A Joint Linearity-Efficiency Model of Radio Frequency Power Amplifiers

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Motivation

- Behavioral Model of Linearity and Efficiency
  - Based on measurements
  - Based on theoretical model $\Leftarrow$ subject of this work

![Behavioral Model Diagram]

- Diagram showing input and output signals of a power amplifier (PA) with DC current $i_{DC}(t)$ and DC voltage $V_{DC}$.
Power Amplifier Circuit Model

\[ g_m = \frac{\partial i_d}{\partial v_g} \]

Current: \( i_d \)
Voltage: \( v_g \)
Maximum current: \( I_{\text{max}} \)
Threshold voltage: \( V_{\text{th}} \)
Saturation voltage: \( V_{\text{sat}} \)

Output Voltage: \( v_{\text{out}} \)
Input Voltage: \( v_{\text{in}} \)
Output Match: \( 50\Omega \)
Input Match: \( 50\Omega \)
Load Resistance: \( R_L \)
DC Current: \( i_{\text{DC}} \)
Waveform of Drain Current

Conduction Angle

\[ \alpha = \begin{cases} 
0 & Q \leq 0 \land A_{in} \leq \frac{Q}{Q-1} \\
2\pi & Q > 0 \land A_{in} \leq \frac{Q}{Q-1} \\
2\arccos\left(\frac{Q}{A_{in}(Q-1)}\right) & \text{otherwise}
\end{cases} \]

Saturation Angle

\[ \beta = \begin{cases} 
0 & Q \leq 1 \land A_{in} \leq 1 \\
2\pi & Q > 1 \land A_{in} \leq 1 \\
2\arccos\left(\frac{1}{A_{in}}\right) & \text{otherwise}
\end{cases} \]

Quiescent Point

\[ Q \]

Input Amplitude

\[ A_{in} \]

normalized to fullscale

"Cubic Model"

\[ s_{out} = 3s_{in}^2 - 2s_{in}^3 \]

"Linear Model"

\[ s_{out} = s_{in} \]
Fourier Series of Drain Current

**Linear Model**

\[
\frac{I_k}{I_{\text{max}}} = \frac{2}{\pi} \left[ \int_0^{\frac{\beta}{2}} \cos(k\theta) \, d\theta + Q \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos(k\theta) \, d\theta + \left[1 - Q \right] A_{\text{in}} \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos(\theta) \cos(k\theta) \, d\theta \right]
\]

**Cubic Model**

\[
\frac{I_k}{I_{\text{max}}} = \frac{2}{\pi} \left[ \int_0^{\frac{\beta}{2}} \cos(k\theta) \, d\theta + [3Q^2 - 2Q^3] \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos(k\theta) \, d\theta + \left[6Q - 6Q^2 \right][1 - Q] A_{\text{in}} \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos(\theta) \cos(k\theta) \, d\theta + \left[3 - 6Q \right][1 - Q]^2 A_{\text{in}}^2 \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos^2(\theta) \cos(k\theta) \, d\theta - 2[1 - Q]^3 A_{\text{in}}^3 \int_{\frac{\alpha}{2}}^{\frac{\beta}{2}} \cos^3(\theta) \cos(k\theta) \, d\theta \right]
\]

\[i_d(\theta) = I_{\text{DC}} + \sum_{k=1}^{\infty} I_k \cos(k\theta) \quad I_{\text{DC}} = \frac{I_0}{2}\]
Harmonics of Drain Current

Linear Model

Class A

Class B

Class AB

Class C

Cubic Model

$A_{in} = 1$  Sweep over $Q$
From Current to Power

Assumptions

- Ideal output matching
  - Match at first harmonic
  - Short at higher harmonics
- Ideal transistor output
  - Zero knee voltage
  - Zero output conductance

\[
\frac{P_{DC}}{P_{\text{ref}}} = 4 \frac{I_{DC}}{I_{\text{max}}}
\]

\[
\frac{P_{RF}}{P_{\text{ref}}} = 4 \left( \frac{I_1}{I_{\text{max}}} \right)^2 \frac{R_L}{R_{\text{ref}}}
\]

\(P_{\text{ref}}\) ... Full scale output in class A

\(R_L\) ... Load-line match for \(P_{\text{ref}}\)
Linearity and Efficiency

**AM-AM Characteristic**

\[ A_{\text{out}} = \sqrt{\frac{P_{RF}}{P_{\text{ref}}}} \]

**Instantaneous Efficiency**

\[ \eta_{\text{inst}} = \frac{P_{RF}}{P_{\text{DC}}} \]
Average Efficiency

Definition

\[ \eta_{\text{avg}} = \frac{\mathbb{E}\{P_{\text{RF}}\}}{\mathbb{E}\{P_{\text{DC}}\}} \]

Time-Domain

\[ \eta_{\text{avg}} = \frac{\int P_{\text{RF}}(t) \, dt}{\int P_{\text{DC}}(t) \, dt} \]

Ensemble-Domain

\[ \eta_{\text{avg}} = \frac{\int a^2 f_A(a) \, da}{\int \frac{a^2}{\eta_{\text{inst}}(a)} f_A(a) \, da} \]

\[ \eta_{\text{avg}} \neq \mathbb{E}\{\eta_{\text{inst}}\} \text{ in general} \]

(only valid for ideal class A)
Application Example

Setup
- Class AB model
- Various signals
- With / without DPD

Observations
- Average efficiency is mainly defined by required backoff
- With DPD, the backoff can be reduced
Matlab Code

http://www.mathworks.com/matlabcentral/fileexchange/53413

calcPaModel

\[ Q \rightarrow \text{calcPaModel} \]

\[ A_{in} \rightarrow P_{RF} \]
\[ A_{in} \rightarrow P_{DC} \]

PA-Model, consisting of 2 memoryless lookup-tables

\[ \tilde{v}_{in}(t) \rightarrow \text{calcPaSignal} \]
\[ \tilde{v}_{out}(t) \quad P_{RF}(t) \quad P_{DC}(t) \]

demo_calcPaModel

Demo-Script for calculation of PA characteristics

demo_calcPaSignal

Demo-Script for simulation of input-output behavior
Conclusion

- Theoretical Model of Linearity and Efficiency
  - Includes strong nonlinearity due to current clipping
  - Includes weak nonlinearity due to transconductance variation
  - Does not include nonlinear memory effects

- Potential Applications
  - High-level system simulations
  - Study of new transmitter architectures
  - Education