Baseband Volterra Filters with Even-Order Terms: Theoretical Foundation and Practical Implications

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Abstract

- **The conventional baseband Volterra series includes only odd-order terms.**
- **We derive an extended baseband Volterra series which also includes even-order terms.**

1. Introduction

- Continuous-time Volterra series in vector notation

$$y(t) = \sum_{p=1}^{P} \int_{\mathbb{R}^p} h_p(\boldsymbol{\tau}_p)\, \Phi_p(t, \boldsymbol{\tau}_p)\, \mathrm{d}\boldsymbol{\tau}_p$$

$$\boldsymbol{\tau}_p = [\tau_1, \tau_2, \ldots, \tau_p]^{\mathrm{T}} \quad \Big| \quad \Phi_p(t, \boldsymbol{\tau}_p) = \prod_{i=1}^{p} x(t - \tau_i)$$

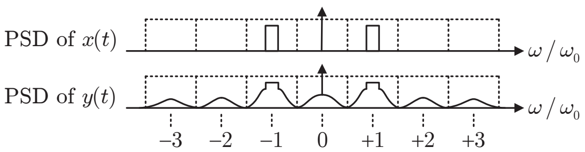
- If $x(t)$ and $y(t)$ are modulated radio-frequency signals, signal processing is more efficient in equivalent baseband.

2. Baseband Volterra Series

- Response to a passband input signal

Input signal: $$x(t) = \frac{1}{2}\tilde{x}(t)e^{+j\omega_0 t} + \frac{1}{2}\tilde{x}^*(t)e^{-j\omega_0 t}$$

Output signal: $$y(t) = \sum_{p=1}^{P} \sum_{\substack{k=-p \\ \text{steps of 2}}}^{+p} \frac{1}{2}\tilde{y}_{p,k}(t)e^{jk\omega_0 t}$$



- Modulation of output signal harmonics

$$\tilde{y}_{p,k}(t) = \int_{\mathbb{R}^p} \tilde{h}_{p,k}(\boldsymbol{\tau}_p)\, \tilde{\Phi}_{p,k}(t, \boldsymbol{\tau}_p)\, \mathrm{d}\boldsymbol{\tau}_p$$

$$\tilde{\Phi}_{p,k}(t, \boldsymbol{\tau}_p) = \prod_{i=1}^{\frac{p+k}{2}} \tilde{x}(t - \tau_i) \prod_{l=\frac{p+k}{2}+1}^{p} \tilde{x}^*(t - \tau_l)$$

- The order p and harmonic k must be of the same parity.

3. Baseband Volterra Series with Even-Order Terms

- Modified basis functional in passband for even p

$$\Phi_p(t, \boldsymbol{\tau}_p) = \prod_{i=1}^{p-1} x(t - \tau_i)\, |x(t - \tau_p)|$$

- Modulation of output signal harmonics for even p

$$\tilde{y}_{p,k}(t) = \sum_{\substack{k_1=-(p-1) \\ \text{steps of 2}}}^{+(p-1)} \int_{\mathbb{R}^p} \tilde{h}^{(k_1)}_{p,k}(\boldsymbol{\tau}_p)\, \tilde{\Phi}^{(k_1)}_{p,k}(t, \boldsymbol{\tau}_p)\, \mathrm{d}\boldsymbol{\tau}_p$$

$$\tilde{\Phi}^{(k_1)}_{p,k}(t, \boldsymbol{\tau}_p) = \tilde{\Phi}_{p-1,k_1}(t, \boldsymbol{\tau}_{p-1})\, |\tilde{x}(t - \tau_p)|\, e^{j(k-k_1)\angle\tilde{x}(t-\tau_p)}$$

- Discrete-time implementation (we choose $k_1 = k$)

$$\tilde{y}[n] = \sum_{p \in \mathcal{P}} \sum_{\boldsymbol{m}_p \in \mathcal{M}_p} \tilde{h}_p[\boldsymbol{m}_p]\, \tilde{\Phi}_p[n, \boldsymbol{m}_p]$$

$$\tilde{\Phi}_1[n, \boldsymbol{m}_1] = \tilde{x}_{n-m_1}$$

$$\tilde{\Phi}_2[n, \boldsymbol{m}_2] = \tilde{x}_{n-m_1}\, |\tilde{x}_{n-m_2}|$$

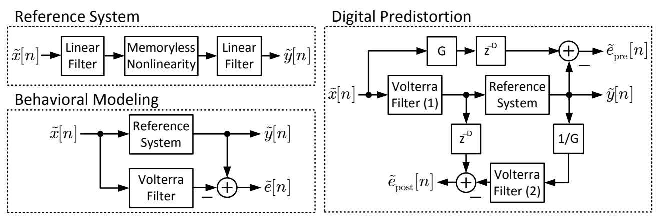
$$\tilde{\Phi}_3[n, \boldsymbol{m}_3] = \tilde{x}_{n-m_1}\, \tilde{x}_{n-m_2}\, \tilde{x}^*_{n-m_3}$$

$$\tilde{\Phi}_4[n, \boldsymbol{m}_4] = \tilde{x}_{n-m_1}\, \tilde{x}_{n-m_2}\, \tilde{x}^*_{n-m_3}\, |\tilde{x}_{n-m_4}|$$

$$\tilde{\Phi}_5[n, \boldsymbol{m}_5] = \tilde{x}_{n-m_1}\, \tilde{x}_{n-m_2}\, \tilde{x}_{n-m_3}\, \tilde{x}^*_{n-m_4}\, \tilde{x}^*_{n-m_5}$$

$$\vdots$$

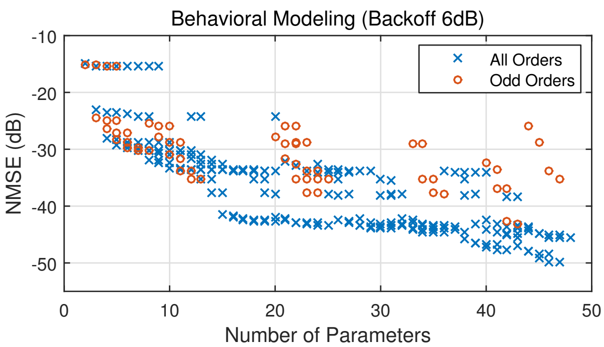
4. Simulation Setups

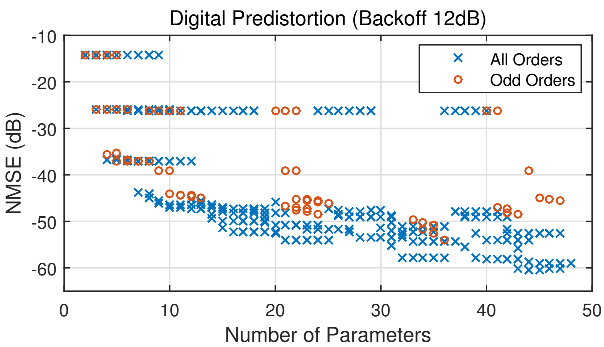


Conclusion

- **Even-order terms are mathematically justified.**
- **Simulations demonstrate improved performance.**

5. Simulation Results





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