Behavioral Modeling and Digital Predistortion of Radio Frequency Power Amplifiers

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PhD Defense
06.03.2018
Overview

1. The linearity-efficiency trade-off
   Joint linearity-efficiency model of radio frequency power amplifiers (RF-PAs)

2. Behavioral modeling of RF-PAs
   Even-order terms in polynomial baseband models

3. Digital predistortion of RF-PAs
   Dual-band digital predistortion (DPD) of an envelope tracking power amplifier
The Linearity-Efficiency Trade-off

Motivation

Joint model

Linearity  ↔  Efficiency

Research Question

What are the limitations of linearity and efficiency of RF-PAs?

Methodology

- Extend the classical efficiency analysis of RF-PAs
- Apply linearity and efficiency quantification methods
- Explore the linearity-efficiency trade-off by simulations
The linearity and efficiency characteristics can be derived by a Fourier series analysis of the drain current waveform.
Linearity & Efficiency Behavior

**Piecewise linear** transistor model

**Linearity:** Amplitude modulation to amplitude modulation (AM-AM)

**Efficiency:** Instantaneous drain efficiency

The **piecewise cubic** transistor model produces realistic **linearity** and **efficiency** characteristics.
Nonlinearity metrics strongly depend on the signal statistics. Average drain efficiency mainly depends on the output power backoff.
Behavioral Modeling of RF-PAs

Motivation

Conventional theory: Only odd-order terms in RF-PA baseband models
Experimental evidence: Even-order terms can improve the accuracy

Research Question

What are the foundations of complex baseband models of RF-PAs?

Methodology

- Derive passband-baseband model pairs with even-order terms
- Analyze the mathematical properties of complex baseband models
Spectral Analysis of a Polynomial Model

Passband signal

\[ x(t) = \text{Re}\{\tilde{x}(t) e^{i\omega t}\} \]

Passband model

\[ y(t) = \sum_{p=1}^{P} \alpha_p x^p(t) \]

Only **odd-order** monomials produce output in the **first spectral zone**.
Analysis of Even-Order Terms

**Passband model**

\[ y(t) = \sum_{p=1}^{P} \alpha_p x^p(t) \]

Polyomial basis functions

**Baseband model**

\[ \tilde{y}(t) = \sum_{p=1}^{P} \tilde{\alpha}_p \tilde{x}(t) |\tilde{x}(t)|^{p-1} \]

Even-order terms in the baseband model can be explained by odd-symmetric magnitude-power functions in the passband model.

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**Magnitude Power Functions**

**Odd-symmetric**

\[ f(x) = \text{sign}(x) |x|^p \]

- \( p = 1 \)
- \( p = 3 \)

**Even-symmetric**

\[ f(x) = |x|^p \]

- \( p = 1 \)
- \( p = 2 \)
- \( p = 3 \)
- \( p = 4 \)

Spectral characteristics correlate with **symmetry** of basis functions, not with **order**.
### Polynomial Models with Memory

**Volterra series**

\[ y(t) = \sum_{p=1}^{P} \int_{\mathbb{R}^p} h_p(\tau_p) \Psi_p(t, \tau_p) \, d\tau_p \]

**Basis functionals**

#### Passband

\[ \prod_{i=1}^{p} x(t - \tau_i) \]

- *p-fold product*

\[ \prod_{i=1}^{p-1} x(t - \tau_i) |x(t - \tau_p)| \]

- *(p–1) terms*

- *1 term*

- *p-fold product, p is even*

#### Baseband

\[ q = \left\lfloor (p + 1)/2 \right\rfloor \]

- \[ \prod_{i=1}^{q} \tilde{x}(t - \tau_i) \prod_{l=q+1}^{2q-1} \tilde{x}^*(t - \tau_i) \]

- *q terms*

- *(q–1) terms*

- *p-fold product, p is odd*

- **Even-order terms in baseband** can also be derived for **Volterra** series models.

