac{\rho d_{min}^2}{4M} \right)^{-M}$$

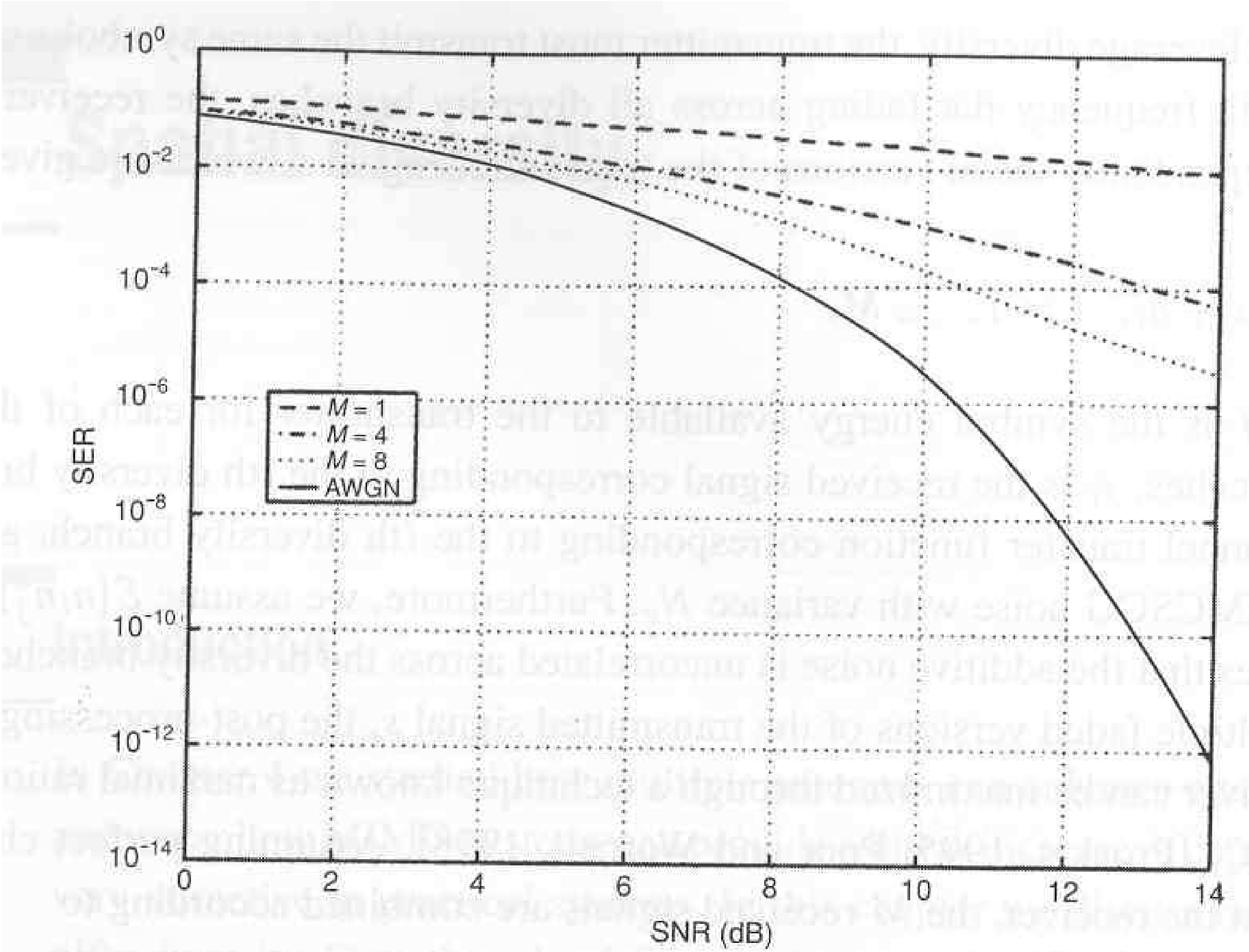
$\overline{N_e}$ - number of nearest neighbors

d_{min}^2 – minimum separation of the symbol constellation.

