Geophysical Unsupervised Signal Processing

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Advanced Signal Processing 2
Outline

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  Summary

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  Wiener Filter and Unsupervised Processing
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Degenerate Unmixing Estimation Technique – DUET
Reflection Seismic
Reflectivity Function

→ Earth response to an ideal impulsive, seismic source
How?

- Information on the subsurface of an area under analysis
- Produce seismic waves (dynamite, air-guns, ...)
- Reflection on subsurface
- Sensor grids located in the surface measure reflections

Why?

- Exploration and monitoring of hydrocarbon reservoirs
- Assessing sites for CO$_2$ sequestration
- Nuclear waste deposition
- Information about structure of earth
Model Description

Source Signals

- $s_1$
- $s_2$
- $s_3$
- $s_m$

Mixing Matrix $A$

$$A = \begin{bmatrix}
a_{11} & a_{21} & \ldots & a_{n1} \\
a_{12} & a_{22} & \ldots & a_{n2} \\
& & \ddots & \vdots \\
a_{1m} & a_{2m} & \ldots & a_{mm}
\end{bmatrix}$$

Observed Signals

- $x_1$
- $x_2$
- $\ldots$
- $x_m$

Different Goals

- Finding $s_1(n)$ – Reflectivity Function
- Finding $A$ – Wavelets
- Separation of $s_i(n)$
Problems:

- Seismic source and subsurface propagation not ideal
- Output is mixture of different waves which must be identified and separated

Hypothesis:

- Linear distortion and mixtures
- Convolution relationship – source signal and propagation environment are not ideal
- Mixing – linear combination
Summary

- Geophysical applications include
  - Estimation of different sources
  - Estimation of reflectivity function
- Problems due to the not ideal environment conditions
- Prior assumption of model to be able to use known algorithms
Approaches for Unsupervised Processing [1]

Wiener Filter and Unsupervised Processing

- Geophysical SP - prominent role to develop unsupervised methods
- Wieners theory to seismology (Enders A. Robinson 1954 [2])
Question

- Additional information to the measured data about input sources and the convolution/mixing system
- If one of them is known – supervised methods

Assumption

- No additional information available
**Source Signature – known**

- Deterministic supervised deconvolution in seismology
- e.g. estimate reflectivity function through linear filtering, using the Wiener-Levinson minimization of the MSE

**Source Signature – unknown**

- Unsupervised task
- Exploiting prior knowledge about structure of source signature and the reflectivity
Task

- Get reflectivity function from measured data using deconvolution

Predictive Deconvolution

- Wiener theory on prediction and filtering could be used for predictive, unsupervised deconvolution [2]

Two Hypothesis

1. The seismic wavelet is the impulse response of an all-pole, minimum phase system.

2. The impulse response of the layered earth model behaves like a decorrelated (white) signal, so that it has a flat frequency spectrum.
Predictive Deconvolution

- Deconvolution using prediction-error filter uses only SOS – simplification of task BUT
- Prediction-error filter acts as a whitening filter – ideally recovers uncorrelated signals
- Problem if seismic wavelet cannot be modeled as an all-pole, minimum phase filter
Limitation of linear predictive deconvolution

Next step – from SOS to HOS
Blind Source Separation

![Diagram showing blind source separation](image)
Blind Source Separation

Source Signals

\[ s_1 \]
\[ s_2 \]
\[ s_3 \]
\[ \vdots \]
\[ s_m \]

Mixing Matrix \( A \)

\[
\begin{bmatrix}
    a_{11} & a_{21} & \cdots & a_{n1} \\
    a_{12} & a_{22} & \cdots & a_{n2} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{1m} & a_{2m} & \cdots & a_{mm}
\end{bmatrix}
\]

Observed Signals

\[ x_1 \]
\[ x_2 \]
\[ \vdots \]
\[ x_m \]

\[
x(n) = [x_1(n) \quad x_2(n) \quad \ldots \quad x_M(n)]^T
\]

\[
s(n) = [s_1(n) \quad s_2(n) \quad \ldots \quad s_N(n)]^T
\]

\[
x = A \cdot s \rightarrow \hat{s} = W \cdot x \quad \text{with} \quad W = A^{-1}
\]
Goal: Estimate all source signals

- $W$ can recover sources if mixing system is linear, memory-less and time-invariant
- SOS: $W$ is adjusted so as to decorrelate the recovered signals $\hat{s}$ (using PCA)
- HOS: $W$ is adjusted so that recovered signals become mutually independent
Blind Source Separation using SOS and HOS
ICA-Based seismic Deconvolution

