

# Behavioral Modeling and Digital Predistortion of Radio Frequency Power Amplifiers

Harald Enzinger

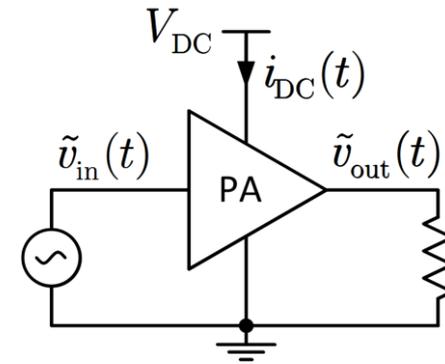
**PhD Defense**

06.03.2018

# Overview

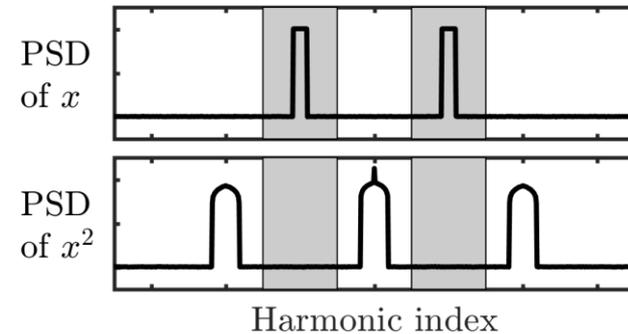
## 1. The linearity-efficiency trade-off

Joint linearity-efficiency model of radio frequency power amplifiers (RF-PAs)



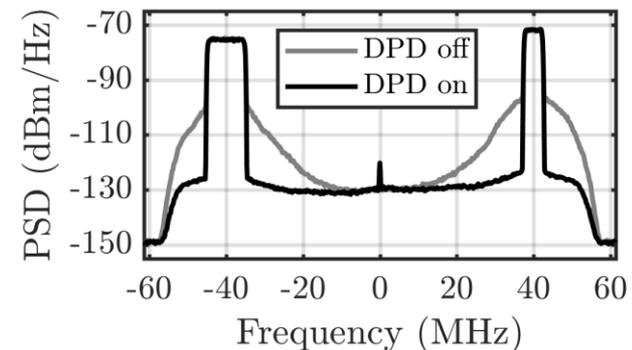
## 2. Behavioral modeling of RF-PAs

Even-order terms in polynomial baseband models



## 3. Digital predistortion of RF-PAs

Dual-band digital predistortion (DPD) of an envelope tracking power amplifier



# The Linearity-Efficiency Trade-off

## Motivation



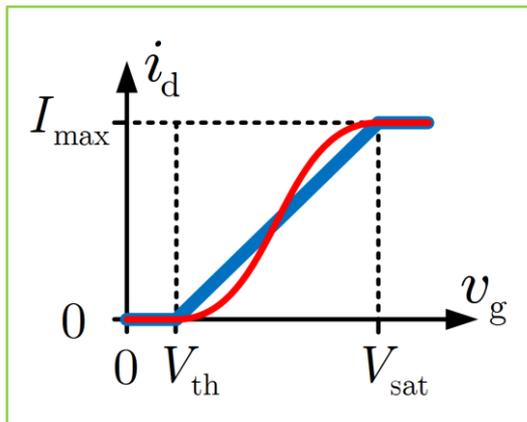
## Research Question

What are the limitations of linearity and efficiency of RF-PAs?

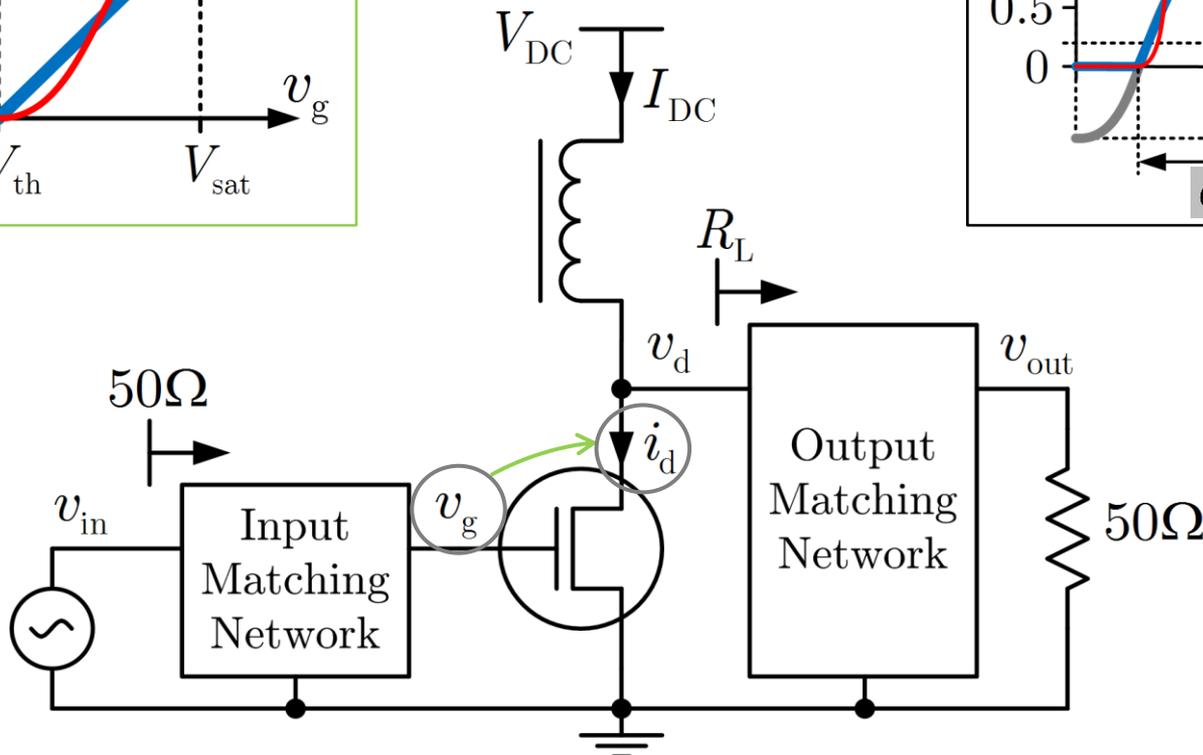
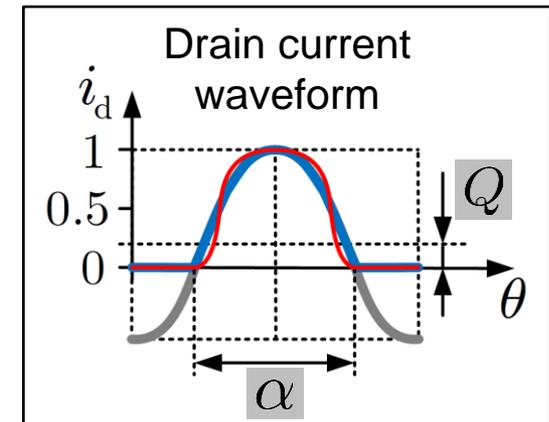
## Methodology