$\rho = \frac{E_s}{N_0}$ – averaged SNR at the receiver antenna.

Diversity Gain, cont'd

Effect of diversity on the SER performance in fading channels.

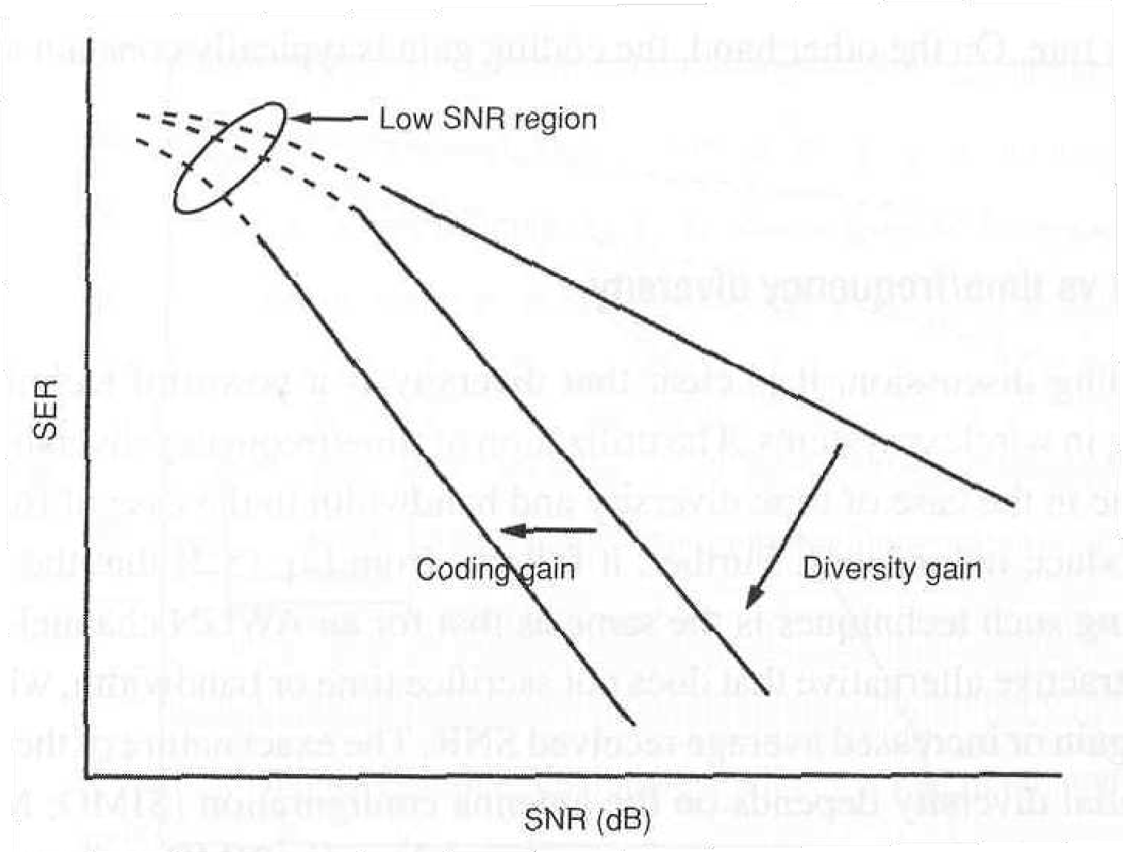


Diversity affects the slope of the SER-SNR curve.

Coding gain vs diversity gain

Approximate $\overline{P}_e$ can be expressed

$$\overline{P}_e \approx \frac{c}{(\gamma_c \rho)^M}$$



c – modulation/channel constant
$\gamma_c \geq 1$ – coding gain
M – diversity order

Spatial vs Time/Frequency diversity

- Spatial diversity
  - No additional bandwidth required
  - Increase of average SNR is possible
  - Additional array gain is possible
- Time/Frequency diversity
  - Time/frequency is sacrificed
  - No array gain.
  - Averaged receive SNR remains as that for AWGN channel.

