Fingerprints

1. Historical Overview

2. The Individuality of Fingerprints

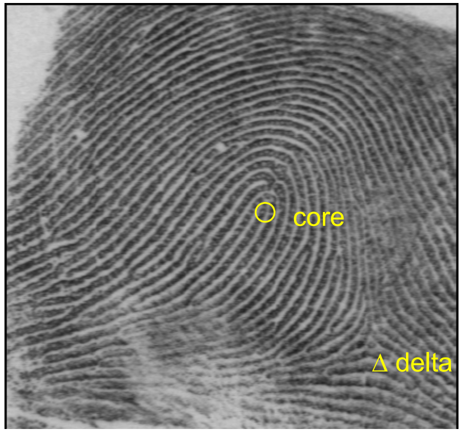
3. A authentications System

Markus Guldenschuh

History of fingerprints

-The Formation depends on the initial conditions of the embryonic mesoderm.

- The ridge structure is permanent and unchanging.



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- The first scientific paper about fingerprints was published in England from plant morphologist Nehemia Grew in 1684.

- In 1880 Henry Fauld first suggested the individuality and uniquness of a fingerprint.

- Since 1899 the „Henry System" is a standard in figerprint identification.

Individuality of fingerprints

Probability of occurrene of a special fingerprint

$1/16 * 1/256 * (p)^{24} = 1.45*10^{-11}$ (eg p=0.5) — Galton, Roxburgh, Pearson, Kingston

p^N N … Number of minutiae, (e.g. p=1/4) — Henry, Balthazard, Bose Wentworth, Wilder and ohters

$C/Pt *(Q/RT)^N$ — Roxburgh

- Pt … Probabilityfactor of fingerprint- and core type
- Q … Measure of image quality
- R … Number of semicircular ridges
- T … corrected number of minutiae types
- C … possible positions of the configuration

$P_{(\text{false association})} = 1 – (1 - P)^k$ — Amy

- P … Porbability of occurrence of a special fingerprint
- k … Number of feature comparisons

$$(e^{-y})(y^N/N!)(P_1)\prod_{i=2}^{N}(P_i)\frac{(0.082)}{[S-(i-1)(0.082)]}$$ — Kingston

- y … expected number of minutiae in the region S (in mm)
- P … Probability of occurrence of the specified minutia

$P_{(\text{false association})} = 1 - (1 - 0.6*(0.0005^{(N-1)})^{(N/5)}$ — Stoney

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Individuality of fingerprints

Comparison of Probability of a Particular Fingerprint Configuration Using Different Models

Author	P(Fingerprint Configuration)	N=36,R=24,M=72 (N=12,R=8,M=24)
Galton (1892)	$\frac{1}{16} \times \frac{1}{256} \times \left(\frac{1}{2}\right)^R$	1.45×10^{-11} (9.54×10^{-7})
Pearson (1930)	$\frac{1}{16} \times \frac{1}{256} \times \left(\frac{1}{36}\right)^R$	1.09×10^{-41} (8.65×10^{-17})
Henry (1900)	$\left(\frac{1}{4}\right)^{N+2}$	1.32×10^{-23} (3.72×10^{-9})
Balthazard (1911)	$\left(\frac{1}{4}\right)^N$	2.12×10^{-22} (5.96×10^{-8})
Bose (1917)	$\left(\frac{1}{4}\right)^N$	2.12×10^{-22} (5.96×10^{-8})
Wentworth & Wilder (1918)	$\left(\frac{1}{50}\right)^N$	6.87×10^{-62} (4.10×10^{-21})
Cummins & Midlo (1943)	$\frac{1}{31} \times \left(\frac{1}{50}\right)^N$	2.22×10^{-63} (1.32×10^{-22})
Gupta (1968)	$\frac{1}{10} \times \frac{1}{10} \times \left(\frac{1}{10}\right)^N$	1.00×10^{-38} (1.00×10^{-14})
Roxburgh (1933)	$\frac{1}{1000} \times \left(\frac{1.5}{10 \times 2.412}\right)^N$	3.75×10^{-47} (3.35×10^{-18})
Trauring (1963)	$(0.1944)^N$	2.47×10^{-26} (2.91×10^{-9})
Osterburg et al. (1977)	$(0.766)^{M-N}(0.234)^N$	1.33×10^{-27} (1.10×10^{-9})
Stoney (1985)	$\frac{N}{5} \times 0.6 \times (0.5 \times 10^{-3})^{N-1}$	1.2×10^{-80} (3.5×10^{-26})