- Extend the classical efficiency analysis of RF-PAs
- Apply linearity and efficiency quantification methods
- Explore the linearity-efficiency trade-off by simulations

# Circuit of a typical RF-PA



— piecewise linear  
— piecewise cubic



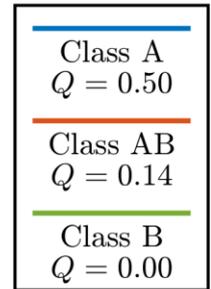
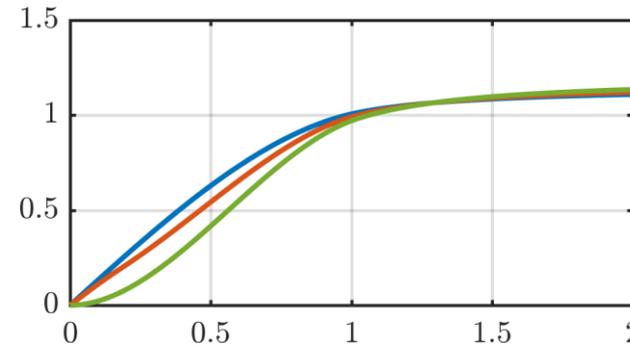
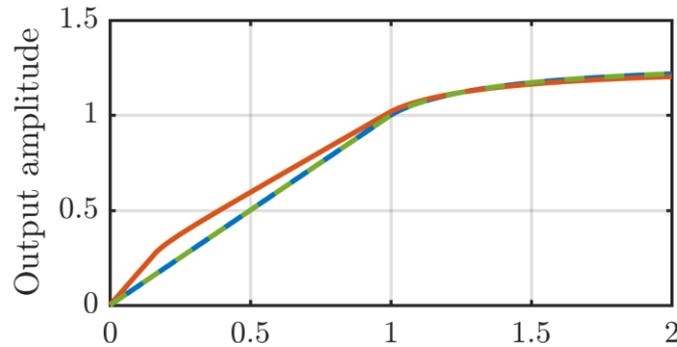
The **linearity** and **efficiency** characteristics can be derived by a **Fourier series analysis** of the **drain current waveform**.

# Linearity & Efficiency Behavior

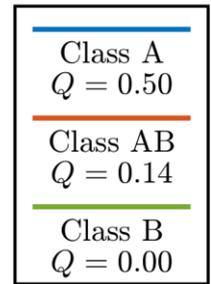
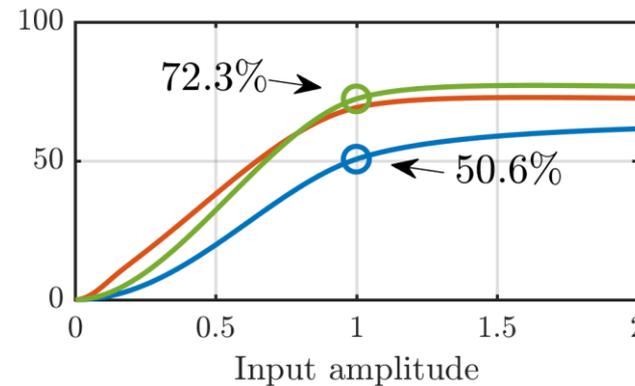
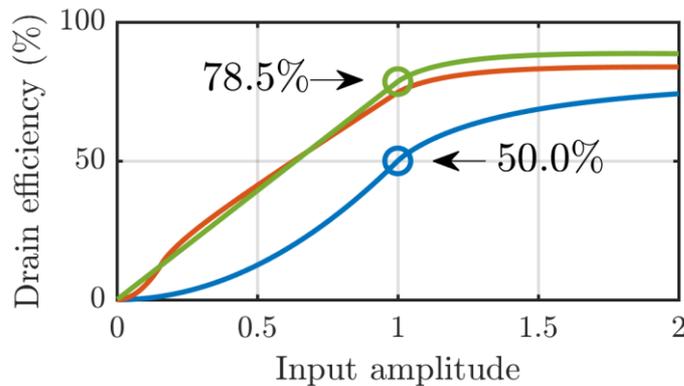
**Piecewise linear** transistor model

**Piecewise cubic** transistor model

**Linearity:** Amplitude modulation to amplitude modulation (AM-AM)

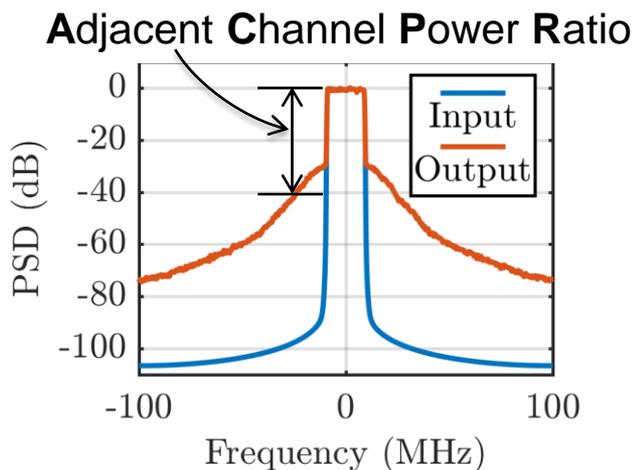


**Efficiency:** Instantaneous drain efficiency



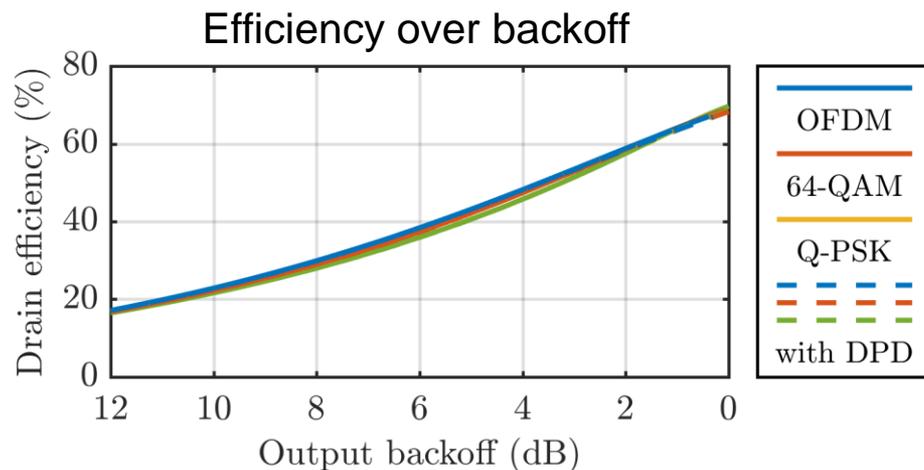
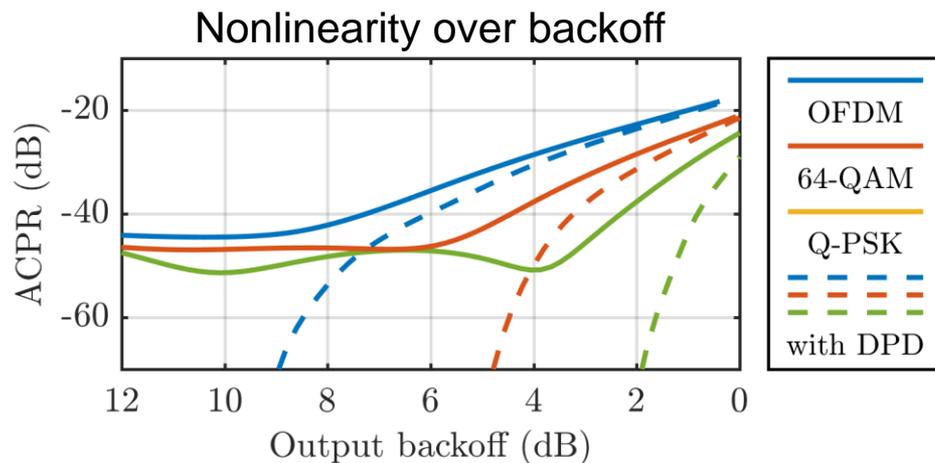
The **piecewise cubic** transistor model produces realistic **linearity** and **efficiency** characteristics.