ICA – Independent Component Analysis

- Computational model for separating multiple sources
- Produces additive subcomponents
- Assumption:
  - Mutual statistical independence
  - Source signals are non-Gaussian and i.i.d (satisfied by reflectivity)
Model

\[ x = As \]

\[
\begin{pmatrix}
  x(0) \\
  x(1) \\
  \vdots \\
  x(N-1)
\end{pmatrix}
= A
\begin{pmatrix}
  \rho(0) \\
  \rho(1) \\
  \vdots \\
  \rho(N-1)
\end{pmatrix}
\]

\[
A = \begin{bmatrix}
  h(0) & 0 & \cdots & 0 & 0 & \cdots & 0 \\
  h(1) & h(0) & \cdots & 0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  h(N_h - 1) & \vdots & \cdots & h(0) & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \cdots & h(1) & h(0) & \cdots & \vdots \\
  0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
  0 & 0 & \cdots & h(N_h - 1) & h(N_h - 2) & \cdots & h(0)
\end{bmatrix}
\]

→ Single Snapshot → delayed versions of \( x(n) \) and \( \rho(n) \) – \( N \) snapshots
Step 1 – Rearrangement

- Obtaining of $M < N$ mixtures
- Improvement of statistical properties of the mixture matrix

\[
X = \begin{bmatrix}
0 & 0 & \ldots & 0 & x(0) & \ldots & x(N - M) \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & x(0) & \ldots & \ldots & x(M - 2) & \ldots & x(N - 2) \\
x(0) & x(1) & \ldots & \ldots & x(M - 1) & \ldots & x(N - 1)
\end{bmatrix}
\]
Step 2 – Data Whitening

- Decorrelation of the signals
- Equalize power

\[ z(n) = W_{SOS}x(n) \]

\( W_{SOS} \) obtained with SOS

Step 3 – Dimension Reduction

- Linear transformation – new set of \( N_h \) mixtures

\[ \tilde{x}(n) = N_k^T W_{SOS}^T z(n) \]
Step 4 – ICA

\[ \hat{s}(n) = \tilde{W}_{HOS} \tilde{x}(n) \]

\[ \tilde{W}_{HOS} = \tilde{Q} \tilde{W}_{SOS} \]

with \[ \tilde{Q} \]
given by ICA

Step 5 – Wavelet and Reflectivity Estimation

\[ \hat{s}_i = h_i^T \tilde{x}(n) \]

- Optimal Pair \((\hat{s}_* (n), h_*)\) gives original reflectivity and wavelet
- \(h_i\) from \(\tilde{W}_{HOS}\)
Results I – Synthetic Data

(a) Noiseless synthetic trace; (b) Synthetic reflectivity function; (c) Linear prediction error filter; (d) Estimated reflectivity based on estimated wavelet
(a) 35-points Berlage wavelet; (b) estimated wavelet by B-ICA; (c) zero-pole plot for the original wavelet; (d) zero-pole plot for estimated wavelet
Summary

- Differences between supervised and unsupervised processing
- Predictive deconvolution to estimate reflectivity function
- Limitations of predictive deconvolution – go to HOS
- Blind source separation to estimate sources
- Combination of deconvolution and BSS – ICA-Based seismic Deconvolution
Volcano Hazards

- many villages close to volcanos
- predict eruptions, tremors, earthquakes to avoid casualties

Problem:
lack of prediction knowledge
  - number of sources?
  - types of sources?

Solution:
use blind source separation for data-mining

Landscape after ashfall.

How to improve knowledge of volcanic behaviour?
DUET: Degenerate Unmixing Estimation Technique

Features

- for blind separation of multiple sources
  - Volcano Tectonic (VT) events
  - Long Period (LP) events
- based on pair of seismograph sensors (stations)
- second sensor enables use of a time-frequency–lattice \((\tau, \omega)\)
- stations separated by less than half a wavelength of interest
- performs best if there’s no signal overlap between sources
- **robust** behaviour in echoic mixtures
DUET: Degenerate Unmixing Estimation Technique

Seismic sources & absorbing sensors.

Are there any assumptions?
Assumptions

- Anechoic Mixing Model
- W-Disjoint Orthogonality
Assumptions: Anechoic Mixing Model

- no realistic model
- direct paths only (non-echoic signals)
- stations provide attenuation & delay parameters of waves

Anechoic (Ideal) Mixing Model.