For a fair comparison, we do not distinguish between minutiae types. By assuming that an average size fingerprint has 24 regions ($R = 24$) as defined by Galton, 72 regions ($M = 72$) as defined by Osterburg et al., and has 36 minutiae on average ($N = 36$), we compute the probability of observing a given fingerprint configuration in the third column of the table.The probability of observing a fingerprint configuration with $N = 12$ and equivalently, $R = 8$ and $M = 24$ is given in braces in the third column. Note that all probabilities represent a full (N minutiae) match as opposed to a partial match (see Table 3).

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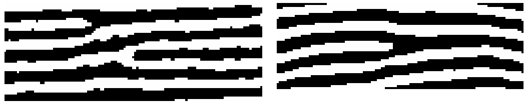
Sharat Pankanti, Sali Prabhakar, Anil K. Jain

A model of fingerprint individuality

Assumptions



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1) Only ridge endings and ridge bifurcations are regarded.
2) Minutiae are uniformely distributed but not too close together.
3) All minutiae-correspondences are equally important.
4) Image quality is not considered.
5) Ridge width is assumed to be the same across the population and the fingerprint.
6) There is only one alignment between the template and the input.
7) The minutiae directions are independent from the location.

Matching Conditions

- extracted features
    - x and y coordinates of the minutiae
    - angle of the corresponding ridge

- matching conditions

$$\sqrt{(x_i' - x_j)^2 + (y_i' - y_j)^2} \leq r_0, \quad \text{and}$$
$$\min\left(\left|\theta_i' - \theta_j\right|, 360 - \left|\theta_i' - \theta_j\right|\right) \leq \theta_0,$$

Correspondence Probability

1) Location

Given m minutiae in the templat, the probability that one input minutiae matches one of the templates is:

m*C/A

C … area of tolerance ($r_0^2 * \pi$, $2 r_0 w$ respectively, w … ridge periode)

A … total area of overlap

Now given n input minutia the probability that there is a matching of ρ minutiae is:

$$p(A,C,m,n,\rho) =$$

$$\binom{n}{\rho} \underbrace{\left(\frac{mC}{A}\right)\left(\frac{(m-1)C}{A-C}\right)\cdots\left(\frac{(m-\rho-1)C}{A-(\rho-1)C}\right)}_{\rho \text{ terms}} \times$$

$$\underbrace{\left(\frac{A-mC}{A-\rho C}\right)\left(\frac{A-(m-1)C}{A-(\rho+1)C}\right)\cdots\left(\frac{(A-(m-(n-\rho+1))C}{A-(n-1)C}\right)}_{n-\rho \text{ terms}}$$

Correspondence Probability

2) Direction or the minutia

$l = P(\min(|\theta'_i - \theta_j|, 360 - |\theta'_i - \theta_j|) <= \theta_0)$

The probability that q minutiae among the ρ correspondences have the same direction is given by:

$$\binom{\rho}{q}(l)^q(1-l)^{\rho-q},$$

The final correspondence probability follows as:

$$p(M,m,n,q) = \sum_{\rho=q}^{\min(m,n)} \left(\frac{\binom{m}{\rho}\binom{M-m}{n-\rho}}{\binom{M}{n}} \times \binom{\rho}{q}(l)^q(1-l)^{\rho-q} \right)$$

where M = A/C

Correspondence Probability

3) Results

TABLE 3
Fingerprint Correspondence Probabilities Obtained from the Proposed Individuality Model for Different Sizes of Fingerprint Images Containing 26, 36, or 46 Minutiae