# Linearity & Efficiency Evaluation



Average drain efficiency

$$\eta_{\text{drain,avg}} = \frac{\mathbb{E}\{P_{\text{RF}}\}}{\mathbb{E}\{P_{\text{DC}}\}}$$



**Nonlinearity metrics** strongly depend on the **signal statistics**.  
**Average drain efficiency** mainly depends on the **output power backoff**.

# Behavioral Modeling of RF-PAs

## Motivation

Conventional theory: Only **odd-order** terms in RF-PA baseband models  
Experimental evidence: **Even-order** terms can improve the accuracy

## Research Question

What are the foundations of complex baseband models of RF-PAs?

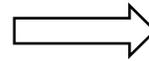
## Methodology

- Derive passband-baseband model pairs with even-order terms
- Analyze the mathematical properties of complex baseband models

# Spectral Analysis of a Polynomial Model

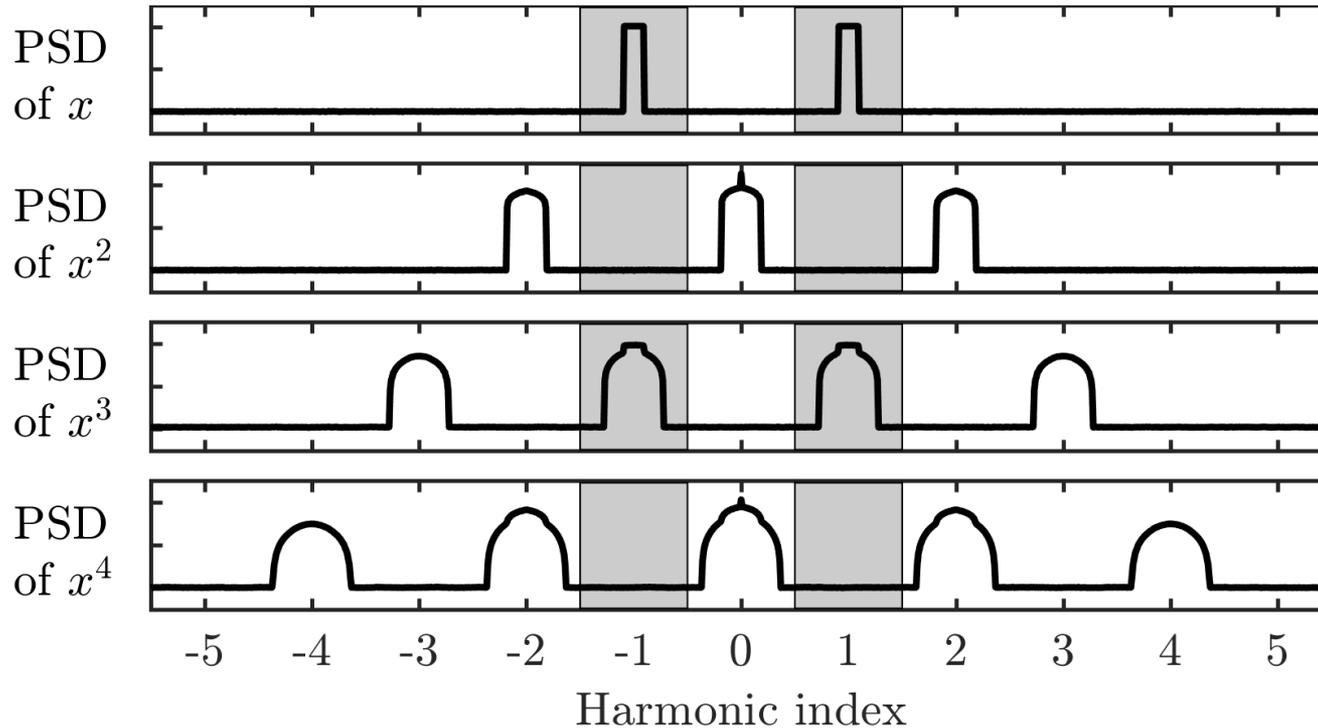