Implementing Spacial diversity

- Receiver diversity – SIMO
- Transmit diversity– MISO
- RX/TX diversity – MIMO

Receive antenna diversity

- Let us assume flat fading

$$\boldsymbol{y} = \sqrt{E_s}\boldsymbol{h}s + \boldsymbol{n}, \quad \boldsymbol{h} = [h_1, \ldots, h_{M_R}]^T$$

- Maximum Ratio Combining at the receiver

$$z = \boldsymbol{h}^H \boldsymbol{y} = \sqrt{E_s}\boldsymbol{h}^H \boldsymbol{h}s + \boldsymbol{h}^H \boldsymbol{n}$$

MRC assumes perfect channel knowledge at RX.

Receive antenna diversity, cont'd

- Averaged probability of symbol error is given as (at high SNR regime and rich scattering)

$$\overline{P}_e \geq \overline{N}_e \left(\frac{\rho d_{min}^2}{4} \right)^{-M_R}$$

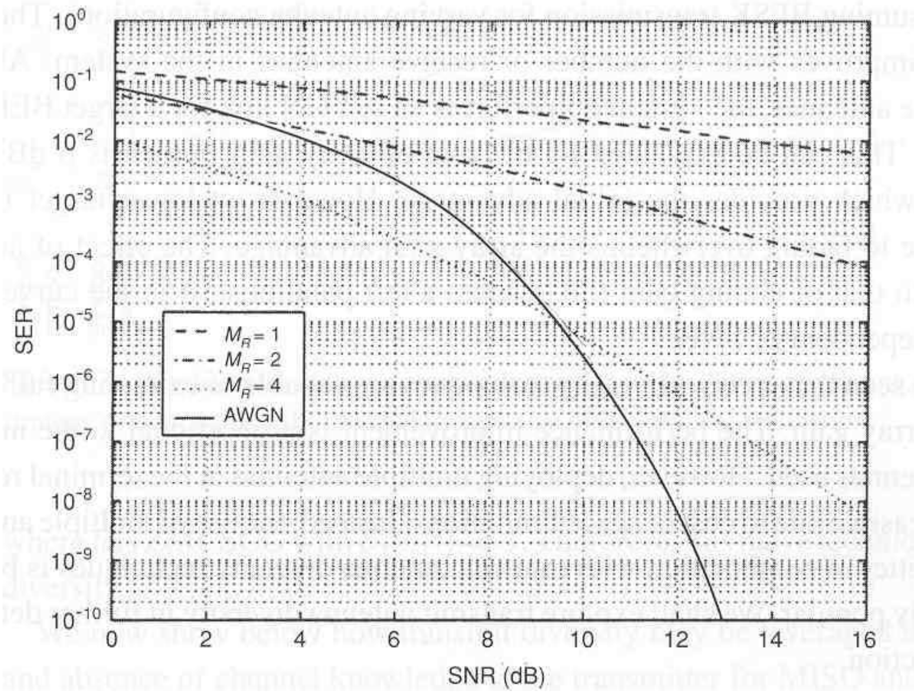
- Averaged SNR at the receiver

$$\overline{\eta} = M_R \cdot \rho$$

Array gain is M_R ($10 \log_{10} M_R$ [dB])

Receive antenna diversity - performance

- Can be better than AWGN due to array gain
- For BER$> 10^{-5}$ outperforms AWGN due to array gain.



RX diversity extracts full diversity and array gain!

Transmit antenna diversity

Straightforward approach is useless:

- Signal model (for $M_T = 2$); $h_1, h_2 \sim \mathcal{N}(0,1)$

$$y = \sqrt{\frac{E_s}{2}}(h_1 + h_2)s + n$$

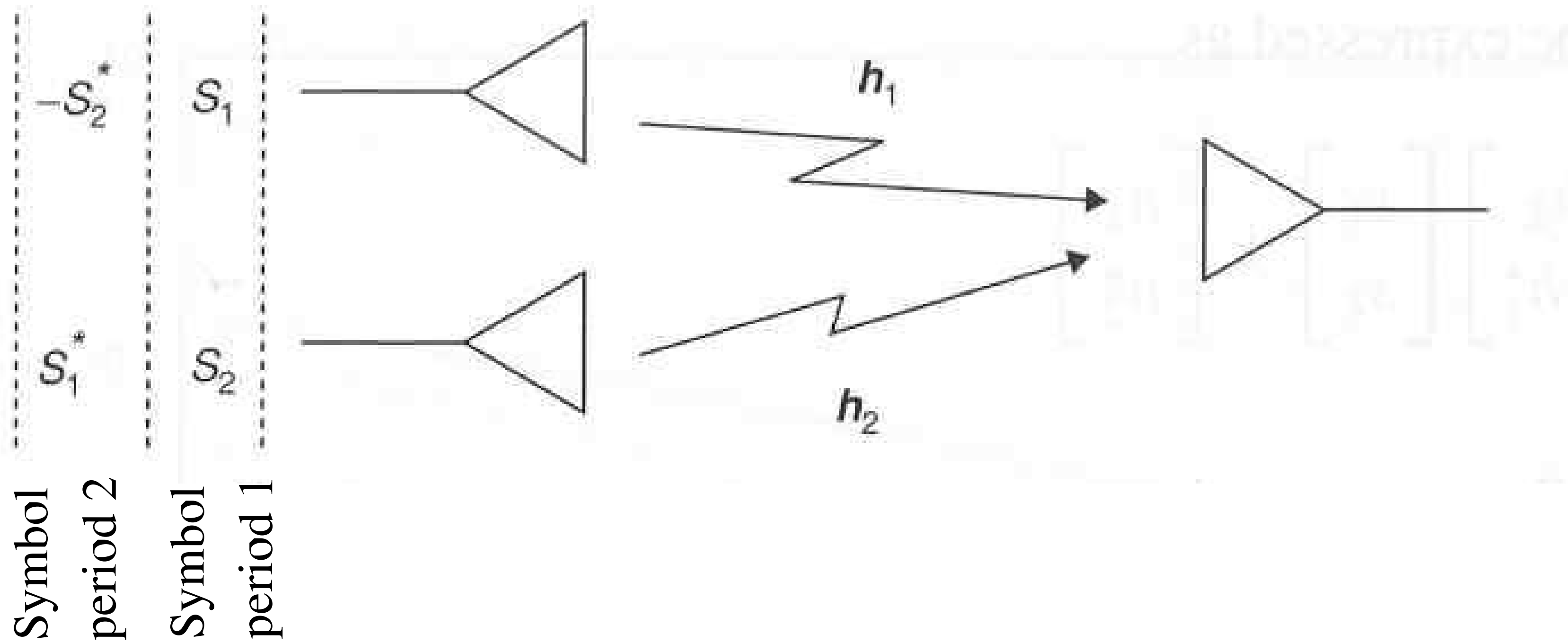
- Equivalently, $h \equiv (h_1 + h_2)/\sqrt{2}$, and $h \sim \mathcal{N}(0,1)$

$$y = \sqrt{E_s}hs + n$$

Unlike RX diversity, pre-processing is needed.

TX diversity – Alamouti(MISO)

Preprocessing without channel knowledge.



Alamouti scheme, cont'd

- Channel is constant over two symbols intervals and flat-fading

$$\boldsymbol{h} = [h_1, h_2]$$

- Two received symbols are:

$$y_1 = \sqrt{\frac{E_s}{2}} h_1 s_1 + \sqrt{\frac{E_s}{2}} h_2 s_2 + n_1$$
$$y_2 = -\sqrt{\frac{E_s}{2}} h_1 s_2^* + \sqrt{\frac{E_s}{2}} h_2 s_1^* + n_2$$

Alamouti scheme, cont'd

- Now, received vector is formed as $\boldsymbol{y} = [y_1, y_2^*]^T$

$$\boldsymbol{y} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \mathbf{H}_{\text{eff}} \boldsymbol{s} + \boldsymbol{n}$$

- MRC gives (using $\mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} = \|\boldsymbol{h}\|^2 \mathbf{I}$)

$$\boldsymbol{z} = \mathbf{H}_{\text{eff}}^H \boldsymbol{y} = \sqrt{\frac{E_s}{2}} \|\boldsymbol{h}\|^2 \mathbf{I} \boldsymbol{s} + \tilde{\boldsymbol{n}}$$