M, m, n, q	P(Fingerprint Correspondence)
104, 26, 26, 26	5.27×10^{-40}
104, 26, 26, 12	3.87×10^{-9}
176, 36, 36, 36	5.47×10^{-59}
176, 36, 36, 12	6.10×10^{-8}
248, 46, 46, 46	1.33×10^{-77}
248, 46, 46, 12	5.86×10^{-7}
70, 12, 12, 12	1.22×10^{-20}

The entry (70, 12, 12, 12) corresponds to the 12-point guideline. The value of M for this entry was computed by estimating typical print area manifesting 12 minutia in a 500 dpi optical fingerprint scan.

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Design of the Fingerprint Verification System

1) Acquisition
2) Representation
3) Feature Extracion
4) Matching

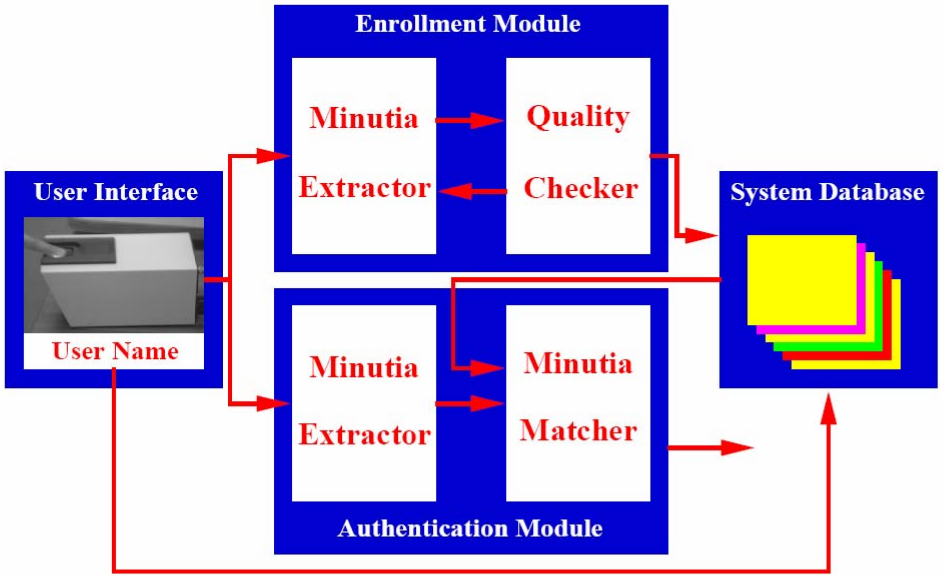

Figure 6. Architecture of the automatic identity authentication system

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1) Acquisition

a) inked (off-line)

b) live scan (ink-less)

- optical frustrated total internal reflection FTIR

  finger is layed on a glass plate

  the parts where ridges touch the plate are randomly scattered, while the areas, under the valleys are totally reflected

- Alternative methods:
  - thermal sensing
  - differential capacities
  - non contact 3D scanning

2) Representation

a) grey scale representation

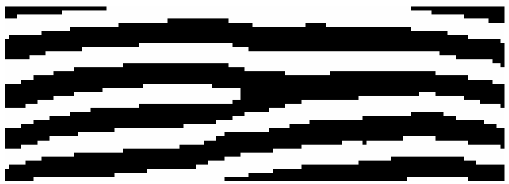
larger data

b) landmark extraction

more privacy (fingerprints cannot be reconstructed from landmark information alone)

e.g.: minutiae, their location (x, y coordinates), orientation, parts of the ridge itself