Passband signal

$$x(t) = \text{Re}\{\tilde{x}(t) e^{j\omega t}\}$$



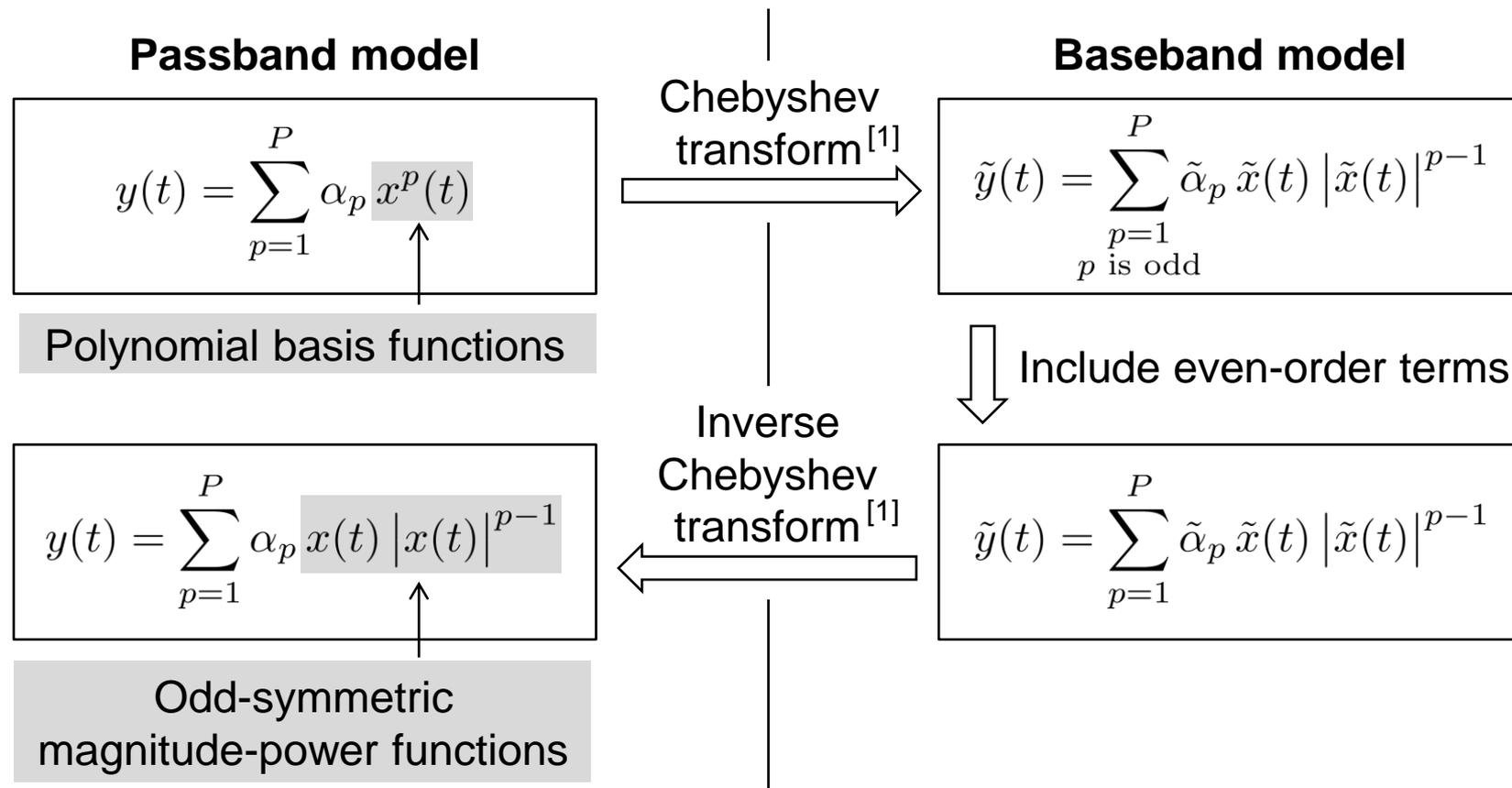
Passband model

$$y(t) = \sum_{p=1}^P \alpha_p x^p(t)$$



Only **odd-order** monomials produce output in the **first spectral zone**.

# Analysis of Even-Order Terms



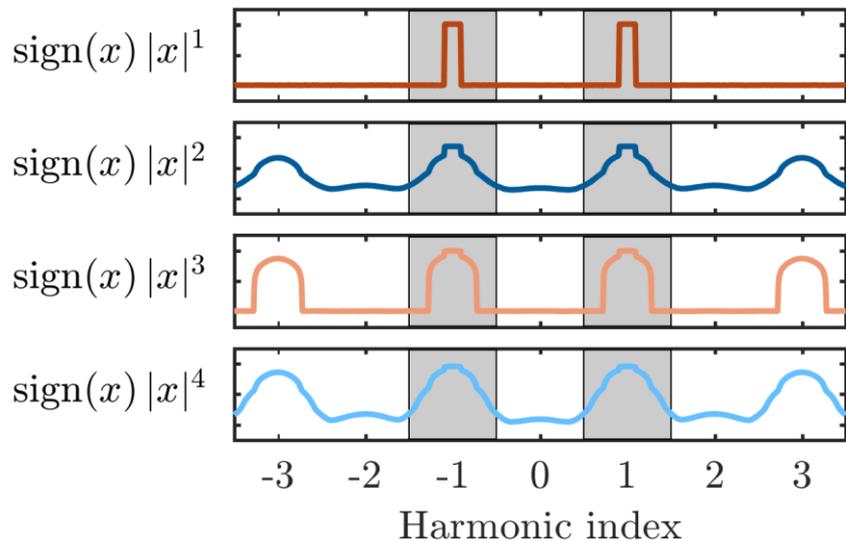
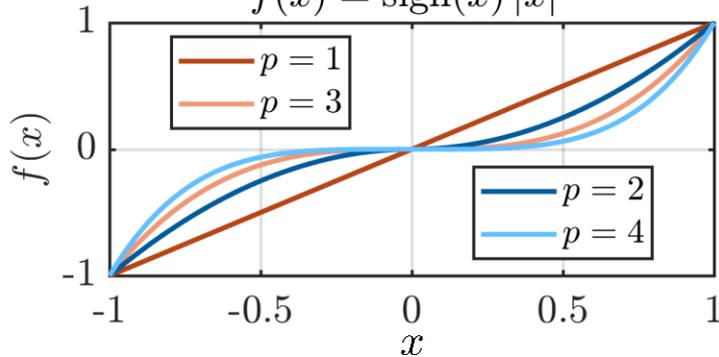
**Even-order terms in the baseband model can be explained by odd-symmetric magnitude-power functions in the passband model.**

[1] N. Blachman, "Detectors, bandpass nonlinearities, and their optimization: Inversion of the Chebyshev transform", *IEEE Transactions on Information Theory*, volume 17, number 4, pages 398–404, July 1971.

# Magnitude Power Functions

## Odd-symmetric

$$f(x) = \text{sign}(x) |x|^p$$

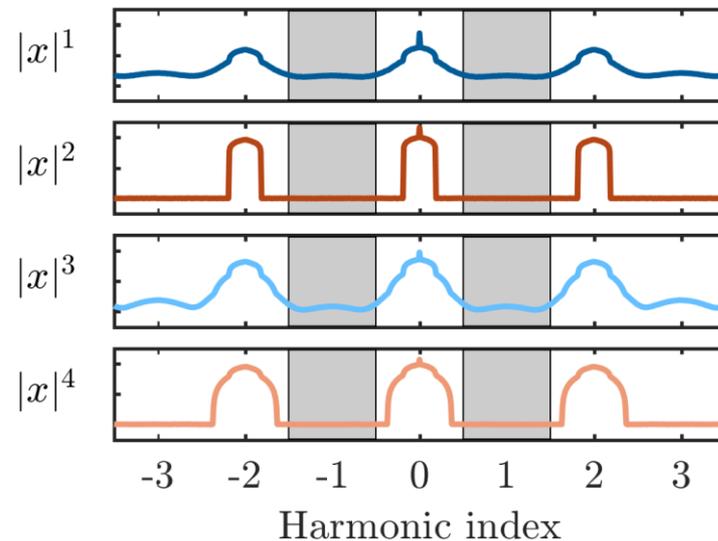
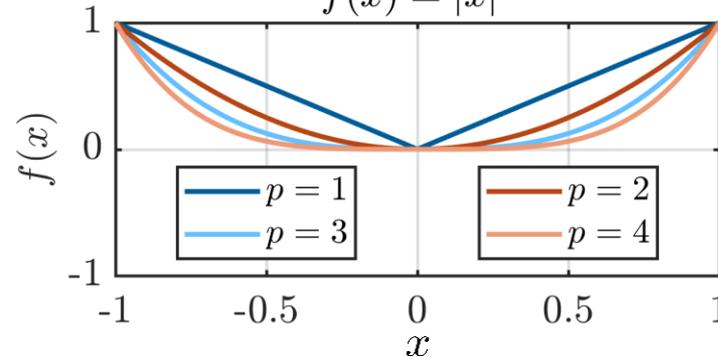


