MU Detection III
Sub-optimum approaches to MU Detection

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January 26, 2005

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Outline

- 0. Recapitulation of MU Detection I + II
  - Linear CDMA Models
  - Conventional Detection: Matched Filters
  - (Asymptotic) multiuser efficiency
  - Optimum MU Detection
- 1. Decorrelating Detector
  - Derivation
  - Performance Analysis

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Outline

- 2. Nondecorrelating Linear Multiuser Detection
  - A. Optimum Linear Multiuser Detection
  - B. MMSE Linear Multiuser Detection
- 3. Decision-Driven Multiuser Detectors (Principles)
  - A. Successive Cancellation
  - B. Multistage Detection

0. Recapitulation of MU Detection I + II

Linear CDMA Models

Basic synchronous CDMA K-user channel model:

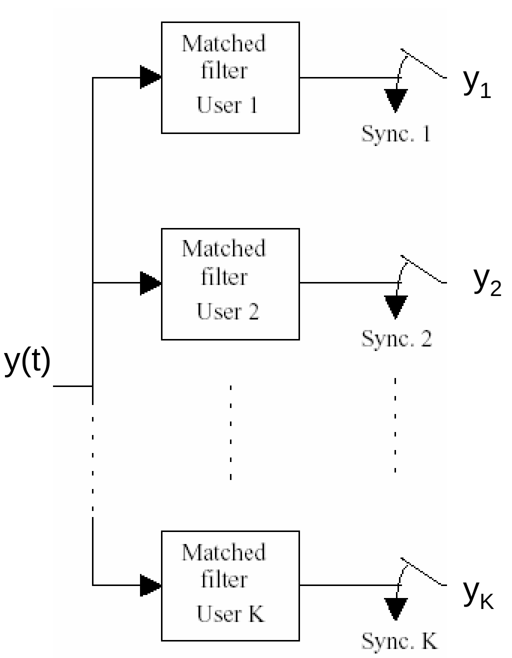
$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T]$$

Basic asynchronous CDMA K-user channel model:

$$y(t) = \sum_{k=1}^{K} \sum_{i=-M}^{M} A_k b_k[i] s_k(t - iT - \tau_k) + \sigma n(t), \quad t \in [0, T]$$

Recapitulation of MU Detection I + II

Conventional Detection: Matched Filters



$$y_1 = \int_0^T y(t) s_1(t) dt,$$

$$\vdots$$

$$y_K = \int_0^T y(t) s_K(t) dt,$$

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k$$

$$n_k = \sigma \int_0^T n(t) s_k(t) dt$$

$$\rho_{jk} = \int_0^T s_j(t) s_k(t) dt$$

Conventional Detection: Matched Filters

- Conventional detector:
  - Bank of matched filters.
  - Output sampled at bit-rate.
  - Sign of output for decision.
- Single user detection strategy:
  - Detect one user without regard to others.
  - Optimized to fight background noise exclusively.
- Low complexity & low performance.

(Asymptotic) multiuser efficiency

- Alternative to bit-error-rate.
- Ratio between:
  - SNR required to achieve same BER in absence of interfering users and … actual SNR.
- Asymptotic:
  - Background noise goes to zero.
  - Bit errors occur only because of interfering users.
  - Always in [0, 1].
    - 0 … probability of bit error is non-zero.
    - >0 … bit error rate vanishes.