Phase Homogeneity

Passband model
\[ \mathcal{N} : x(t) \rightarrow y(t) \]

Baseband model
\[ \tilde{\mathcal{N}}_k : \tilde{x}(t) \rightarrow \tilde{y}_k(t) \]

If the passband model is time-invariant, the baseband model must obey phase homogeneity:
\[ \tilde{\mathcal{N}}_k \left\{ e^{j\xi} \tilde{x}(t) \right\} = e^{jk\xi} \tilde{\mathcal{N}}_k \left\{ \tilde{x}(t) \right\} \]

e.g. baseband Volterra series (1\textsuperscript{st} harmonic, k=1)

<table>
<thead>
<tr>
<th>Order</th>
<th>( \tilde{x}(t - \tau_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st}</td>
<td>( \tilde{x}(t - \tau_1) )</td>
</tr>
<tr>
<td>2\textsuperscript{nd}</td>
<td>( \tilde{x}(t - \tau_1) ) (</td>
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<tr>
<td>3\textsuperscript{rd}</td>
<td>( \tilde{x}(t - \tau_1) ) ( \tilde{x}(t - \tau_2) ) ( \tilde{x}^*(t - \tau_3) )</td>
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<td>4\textsuperscript{th}</td>
<td>( \tilde{x}(t - \tau_1) ) ( \tilde{x}(t - \tau_2) ) ( \tilde{x}^*(t - \tau_3) ) (</td>
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<td>5\textsuperscript{th}</td>
<td>( \tilde{x}(t - \tau_1) ) ( \tilde{x}(t - \tau_2) ) ( \tilde{x}(t - \tau_3) ) ( \tilde{x}^<em>(t - \tau_4) ) ( \tilde{x}^</em>(t - \tau_5) )</td>
</tr>
</tbody>
</table>

Phase homogeneity is a necessary symmetry of all complex baseband models of time-invariant passband systems.
Digital Predistortion of RF-PAs

Motivation

Improve the performance of a practical wireless transmitter

Research Question

Which methods give the best results in practical DPD applications?

Methodology

- Student design competition “PA linearization through DPD”
- Remote measurement setup
- Benchmarking of DPD methods
Setup of DPD Design Competition 2017

Aim: Produce the highest output power for given linearity requirements.
Crest Factor Reduction

Crest factor reduction simplifies the linearization by digital predistortion.

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Structure of the Digital Predistorter

Dual-band vector-switched\textsuperscript{[1]} model

Low-level model structure
- Pruned generalized memory polynomial
- Pruned dual-band memory polynomial
- 72 coefficients per region and band

\textbf{Piecewise models:} higher \textbf{accuracy} by higher \textbf{locality}.

Training of the Digital Predistorter

**Indirect learning**

- Initialize with **indirect learning**, optimize with several iterations of **direct learning**.

**Indirect Learning**

- Indirect Learning
- $y_s \rightarrow x$
- $x = Y_s c_{DPD}$

**Direct Learning**

- Direct Learning
- $u \mapsto e$
- $e = U \cdot c_{error}$
- Iterative coefficient update
- $c_{DPD}^{(i+1)} = c_{DPD}^{(i)} + \mu \cdot c_{error}^{(i)}$

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Measurement Results

Performance at the competition

- ACPR: -49.2 dB
- NMSE: -35.7 dB
- Output power: 24.4 dBm
- Drain efficiency: 22.3%

First place: 71.8 points
Second place: 68.9 points
Third place: 63.2 points

(eight teams participating)

The presented methods were **successfully evaluated** against seven international competitors.
Thesis Summary

1. The linearity-efficiency trade-off
   - Joint linearity-efficiency model of RF-PAs
   - Linearity and efficiency quantification
   - Architectures for highly efficient RF-PAs

2. Behavioral modeling of RF-PAs
   - The first theoretical foundation for even-order terms in polynomial baseband models
   - Phase homogeneity of complex baseband models of time-invariant passband systems

3. Digital predistortion of RF-PAs
   - Dual-band crest factor reduction
   - Dual-band vector-switched digital predistortion
   - Training by indirect and direct learning
List of Publications

1. The Linearity-Efficiency Trade-off


2. Behavioral Modeling of RF-PAs


3. Digital Predistortion of RF-PAs


Related publications, not discussed within the thesis