## Basis Function

## PSD of Output Signal

## Even-symmetric

$$f(x) = |x|^p$$



**Spectral characteristics** correlate with **symmetry** of basis functions, not with **order**.

# Polynomial Models with Memory

**Volterra series**

$$y(t) = \sum_{p=1}^P \int_{\mathbb{R}^p} h_p(\tau_p) \Psi_p(t, \tau_p) d\tau_p$$

**Basis functionals**

**Passband**

$$\underbrace{\prod_{i=1}^p x(t - \tau_i)}_{p\text{-fold product}}$$

Transform<sup>[1]</sup>

**Baseband**

$$q = \lfloor (p + 1)/2 \rfloor$$

$$\underbrace{\prod_{i=1}^q \tilde{x}(t - \tau_i)}_{q \text{ terms}} \underbrace{\prod_{l=q+1}^{2q-1} \tilde{x}^*(t - \tau_l)}_{(q-1) \text{ terms}}$$

$p$ -fold product,  $p$  is odd

Transform and prune

$$\underbrace{\prod_{i=1}^{p-1} x(t - \tau_i)}_{(p-1) \text{ terms}} \underbrace{|x(t - \tau_p)|}_{1 \text{ term}}$$

$p$ -fold product,  $p$  is even

$$\underbrace{\prod_{i=1}^q \tilde{x}(t - \tau_i)}_{q \text{ terms}} \underbrace{\prod_{l=q+1}^{2q-1} \tilde{x}^*(t - \tau_l)}_{(q-1) \text{ terms}} \underbrace{|x(t - \tau_p)|}_{1 \text{ term}}$$

$p$ -fold product,  $p$  is even

**Even-order terms in baseband can also be derived for Volterra series models.**

[1] S. Benedetto, E. Biglieri, and R. Daffara, "Modeling and performance evaluation of nonlinear satellite links – A Volterra series approach", *IEEE Transactions on Aerospace and Electronic Systems*, volume 15, number 4, pages 494–507, July 1979.

# Phase Homogeneity

Passband model

$$\mathcal{N} : x(t) \mapsto y(t)$$

Baseband model

$$\tilde{\mathcal{N}}_k : \tilde{x}(t) \mapsto \tilde{y}_k(t)$$

If the passband model is **time-invariant**,  
the baseband model must obey  
**phase homogeneity**:

$$\tilde{\mathcal{N}}_k \left\{ e^{j k \xi} \tilde{x}(t) \right\} = e^{j k \xi} \tilde{\mathcal{N}}_k \left\{ \tilde{x}(t) \right\}$$

e.g. baseband Volterra series (1<sup>st</sup> harmonic, k=1)

$$1^{\text{st}} \text{ Order } \tilde{x}(t - \tau_1)$$

$$2^{\text{nd}} \text{ Order } \tilde{x}(t - \tau_1) \quad |\tilde{x}(t - \tau_2)|$$

$$3^{\text{rd}} \text{ Order } \tilde{x}(t - \tau_1) \quad \tilde{x}(t - \tau_2) \quad \tilde{x}^*(t - \tau_3)$$

$$4^{\text{th}} \text{ Order } \tilde{x}(t - \tau_1) \quad \tilde{x}(t - \tau_2) \quad \tilde{x}^*(t - \tau_3) \quad |\tilde{x}(t - \tau_4)|$$

$$5^{\text{th}} \text{ Order } \tilde{x}(t - \tau_1) \quad \tilde{x}(t - \tau_2) \quad \tilde{x}(t - \tau_3) \quad \tilde{x}^*(t - \tau_4) \quad \tilde{x}^*(t - \tau_5)$$

**Phase homogeneity** is a **necessary symmetry** of all  
**complex baseband** models of **time-invariant passband** systems.

# Digital Predistortion of RF-PAs

## Motivation

Improve the performance of a practical wireless transmitter

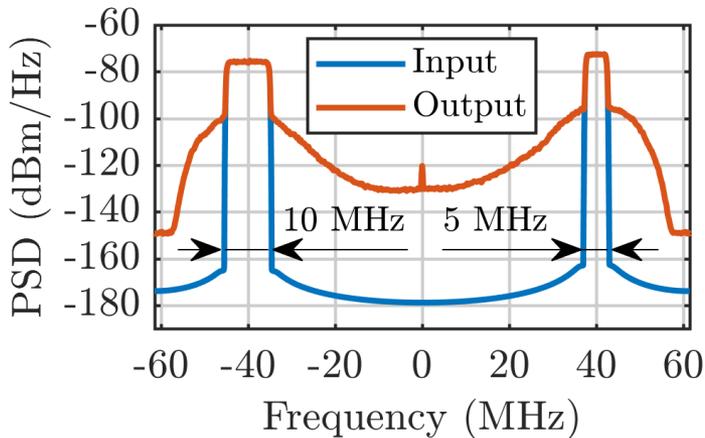
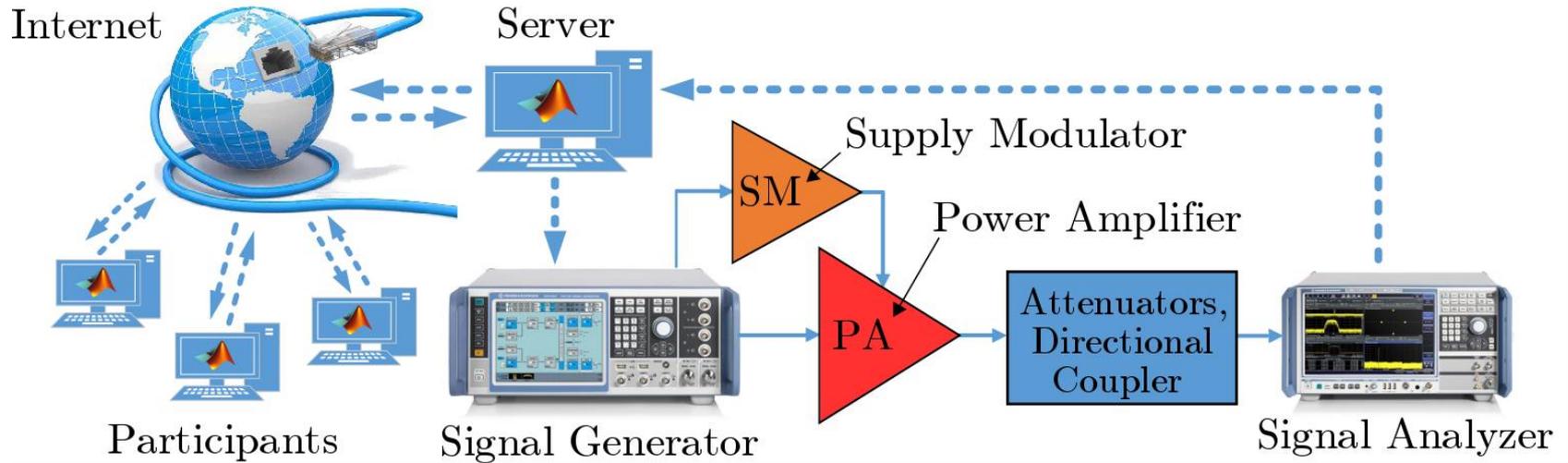
## Research Question

Which methods give the best results in practical DPD applications?