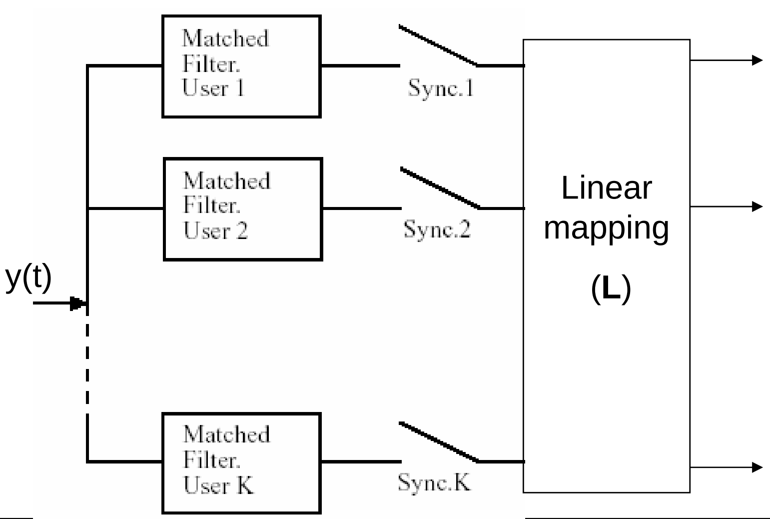
Optimum MU Detection

- Baseline of comparison for suboptimum multiuser detectors.
  - Minimum achievable probability of error.
  - Optimum near-far resistance.
  - Optimum asymptotic multiuser efficiency.

- High (unacceptable) complexity!!
- Therefore: Suboptimal approaches. Looking for performance/complexity tradeoffs.

Linear Detectors

- Linear mapping (**L**) to the output of the matched filters.
- Complexity grows linearly with number of users.
- **Base for all other linear detectors!**



1. Decorrelating Detector

Decorrelating Detector

Principles

- Requires knowledge of all signature waveforms.
- Conventional detector
  - Even error possible, when noise = 0.
    - Near-far problem (high differences in signal energies)
- Decorrelating detector
  - Error-free, when noise = 0.
  - Same property as optimum detector.

Decorrelating Detector

Derivation

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}$$

$$\mathbf{y} = [y_1, \ldots, y_k]^T$$

$$\mathbf{b} = [b_1, \ldots, b_k]^T$$

$$\mathbf{A} = diag\{A_1, \ldots, A_k\}$$

$$E[\mathbf{n}\mathbf{n}^T] = \sigma^2\mathbf{R}$$

- Matched filter equations in matrix form.
- **R**
  - normalized correlation matrix.
  - collecting auto- and crosscorrelations of signature waveforms.

Decorrelating Detector

Derivation (noise-free)

$$\mathbf{L} = \mathbf{R}^{-1}$$

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{R}^{-1}\mathbf{R}\mathbf{A}\mathbf{b} = \mathbf{A}\mathbf{b}$$

$$\hat{b}_k = sign((\mathbf{R}^{-1}\mathbf{y})_k) = sign((\mathbf{A}\mathbf{b})_k) = b_k$$

- K-th component is free from interference!!
  - Perfect demodulation.
- No knowledge of the received amplitudes required.

Decorrelating Detector

Derivation (plus noise)

- K-th component still free from user-interference.
- Only source of interference is background noise.
  - Therefore called *"decorrelating detector"*.
- Independent demodulation of each user.

- Equations:

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n}$$

$$\hat{b}_k = sign((\mathbf{R}^{-1}\mathbf{y})_k)$$

$$= sign((\mathbf{A}\mathbf{b})_k + (\mathbf{R}^{-1}\mathbf{n})_k)$$

Decorrelating Detector

Performance Analysis

- Bit-error-rate is invariant to the amplitudes of the interfering signals.

- Only interferer is the noise term:
  - Gaussian, zero mean, variance equal to the kk-component of the covariance matrix

$$E[(\mathbf{R}^{-1}\mathbf{n})(\mathbf{R}^{-1}\mathbf{n})^T] = E[\mathbf{R}^{-1}\mathbf{n}\mathbf{n}^T\mathbf{R}^{-1}]$$

$$= \sigma^2\mathbf{R}^{-1}\mathbf{R}\mathbf{R}^{-1} = \sigma^2\mathbf{R}^{-1}$$

Decorrelating Detector

Performance Analysis

- Thus, the bit-error-rate is

$$P_k^d(\sigma) = Q\Big(\frac{A_k}{\sigma\sqrt{(\mathbf{R}^{-1})_{kk}}}\Big)$$
$$= Q\Big(\frac{A_k\sqrt{1-\rho^2}}{\sigma}\Big) \quad two-user-case$$

- Decorrelating detector is optimal to:
  - Least-squares.
  - Near-far resistance.
  - ML when the received amplitudes are unknown !!
- Eliminates the multiuser interference.