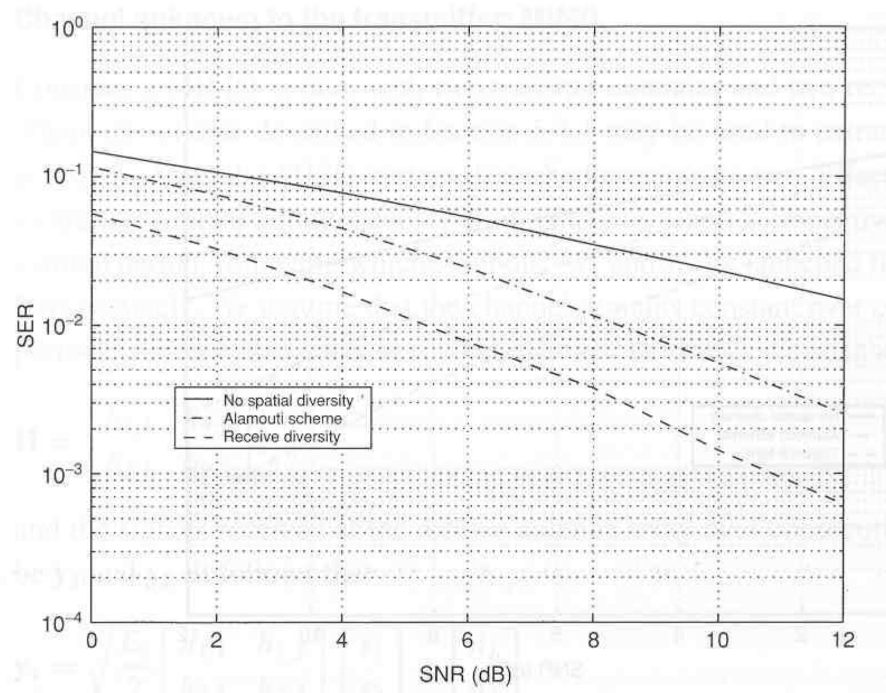
which simplifies to $z_i = \sqrt{\frac{E_s}{2}} \|\boldsymbol{h}\|^2 s_i + \tilde{n}_i$.

Alamouti scheme – Performance

- High SNR regime gives

$$\overline{P}_e \leq \overline{N}_e \left(\frac{\rho d_{min}^2}{4 \cdot 2} \right)^{-2}$$

- No array gain: $\overline{\eta} = \rho$
- Full transmit diversity for $M_R = 2$



How to implement array gain?

MISO with known channel

- Flat-fading channel is given as $\boldsymbol{h} = [h_1, \ldots, h_{M_T}]^T$
- Signal at the receiver