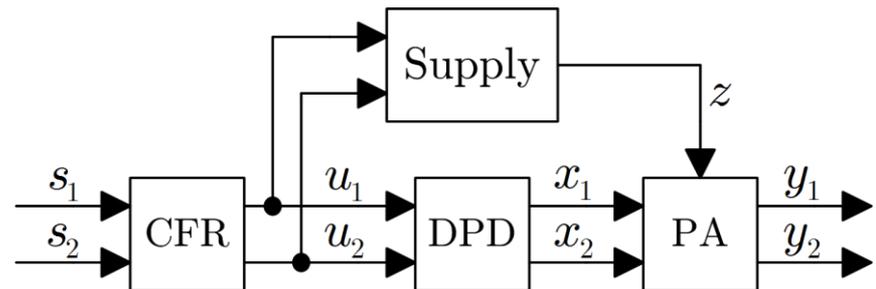
## Methodology

- Student design competition “PA linearization through DPD”
- Remote measurement setup
- Benchmarking of DPD methods

# Setup of DPD Design Competition 2017

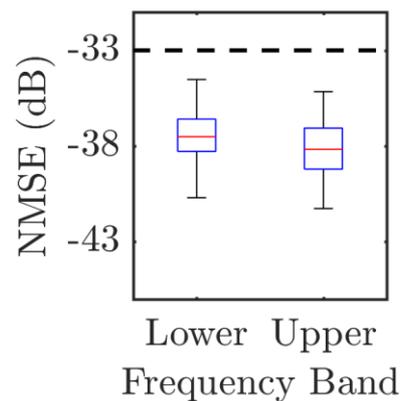
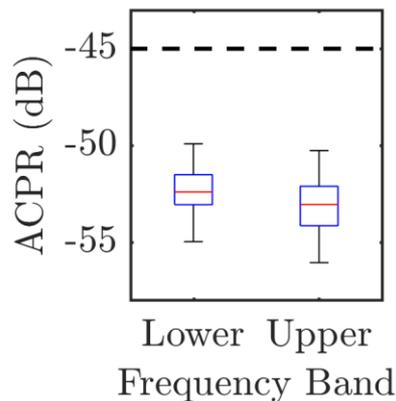
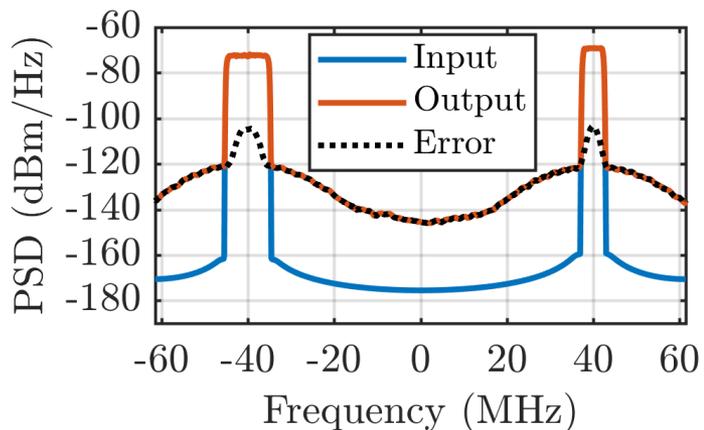
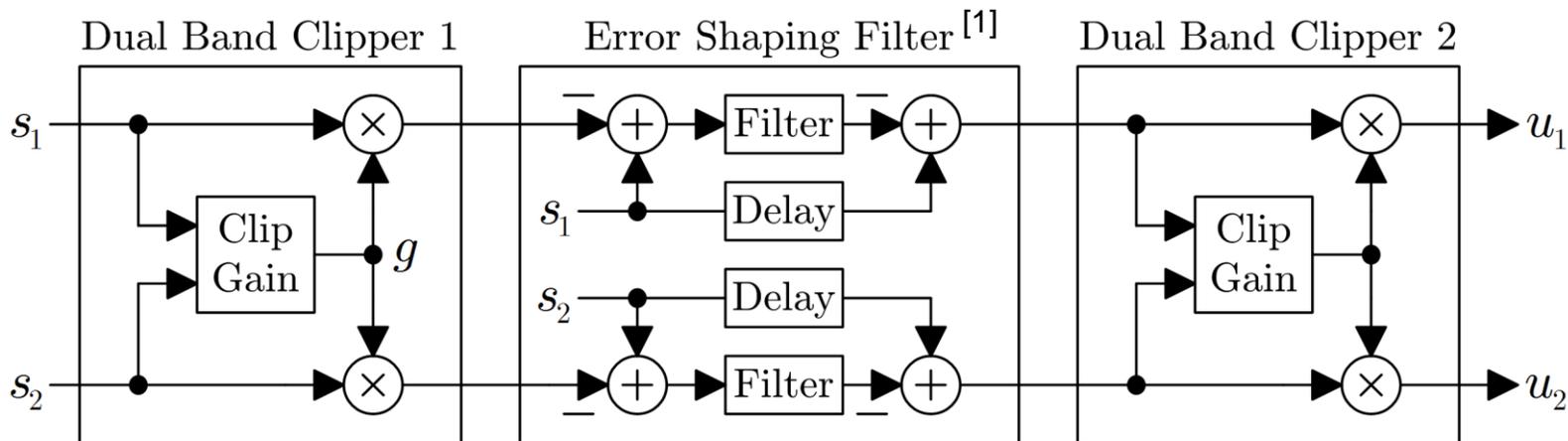