Decorrelating Detector

Performance Analysis

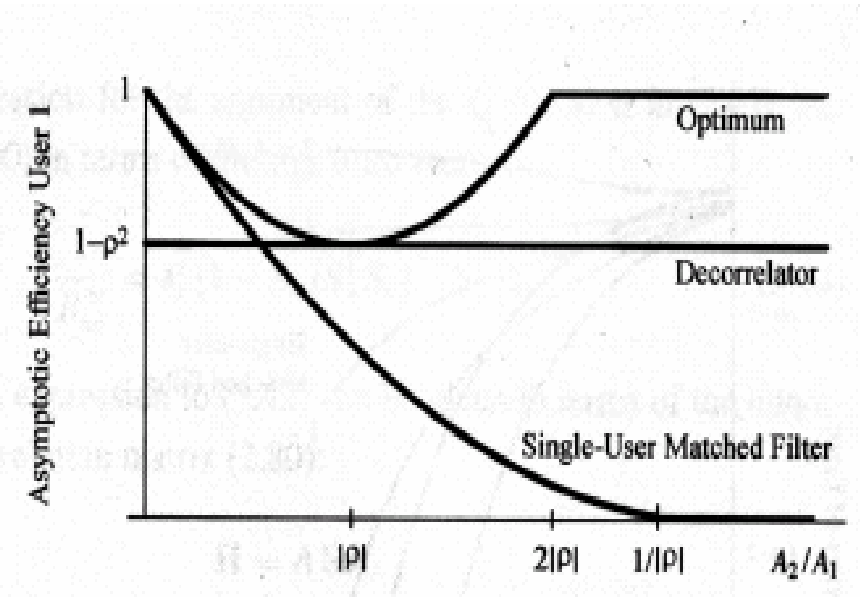


FIGURE 5.8. Asymptotic multiuser efficiencies for two synchronous users.

2. Nondecorrelating Linear Multiuser Detection

Principles

- Improve performance by incorporating received SNRs.
- When received amplitudes are unknown the decorrelating detector is a good choice.
- Single user matched filter performs better than the decorrelating detector for low power (far) interferers -> Performance improvement possible!

2. Nondecorrelating Linear Multiuser Detection

A. Optimum Linear Multiuser Detection

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Principles

- Maximize the asymptotic multiuser efficiency!
- k-th user linear transform: $\hat{b}_k = sign(\mathbf{l}_k^T \mathbf{y})$
- Maximization of function

  $$\eta_k(\mathbf{l}_k) = \ldots$$

  yields the coefficients for $\mathbf{l}_k$.

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Nondecorrelating Linear Multiuser Detection
Optimum Linear Multiuser Detection
Performance