$$y = \sqrt{\frac{E_S}{M_T}}\boldsymbol{h}^H \boldsymbol{w} s + n$$

where $\boldsymbol{w} = \sqrt{M_T}\frac{\boldsymbol{h}}{\|\boldsymbol{h}\|}$

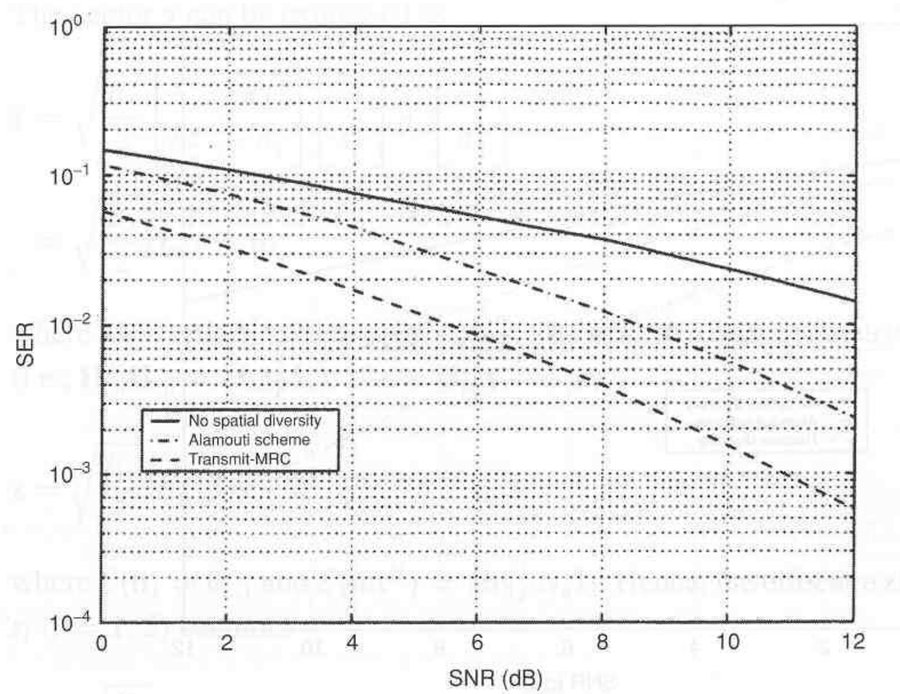
- This scheme is known as transmit MRC.

Transmit MRC – Performance

- High SNR regime gives

$$\overline{P}_e \leq \overline{N}_e \left(\frac{\rho d_{min}^2}{4} \right)^{-M_T}$$

- Array gain in rich scattering $\overline{\eta} = M_T \cdot \rho$
- Equivalent to RX diversity.



Transmitter should be aware of the channel.

Alamouti scheme & MIMO

- Transmit symbols like MISO Alamouti
- Flat-fading channel matrix

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$

- Receive symbols ($M_R = 2$)

$$\boldsymbol{y}_1 = \sqrt{\frac{E_s}{2}}\mathbf{H}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \boldsymbol{n}_1, \quad \boldsymbol{y}_2 = \sqrt{\frac{E_s}{2}}\mathbf{H}\begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix} + \boldsymbol{n}_2,$$

equivalently...

Alamouti scheme & MIMO, cont'd

- By defining $\boldsymbol{y} = [\boldsymbol{y}_1, \boldsymbol{y}_2^*]^T$

$$\boldsymbol{y} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,2}^* & -h_{2,1}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{n_1} \\ \boldsymbol{n_2} \end{bmatrix}$$

- MRC is formed (using $\mathbf{H}_{\text{eff}}^H \mathbf{H}_{\text{eff}} = \|\mathbf{H}\|_F^2 \mathbf{I}$)

$$\boldsymbol{z} = \sqrt{\frac{E_s}{2}} \|\mathbf{H}\|_F^2 \mathbf{I} \boldsymbol{s} + \tilde{\boldsymbol{n}}, \text{ or,}$$

$$z_i = \sqrt{\frac{E_s}{2}} \|\mathbf{H}\|_F^2 s_i + \tilde{n}_i$$

Alamouti scheme & MIMO, cont'd

- High SNR regime gives and rich scattering

$$\overline{P}_e \leq \overline{N}_e \left(\frac{\rho d_{min}^2}{4M_T} \right)^{-M_R M_T}$$

- Diversity order $M_R M_T$
- Array gain $\overline{\eta} = M_R \rho$

Thus, without channel knowledge only array gain is obtained.

MIMO with channel knowledge

- If channel known – transmit MRC is used

$$\boldsymbol{y} = \sqrt{\frac{E_s}{M_T}}\mathbf{H}^H \boldsymbol{w} s + \boldsymbol{n}$$

- Receiver forms weighted sum $z = \boldsymbol{g}^H \boldsymbol{y}$
- Vectors $\boldsymbol{w}$ and $\boldsymbol{g}$ are singular vectors from SVD decomposition of $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H; \quad \boldsymbol{w} \in \mathbf{U}, \boldsymbol{g} \in \mathbf{V}$$

$\boldsymbol{w}$ and $\boldsymbol{g}$ correspond to the max singular value σ_{max}^2 of $\mathbf{H}$