Ridge Ending



Ridge Bifurcation

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3) Feature Extraction

a) Orientation Field Estimation

b) Ridge Extarction

c) Thinning

d) Minutia Extractioin

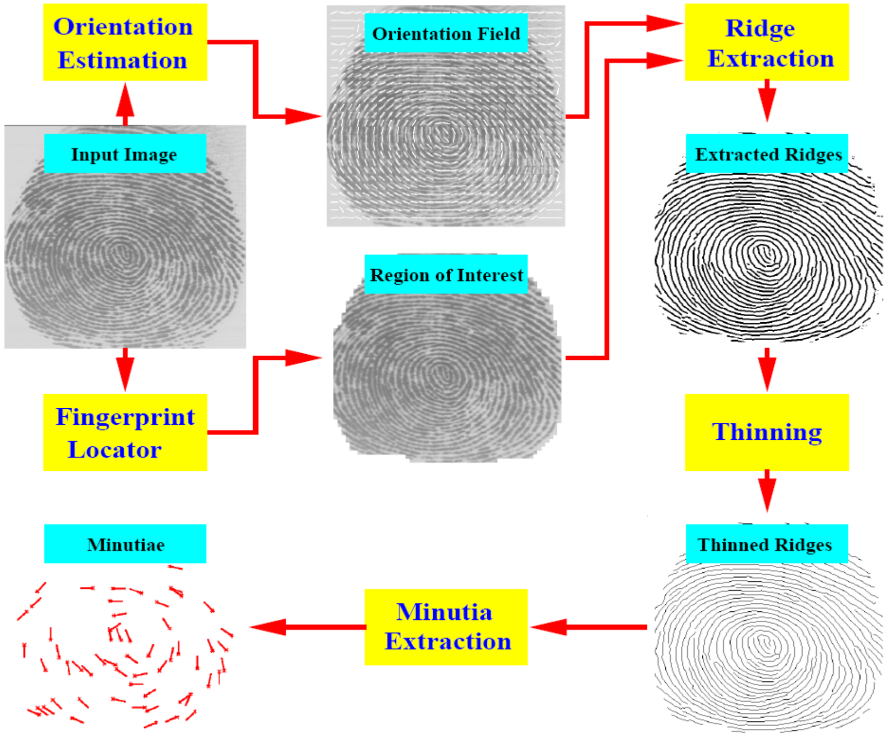

Figure 7. Flowchart of the minutiae extraction algorithm

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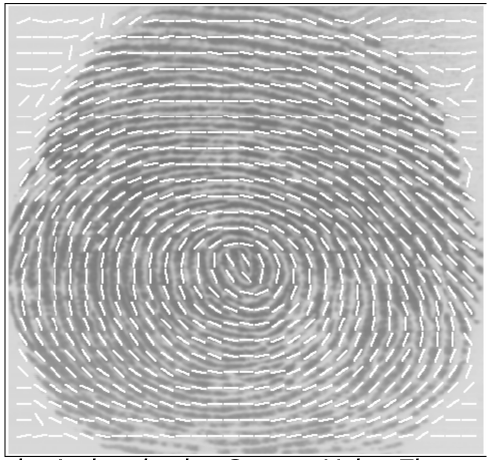
3.a) Orientation Field Estimation

$$V_x(i,j) = \sum_{u=i-\frac{W}{2}}^{i+\frac{W}{2}} \sum_{v=j-\frac{W}{2}}^{j+\frac{W}{2}} 2G_x(u,v)G_y(u,v),$$

$$V_y(i,j) = \sum_{u=i-\frac{W}{2}}^{i+\frac{W}{2}} \sum_{v=j-\frac{W}{2}}^{j+\frac{W}{2}} (G_x^2(u,v) - G_y^2(u,v)),$$

$$\theta(i,j) = \frac{1}{2} tan^{-1}(\frac{V_x(i,j)}{V_y(i,j)}),$$



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Where W is the size of the Window (16 x16) and G_x and G_y are the Gradients in x and y direction.

$$C(i,j) = \frac{1}{N}\sqrt{\sum_{(i',j')\in D} |\theta(i',j') - \theta(i,j)|^2},$$

$$|\theta' - \theta| = \begin{cases} d & if\,(d = (\theta' - \theta + 360)\; mod\, 360) < 180, \\ d - 180 & otherwise, \end{cases}$$

D represents the local neighbourhood around the block (5 x 5).

If C (conistency level) lies above a certain treshhold, the orientations around this region are re-estimated at a lower resolution.