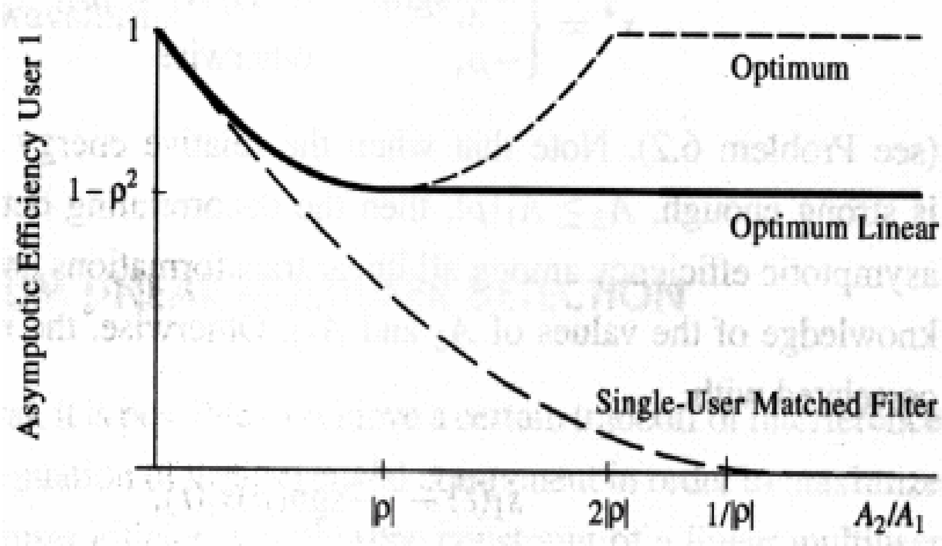

FIGURE 6.2. Asymptotic multiuser efficiencies for two synchronous users.

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Nondecorrelating Linear Multiuser Detection
Optimum Linear Multiuser Detection
Conclusion

- Considering only low background noise.
- Compromise solution between the decorrelating detector and the single-user matched filter.

- Small relative energy of the interferers:
  - Single-user matched filter.

- High relative energy of the interferers:
  - Decorrelating detector.

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2. Nondecorrelating Linear Multiuser Detection

B. MMSE Linear Multiuser Detection

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Nondecorrelating Linear Multiuser Detection

MMSE Linear Multiuser Detection

Principles

- Minimizing the mean squared error of
  - Original k user bit and the estimated bit.
  - Estimated bit:
    - Output of the k-th linear transform.

$$\hat{b}_k = (\mathbf{l}_k^T \mathbf{y})$$

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Optimization

- Outputs a weighted sum of the matched filter outputs -> finite dimensional optimization.
- For user k: $\min_{\mathbf{l}_k} \mathsf{E}[(b_k - \mathbf{l}_k^T \mathbf{y})^2]$
- Result:

$$\mathbf{L} = [\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1}$$

$$\sigma^2 \mathbf{A}^{-2} = diag\left\{\frac{\sigma^2}{A_1^2}, \ldots, \frac{\sigma^2}{A_K^2}\right\}$$

Performance

- SNR goes to infinity => MMSE detector converges to decorrelating detector.
  - MMSE detector has the same asymptotic multiuser efficiency and near-far resistance as the decorrelating detector.
- No bit-error-rate minimization.
- **But** maximizes the signal-to-interference ratio at the output of the linear transformation.

Conclusion

- Considering low and high background noise.
- Compromise between conventional detector and decorrelating detector.

- Special Cases:
  - (A_2, …, A_K) -> 0: Matched filter for user one.
  - Noise variance goes to infinity: Conventional detector.
  - Noise variance goes to zero: Decorrelating detector.

Adaptive Implentation

- Linear detector impulse response:
  - Computational costly. Matrix inversion!!
  - Time varying crosscorrelations & time varying received powers. -> Recalculation necessary!!

- Adaptive implementation of the MMSE detector:
  - Big advantage – low complexity!!
  - Using training sequence.
  - No knowledge of
    - **signature waveforms**
    - **received powers**

3. Decision-Driven Multiuser Detectors

Decision-Driven Multiuser Detectors

Principles

- Nonlinear detectors
- Decisions of bits of interfering users used to demodulate bit of interest.
- Particularly suited to high signal-to-noise ratio channels with power imbalances.

3. Decision-Driven Multiuser Detectors

A. Successive Cancellation

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Principles

- Simple and natural idea:
  - Make decision on an interfering user's bit.
  - Subtract recreated signal.
  - Resulting signal should contain one fewer user.
  - Repeat until all but one user have been demodulated.
- How to obtain the intermediate decisions?
  - single-user matched filter