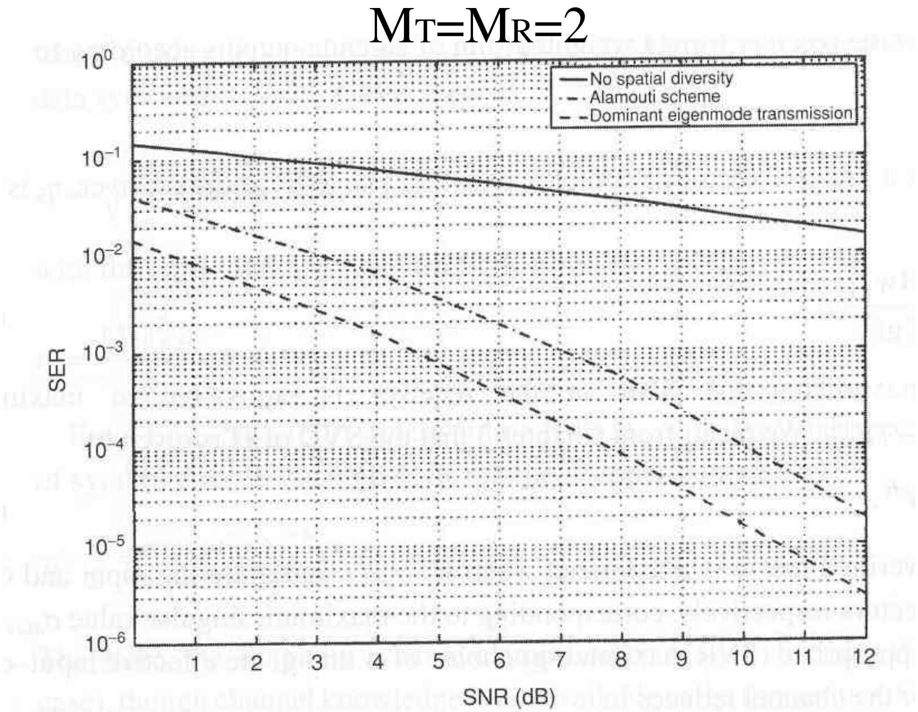
This is known as **dominant eigenmode transmission** – DET

MIMO with DET – Performance

- Diversity order $M_R M_T$
- SNR $\overline{\eta} = E\{\sigma_{max}^2\}\rho$

$$\max(M_T, M_R) \leq E\{\sigma_{max}^2\}$$

$$E\{\sigma_{max}^2\} \leq M_T M_R$$



Channel knowledge increases array gain.

Summary

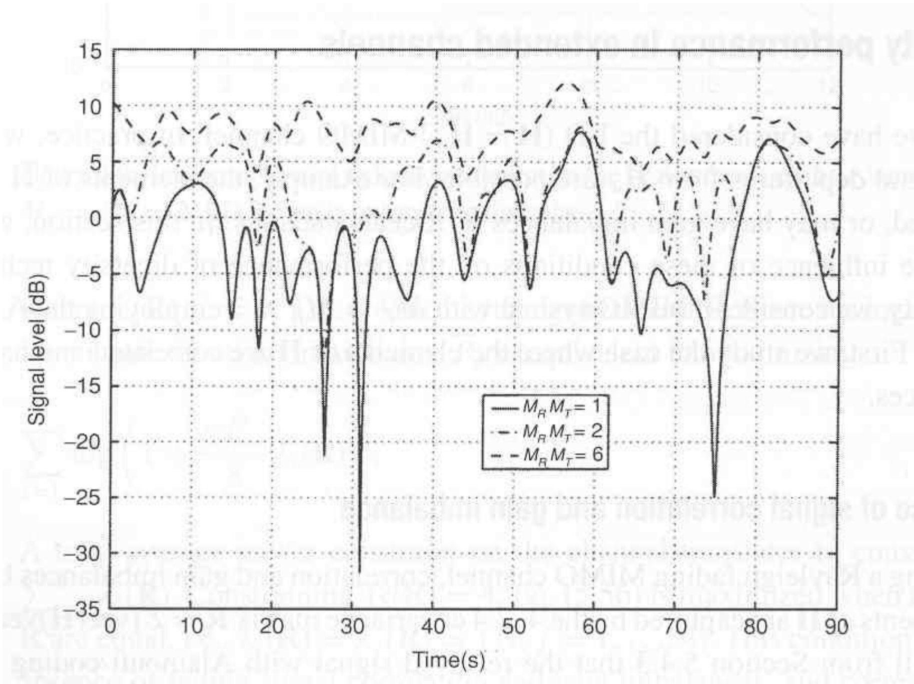
Configuration	Array gain	Diversity gain
SIMO (Ch.Un.)	M_R	M_R
SIMO (Ch.Kn.)	M_R	M_R
MISO (Ch.Un.)	1	M_T
MISO (Ch.Kn.)	M_T	M_T
MIMO (Ch.Un.)	M_R	$M_R M_T$
MIMO (Ch.Kn.)	$\max(M_T, M_R) \leq E\{\sigma^2_{max}\}$, $E\{\sigma^2_{max}\} \leq M_T M_R$	$M_R M_T$