Echoic (Realistic) Mixing Model.
Assumptions: Anechoic Mixing Model cont’d

- model

\[
x_k(t) := \sum_{j=1}^{N} a_{k,j} \cdot s_j(t - \delta_{k,j}) \quad k = \{1, 2\}
\]

where

- \(x_k(t)\) ... mixture of k-th sensor
- \(s_j(t)\) ... source signal
- \(a_{k,j}\) ... attenuation coefficient
- \(\delta_{k,j}\) ... time delay
- \(k\) ... sensor index
- \(j\) ... source index
Assumptions: W-Disjoint Orthogonality (WDO)

- disjoint supports of signals’ windowed Fourier transforms ($\mathcal{F}$)
- every $(\tau, \omega)$-point in a mixture $x_j(t)$ dominated by contribution of at most one source

\[ \Omega_1 \cap \Omega_2 = 0 \]

Disjoint sets $\Omega_1$ & $\Omega_2$.

WDO and non-WDO signals.
Assumptions: W-Disjoint Orthogonality (WDO) cont’d

- given window function $w(t)$ & source signals $s_j(t), s_k(t)$
- given windowed $\mathcal{F}$ of $\hat{s}_j(\tau, \omega), \hat{s}_k(\tau, \omega)$

$$\hat{s}_j(\tau, \omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} w(t - \tau) s_j(t) e^{-i\omega t} dt$$

where $\omega$ is the angular frequency and $\tau$ is the window shift

- WDO if supports of $\hat{s}_j(\tau, \omega), \hat{s}_k(\tau, \omega)$ are disjoint

$$\hat{s}_j(\tau, \omega) \cdot \hat{s}_k(\tau, \omega) = 0 \quad \forall \tau, \omega, \quad \forall j \neq k$$
Assumptions: W-Disjoint Orthogonality (WDO) cont’d

- if $w(t) = 1$
  - simply $\mathcal{F}$ of whole mixture
  - signals have to be frequency-disjoint to satisfy WDO-condition (frequency-division multiplexed signals)
- if $w(t) = \delta(t)$
  - signals have to be time-disjoint to satisfy WDO-condition (time-division multiplexed signals)

How does the algorithm work?
Signal Representation

- mixture signals can be decomposed into weighted & shifted source signals in time & frequency domain

\[
x_k(t) := \sum_{j=1}^{N} a_{k,j} \cdot s_j(t - \delta_{k,j})
\]

\[
\hat{x}_k(\tau, \omega) := \sum_{j=1}^{N} a_{k,j} \cdot \hat{s}_j(\tau, \omega) \cdot e^{-i\omega \delta_{k,j}}
\]

Sources, sensors, and delays.
Signal Representation cont’d

- without loss of generality

\[ a_{1,j} = 1 \quad \delta_{1,j} = 0 \quad a_{2,j} = a_j \quad \delta_{2,j} = \delta_j \]

- interested in the differences of both captured signals
- vectorized mixture signals \((k = \{1, 2\})\)

\[
\begin{bmatrix}
\hat{x}_1(\tau, \omega) \\
\hat{x}_2(\tau, \omega)
\end{bmatrix} =
\begin{bmatrix}
1 & \cdots & 1 \\
a_1 e^{-i\omega\delta_1} & \cdots & a_N e^{-i\omega\delta_N}
\end{bmatrix}
\begin{bmatrix}
\hat{s}_1(\tau, \omega) \\
\cdots \\
\hat{s}_N(\tau, \omega)
\end{bmatrix}
\]

How to decompose the mixture signals?
Binary Mask

- given mixture signal $x_k(t)$
- given $\mathcal{F}$ of $\hat{x}_k(\tau, \omega)$

\[
\hat{x}_k(\tau, \omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sum_{j=1}^{N} a_{k,j} \cdot s_j(t - \delta_{k,j}) e^{-i\omega t} dt
\]

\[
\hat{x}_k(\tau, \omega) := a_1 \cdot \hat{s}_1(\tau, \omega) \cdot e^{-i\omega \delta_1} + a_2 \cdot \hat{s}_2(\tau, \omega) \cdot e^{-i\omega \delta_2} + \ldots + a_N \cdot \hat{s}_N(\tau, \omega) \cdot e^{-i\omega \delta_N}
\]
Binary Mask cont’d

- source separation in case of WDO using an ideal binary mask

\[ \hat{s}_j(\tau, \omega) := M_j(\tau, \omega) \cdot x_k(\tau, \omega), \ \forall \tau, \omega \]

where

\[ M_j(\tau, \omega) := \begin{cases} 1 & \hat{s}_j(\tau, \omega) \neq 0 \\ 0 & \text{otherwise} \end{cases} \]

and

\[ M_j(\tau, \omega) \cdot M_k(\tau, \omega) = 0 \ \forall \tau, \omega \ \forall j \neq k \]
Binary Mask cont’d

Binary mask of a mixture signal for source 2.