## Linearization architecture:



**Aim:** Produce the highest **output power** for given **linearity requirements**.

# Crest Factor Reduction



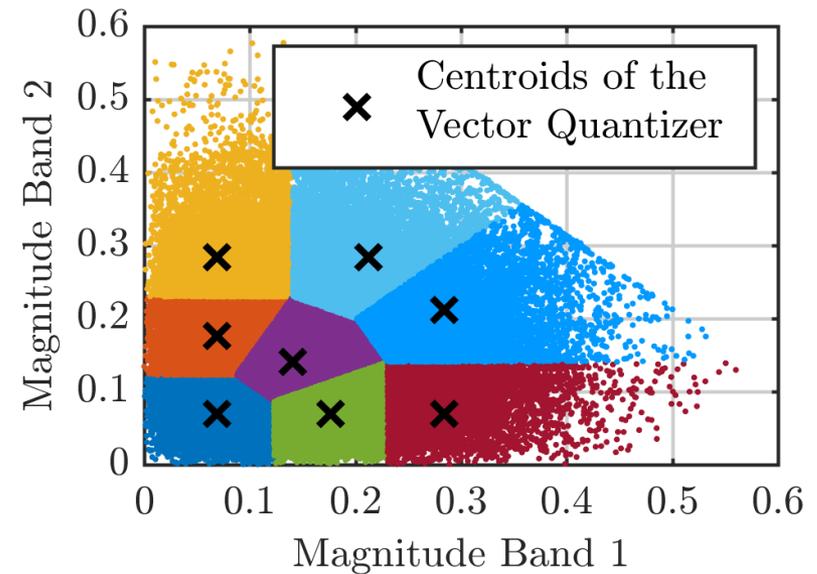
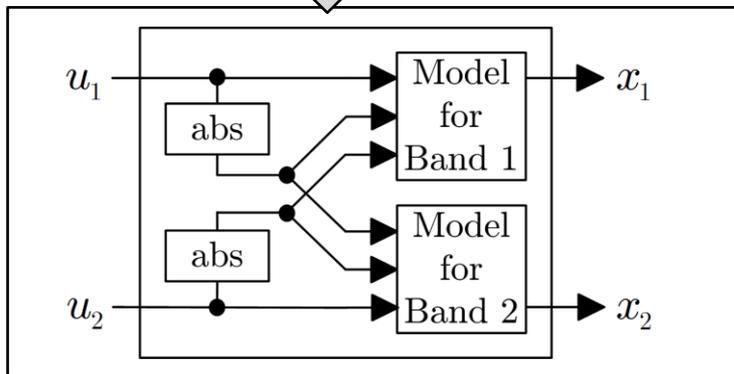
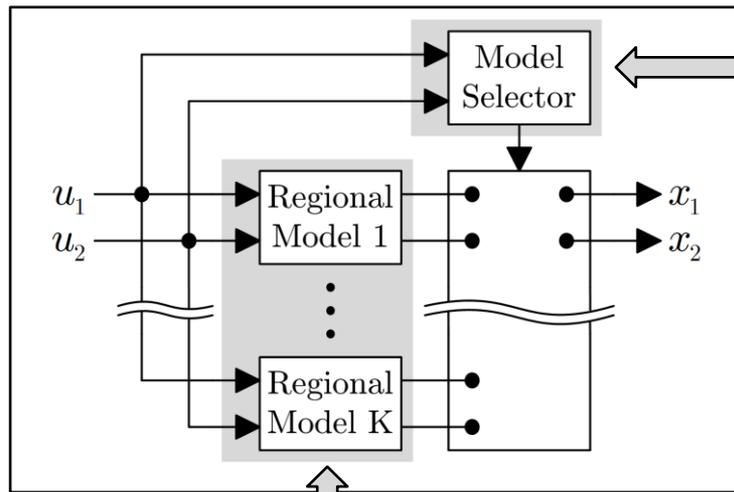
Final crest factor  
8.6 dB

**Crest factor reduction simplifies the linearization by digital predistortion.**

[1] W.-J. Kim, K.-J. Cho, S. P. Stapleton, and J.-H. Kim, "An efficient crest factor reduction technique for wideband applications", *Analog Integrated Circuits and Signal Processing*, volume 51, number 1, pages 19–26, April 2007.

# Structure of the Digital Predistorter

Dual-band vector-switched<sup>[1]</sup> model



## Low-level model structure

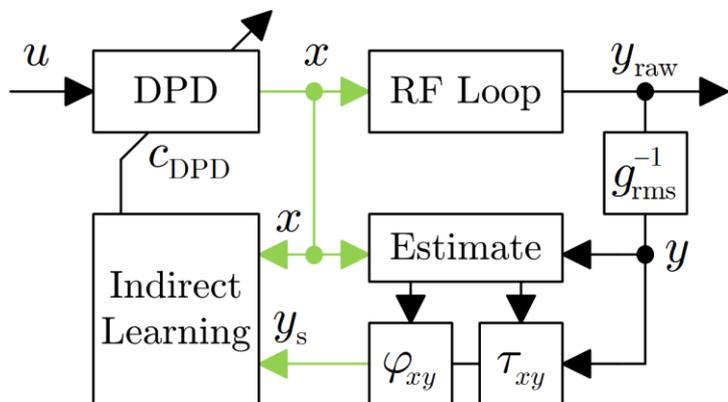
- Pruned generalized memory polynomial
- Pruned dual-band memory polynomial
- 72 coefficients per region and band

**Piecewise models: higher accuracy by higher locality.**

[1] S. Afsardoost, T. Eriksson, and C. Fager, "Digital Predistortion Using a Vector-Switched Model", *IEEE Transactions on Microwave Theory and Techniques*, volume 60, number 4, pages 1166–1174, April 2012.

# Training of the Digital Predistorter

## Indirect learning

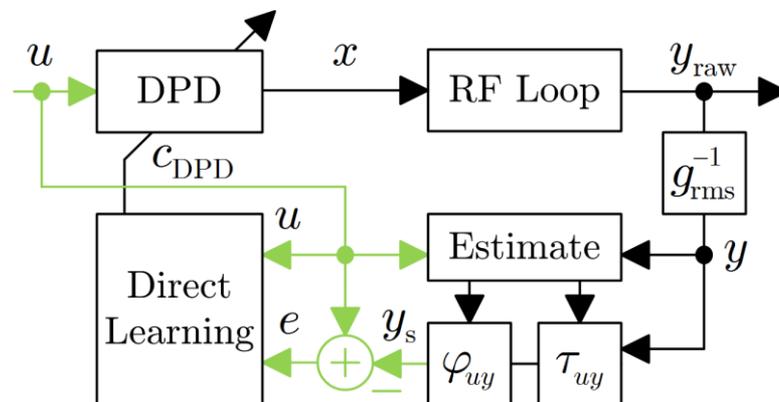


Inverse mapping  
 $y_s \mapsto x$

$$x = Y_s c_{\text{DPD}}$$

$$\begin{bmatrix} x[1] \\ \vdots \\ x[N] \end{bmatrix} = \begin{bmatrix} \mathcal{F}_1\{y_s[1]\} & \dots & \mathcal{F}_K\{y_s[1]\} \\ \vdots & \vdots & \vdots \\ \mathcal{F}_1\{y_s[N]\} & \dots & \mathcal{F}_K\{y_s[N]\} \end{bmatrix} \begin{bmatrix} c_{1,\text{DPD}} \\ \vdots \\ c_{K,\text{DPD}} \end{bmatrix}$$