3.a) Orientation Field Estimation

$$CL(i,j) = \sqrt{\frac{1}{W \times W} \frac{(V_x(i,j)^2 + V_y(i,j)^2)}{V_e(i,j)}}$$

with

$$V_e(i,j) = \sum_{u=i-\frac{W}{2}}^{i+\frac{W}{2}} \sum_{v=j-\frac{W}{2}}^{j+\frac{W}{2}} (G_x^2(u,v) + G_y^2(u,v))$$

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If the certainity level CL lies under a certain treshhold the pixel is marked as background.

3.b) Ridge Detection

-Grey level values of a fingerprint image show local maximae along the direction normal to the local ridge orientation.

- Convolution with masking windows to transform the grey-scaled image into a binary one.

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3.c) Minutiae Detection

Before the Thinning and the Minutiae Detection Algorithm can be applied, speckles and holes due to the imperfectness of the image capturing must be removed.

A pixel has the value 1 (ridge) or 0 (valley). Each pixel has 8 neighbours N.

$\Sigma^{7}_{i=0} N_i = 1$ ridge ending

$\Sigma^{7}_{i=0} N_i > 2$ ridge bifurcation

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4) Minutiae Matching

a) Transformation of the input pattern

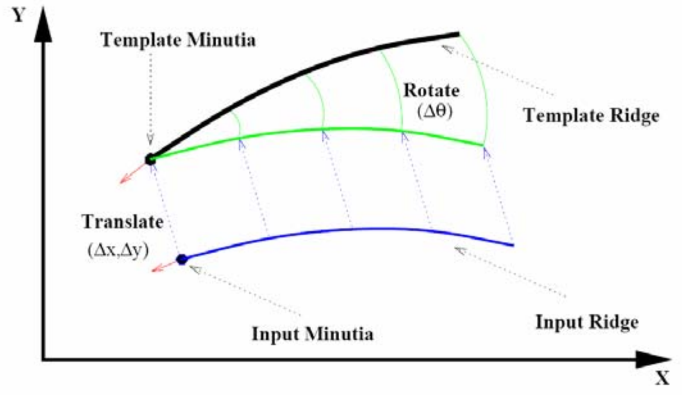

Figure 12. Alignment of the input ridge and the template ridge.

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b) Dynamic Distance Measurement in polar coordinates

c) Adaptation of the Matching Algotrithm

4.a) Transformation

d_i, D_i … Distances from point i on the ridge to the x-axis (given by the orietation of the minutia)

L …….. Minimal length of the ridges

$$S = \frac{\sum_{i=0}^{L} d_i D_i}{\sqrt{\sum_{i=0}^{L} d_i^2 D_i^2}},$$

If S exceeds a certain treshhold the transformation between the 2 ridges is estimated by:

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} x^d \\ y^d \end{pmatrix} - \begin{pmatrix} x^D \\ y^D \end{pmatrix},$$ translation vector

$$\Delta\theta = \frac{1}{L}\sum_{i=0}^{L}(\gamma_i - \Gamma_i),$$ rotation angle

Transformation:

$$\begin{pmatrix} x_i^A \\ y_i^A \\ \theta_i^A \end{pmatrix} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} + \begin{pmatrix} \cos\Delta\theta & \sin\Delta\theta & 0 \\ \sin\Delta\theta & -\cos\Delta\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i - x^d \\ y_i - y^d \\ \theta_i - \theta^d \end{pmatrix},$$

4.b) String Matching

The edit distance is recursively calculated as the entries of a Matix C.

$$C(m,n) = \begin{cases} 0 & \text{if } m = 0 \text{ and } n = 0 \\ \min \left\{ \begin{matrix} C(m-1,n)+\Omega \\ C(m,n-1)+\Omega \\ C(m-1,n-1)+w(m,n) \end{matrix} \right\} & 0 < m \leq M \text{ and } 0 < n \leq N, \end{cases}$$