How do we obtain coefficients $a_j$ & $\delta_j$ in case of WDO-signals?
Parameter Estimation

WDO- & Anechoic Source Signals

- ratio of \((\tau, \omega)\)-representations of mixtures depends on active source component mixing parameters only

\[
R(\tau, \omega) := \frac{\hat{x}_2(\tau, \omega)}{\hat{x}_1(\tau, \omega)}
\]

- assume set of \((\tau, \omega)\) on \(\Omega_j := \{(\tau, \omega) : M_j(\tau, \omega) = 1\}\)

\[
R(\tau, \omega) := a_j \cdot e^{-i\omega\delta_j}
\]

\[
a_j := |R(\tau, \omega)| \quad \delta_j := -\frac{1}{\omega} \arg R(\tau, \omega)
\]

Do we have ideal conditions in real world?
Parameter Estimation cont’d

Approximated WDO- & Echoic Source Signals

- in field scenarios perfect WDO never given
- high degree of WDO approximation per frame if
  - small overlap of source spectra in frequency
  - small overlap of source spectra in time
- increase degree of WDO approximation by
  - changing sensor position
  - changing window type/size (ao)

In case of non-WDO, why is the DUET still suitable?

- Volcano Tectonic and Long Period events ...
  - hardly occur simultaneously and, thus,
  - exhibit high degree of WDO approximation
Parameter Estimation cont’d

- accurate estimation of true coefficients required

\[ \alpha = a_j - \frac{1}{a_j} \]

\[ \hat{\alpha}_j = \frac{\sum_{(\tau, \omega) \in \Omega_j} |\hat{x}_1(\tau, \omega) \cdot \hat{x}_2(\tau, \omega)|^2 \cdot \alpha}{\sum_{(\tau, \omega) \in \Omega_j} |\hat{x}_1(\tau, \omega) \cdot \hat{x}_2(\tau, \omega)|^2} \]

\[ \hat{\delta}_j = \frac{\sum_{(\tau, \omega) \in \Omega_j} |\hat{x}_1(\tau, \omega) \cdot \hat{x}_2(\tau, \omega)|^2 \cdot \delta}{\sum_{(\tau, \omega) \in \Omega_j} |\hat{x}_1(\tau, \omega) \cdot \hat{x}_2(\tau, \omega)|^2} \]

How do we obtain mask \( M_j \) to separate source signals?
Parameter Estimation & Demixing cont’d

1. construct \((\tau, \omega)\)-representations \(\hat{x}_1(\tau, \omega)\) & \(\hat{x}_2(\tau, \omega)\)

2. label each non-zero \((\tau, \omega)\)-point with computed pair \((\hat{\alpha}, \hat{\delta})\)

3. weight each pair with energy of corresponding \((\tau, \omega)\)-point

4. generate histogram of these labels

5. find peaks in histogram for each source

6. construct masks \(M_j\) to determine \(\Omega_j\) for each source

7. apply masks to mixture

\[ \hat{s}_j(\tau, \omega) = M_j(\tau, \omega) \cdot \hat{x}_1(\tau, \omega) \]

8. convert each \(\hat{s}_j(\tau, \omega)\) back into time domain
Results (with both parameters $\alpha$ and $\delta$)

($\alpha, \delta$)-lattice for one (top left) and six (top right) sources. Original, mixed, and separated signals (bottom).
Results (with single parameter $\delta$)

Mixtures of Long Period (LP) and Volcano Tectonic (VT) events at both sensors (top/bottom left). Band-pass filtered mixtures (top/bottom right).
Results (with single parameter $\delta$) cont’d

Band-pass filtered mixtures (top left) and corresponding delay histogram of VT and LP events (top right). Separated signals of VT (bottom top) and LP (bottom bottom).
Conclusion

- DUET applied to geophysical data
- for blind source separation of VT and LP events
- results show that DUET ...
  - is a powerful tool
  - for identifying, separating, and locating VT and LP events
  - to better understand the source mechanism behind tremors, earthquakes, and eruptions caused by volcanic activities
References