## Direct learning<sup>[1]</sup>



Error mapping  
 $u \mapsto e$

$$e = U c_{\text{error}}$$

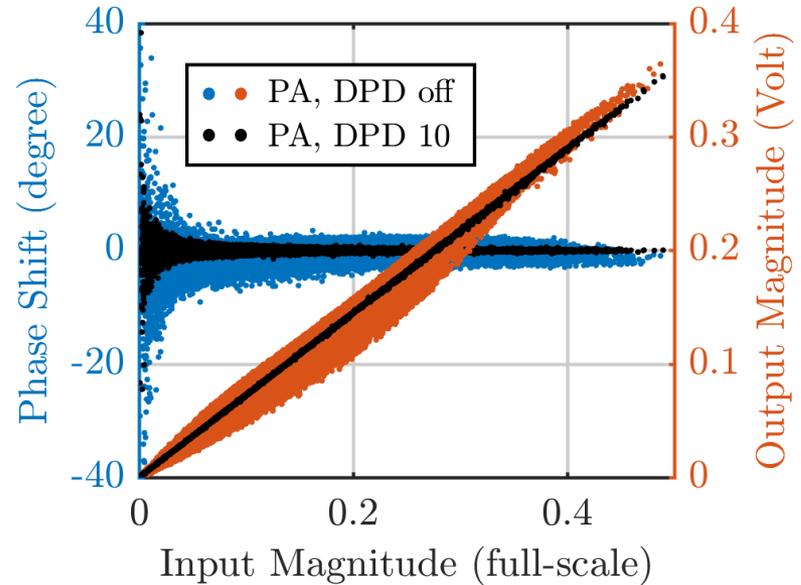
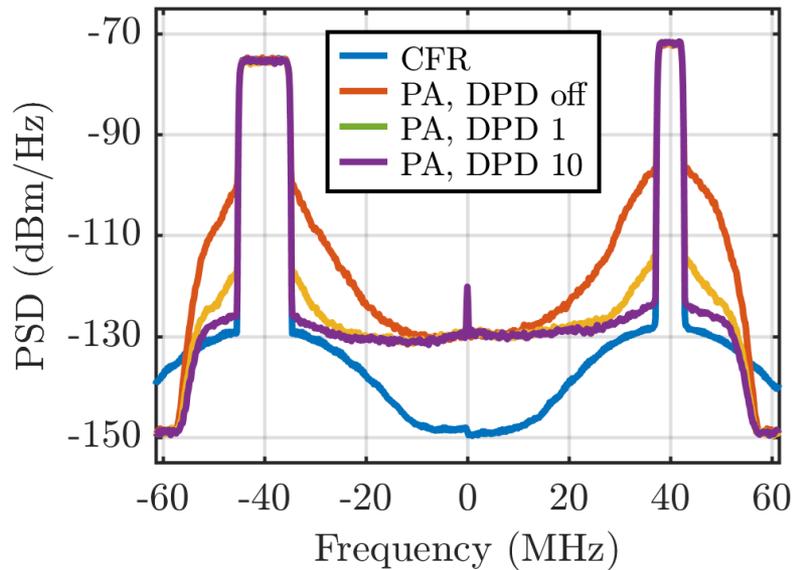
Iterative coefficient update

$$c_{\text{DPD}}^{(i+1)} = c_{\text{DPD}}^{(i)} + \mu c_{\text{error}}^{(i)}$$

Initialize with **indirect learning**, optimize with several iterations of **direct learning**.

[1] L. Guan and A. Zhu, "Dual-loop model extraction for digital predistortion of wideband RF power amplifiers", *IEEE Microwave and Wireless Components Letters*, volume 21, number 9, pages 501–503, September 2011.

# Measurement Results



## Performance at the competition

- ACPR -49.2 dB
- NMSE -35.7 dB
- Output power 24.4 dBm
- Drain efficiency 22.3 %

**First place 71.8 points**  
 Second place 68.9 points  
 Third place 63.2 points  
 (eight teams participating)

The presented methods were **successfully evaluated** against seven international competitors.

# Thesis Summary

## 1. The linearity-efficiency trade-off

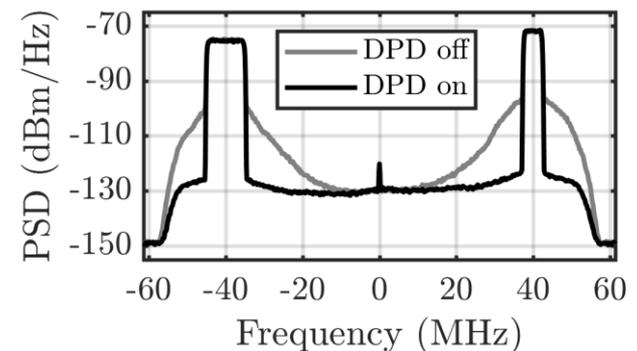
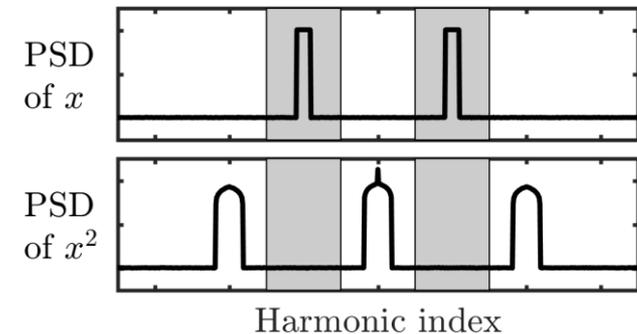
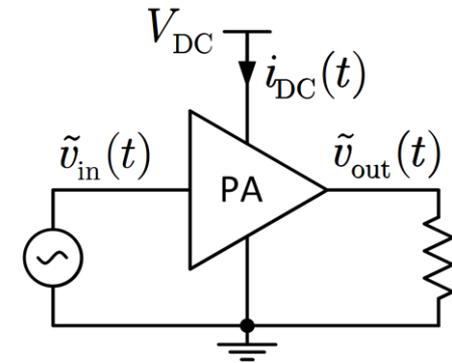
- Joint **linearity-efficiency model** of RF-PAs
- Linearity and efficiency quantification
- Architectures for highly efficient RF-PAs

## 2. Behavioral modeling of RF-PAs

- The first theoretical foundation for **even-order terms** in polynomial baseband models
- **Phase homogeneity** of complex baseband models of time-invariant passband systems

## 3. Digital predistortion of RF-PAs

- Dual-band **crest factor reduction**
- Dual-band vector-switched **digital predistortion**
- Training by **indirect** and **direct learning**



# List of Publications

## 1. The Linearity-Efficiency Trade-off

- [1] **H. Enzinger**, K. Freiberger and C. Vogel, “A joint linearity-efficiency model of radio frequency power amplifiers“, *IEEE International Symposium on Circuits and Systems*, 2016.

## 2. Behavioral Modeling of RF-PAs

- [2] **H. Enzinger**, K. Freiberger and C. Vogel, “Analysis of even-order terms in memoryless and quasi-memoryless polynomial baseband models“, *IEEE International Symposium on Circuits and Systems*, 2015.
- [3] **H. Enzinger**, K. Freiberger, G. Kubin and C. Vogel, “Baseband Volterra filters with even-order terms: Theoretical foundation and practical implications“, *Asilomar Conference on Signals, Systems, and Computers*, 2016.

## 3. Digital Predistortion of RF-PAs

- [4] **H. Enzinger**, K. Freiberger and C. Vogel, “Competitive linearity for envelope tracking: Dual-band crest factor reduction and 2D-vector-switched digital predistortion“, *IEEE Microwave Magazine*, 2018.
- [5] **H. Enzinger**, K. Freiberger, G. Kubin and C. Vogel, “A survey of delay and gain correction methods for the indirect learning of digital predistorters“, *IEEE International Conference on Electronics, Circuits, and Systems*, 2016.

## Related publications, not discussed within the thesis

- [6] **H. Enzinger** and C. Vogel, “Analytical description of multilevel carrier-based PWM of arbitrary bounded input signals“, *IEEE International Symposium on Circuits and Systems*, 2014.
- [7] **H. Enzinger**, K. Freiberger, G. Kubin and C. Vogel, “Fast time-domain Volterra filtering“, *Asilomar Conference on Signals, Systems, and Computers*, 2016.