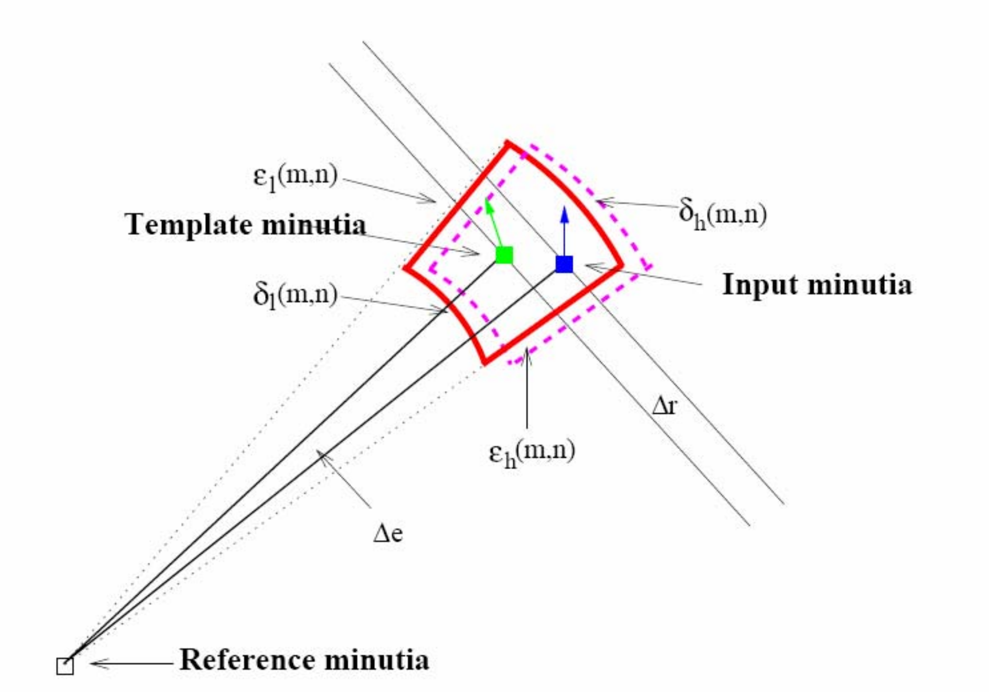
$$w(m,n) = \begin{cases} \alpha \left| r_m^P - r_n^Q \right| + \beta \Delta e + \gamma \Delta\theta & \text{if } \left| r_m^P - r_n^Q \right| < \delta,\ \Delta e < \epsilon \text{ and } \Delta\theta < \varrho \\ \Omega & \text{otherwise,} \end{cases}$$

bounding box (δ,ε,ρ)

$$\Delta e = \begin{cases} a & \text{if } (a = (e_m^P - e_n^Q + 360) \bmod 360) < 180 \\ a - 180 & \text{otherwise,} \end{cases}$$

$$\Delta\theta = \begin{cases} a & \text{if } (a = (\theta_m^P - \theta_n^Q + 360) \bmod 360) < 180 \\ a - 180 & \text{otherwise,} \end{cases}$$