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Successive Cancellation

Principles

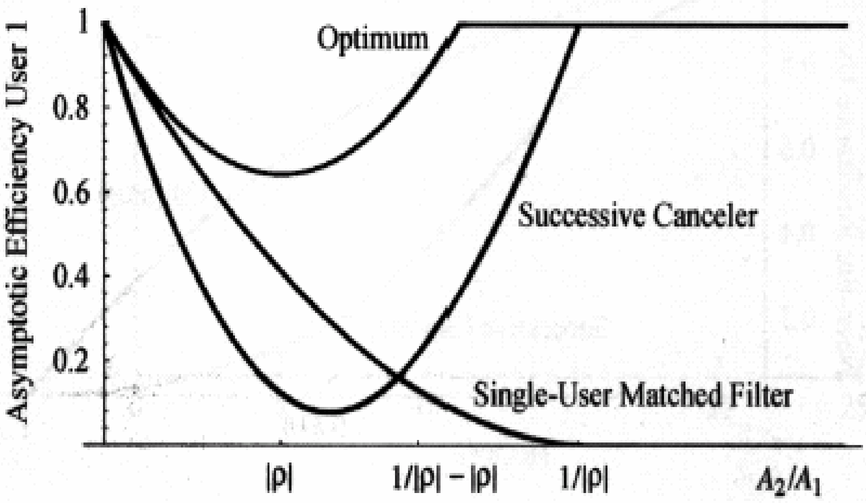
- Order of demodulation?
  - Order of decreasing received power:
    - Popular approach but not always best.
    - Takes not the crosscorrelation among users into account.
  - Order of expected value of squared k-th matched filter output:

$$\mathrm{E}\left[\left(\int_0^T y(t)s_k(t)dt\right)^2\right] = \sigma^2 + A_k^2 + \sum_{j\neq k} A_j^2 \rho_{jk}^2$$

Successive Cancellation

Performance

FIGURE 7.9. Asymptotic multiuser efficiencies for two synchronous users; $|\rho| = 0.6$.



3. Decision-Driven Multiuser Detectors

B. Multistage Detection

Principles

- Various decisions are produced at consecutive stages.
- First stage could be:
  - Conventional bank of single-user matched filters.
  - Decorrelating detector.
- Second stage uses successive cancellation for multiple users.
- Iteration possible to more stages with hopefully, increasingly performance.

Multistage Detection

Conventional 1st Stage

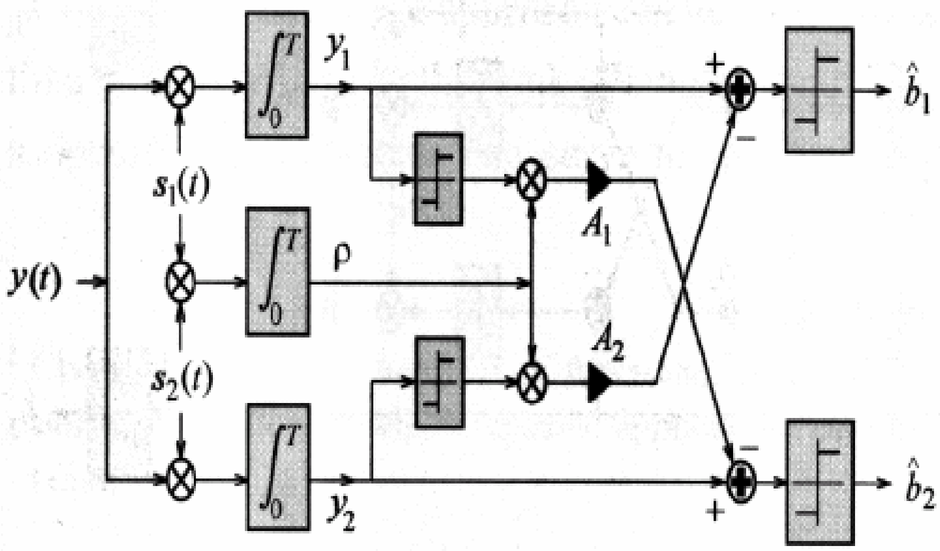


FIGURE 7.14. Two-stage detector for two synchronous users.