Diversity order and channel variability

- Can be quantified as

$$\mu_{var} = \frac{1}{\sqrt{M_R M_T}}$$

- Link stabilizes as
$\mu_{var} \to \infty$



Diversity order for extended channels

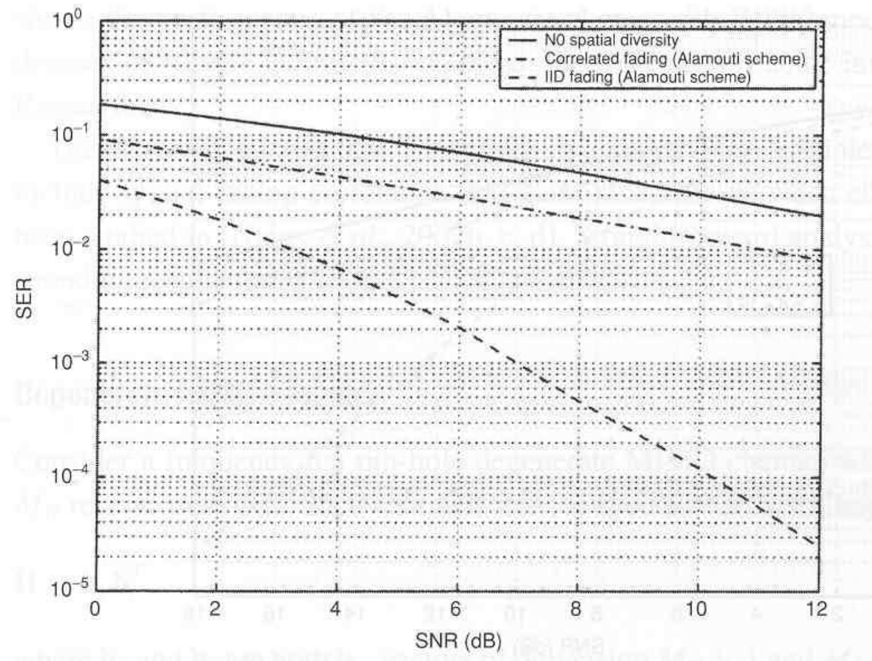
- Elements in channel matrix $\mathbf{H}$ are
  - Correlated
  - Have gain imbalances
  - Fade with Ricean statistics
- Highest possible diversity $\longrightarrow$ rich scattering (IID elements in $\mathbf{H}$)
- Let as consider Alamouti scheme with $M_T = M_R = 2$

Correlations in $\mathbf{H}$

- Diversity order decreases to $r(\mathbf{R})$, where $\mathbf{R}$ is (4×4) covariance Matrix:

$$\mathbf{R} = E\{\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H\}$$

- Rank $r(\mathbf{R}) = 1 \longrightarrow$ full correlation.
- No spatial diversity.
- Array gain is present.



Ricean fading of H

- Ricean fading occurs when LOS is present.
- As $K \to \infty$ link stabilizes. K-factor is given as

$$K = \frac{A_{\mathrm{LOS}}^2}{\sigma_{\mathrm{diffuse}}^2}$$