4.c) Bounding Box Adaptation



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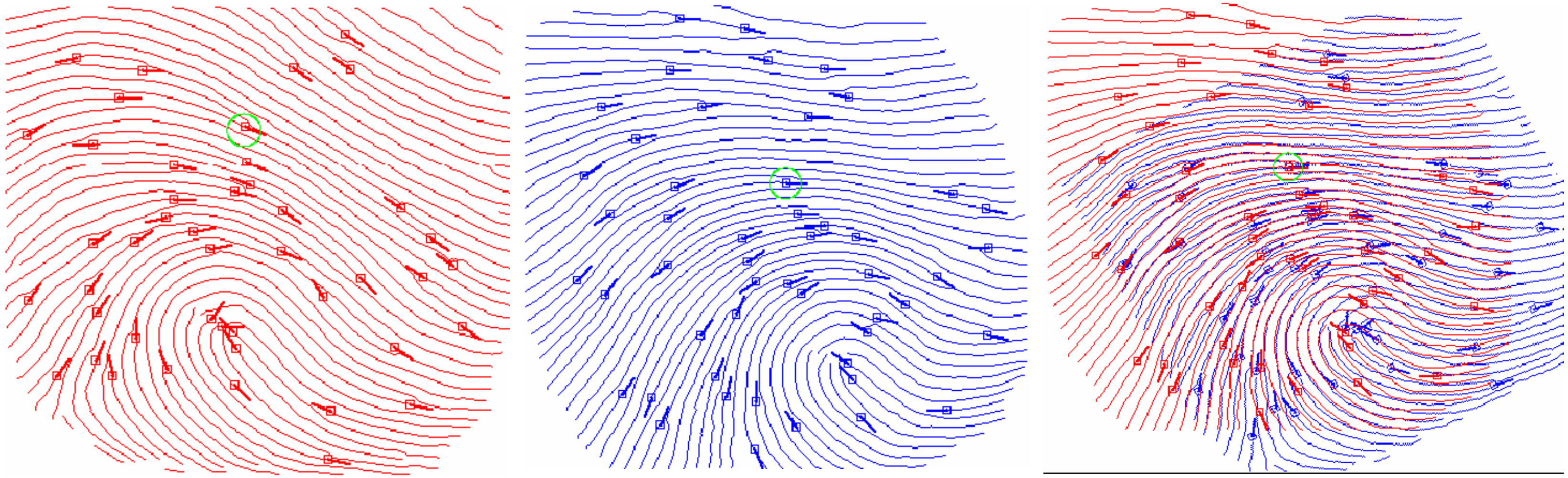
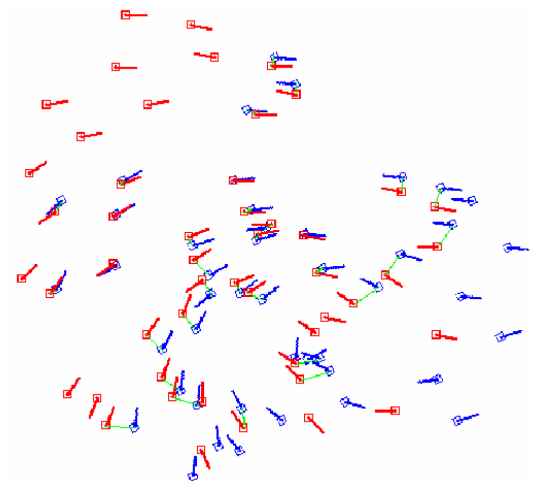
In dependance to the aberration of the input pattern the boundig box is adjusted.

Results of the Matching Algorithm



(a) (b) (c)



(d)

Results of applying the matching algorithm to an input minutiae set and a template;
(a) input minutiae set; (b) template minutiae set;
(c) alignment result based on the minutiae marked with green circles;
(d) matching result where template minutiae and their correspondences are connected by green lines.

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Results of the Matching Algorithm

Threshold Value	False Acceptance Rate (MSU)	False Reject Rate (MSU)	False Acceptance Rate (NIST 9)	False Reject Rate (NIST 9)
7	0.07%	7.1%	0.073%	12.4%
8	0.02%	9.4%	0.023%	14.6%
9	0.01%	12.5%	0.012%	16.9%
10	0	14.3%	0.003%	19.5%

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-MSU:

70 persons

3 templates and 7 test-images each

31 360 matches

-NIST:

900 types

1 template and 1 test-iamge per type

718 200 matches

Literature

-„On the Individuality of Fingerprints“ Sharath Pankanti, Salil Prabhakar and Anil K. Jain (2002)

- „An Identity Authentication System Using Fingerprints“ Anil Jain, Lin Hong, Sharath Pankanti and Ruud Bolle (1997)

- „A systematic method for fingerprint ridge orientation estimation and image segmentation“ En Zhu, Jianping Yin, Chunfeng Hu and Guomin Zhang (2006)

- „A Minutiae-based Fingerprint Individuality Model“ Jiansheng Chen and Yiu-Sang Moon (2007)

- „Individuality of Fingerprints: Comparison of Models and Measurements“ Sargur Srihari and Harisch Srinivasan (2007)