Decision-Driven Multiuser Detectors

Multistage Detection

Decorrelating 1st Stage

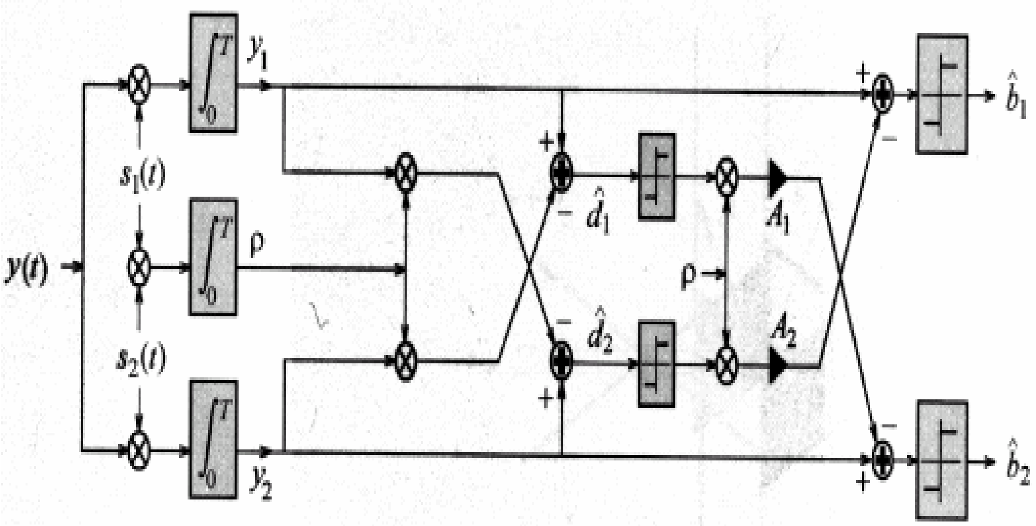


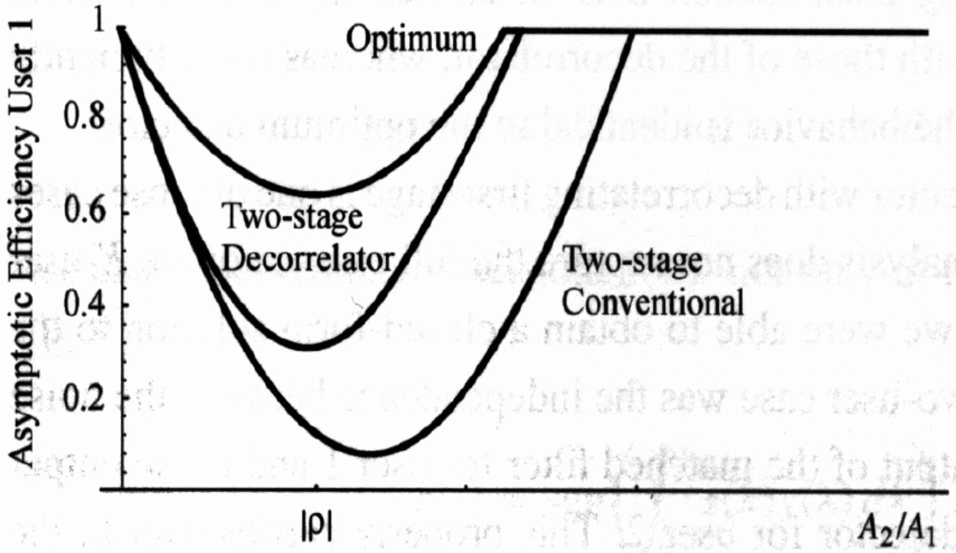
FIGURE 7.18. Two-stage detector with decorrelating first stage.

Decision-Driven Multiuser Detectors

Multistage Detection

Performance

FIGURE 7.21. Asymptotic multiuser efficiencies for two synchronous users; $|\rho| = 0.6$.



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Discussion

- Open questions …
- Comments …
- Otherwise …

THANK YOU FOR YOUR ATTENTION

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