Mobile Radio Systems – Small-Scale Channel Modeling

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Outline

- 3-1 Introduction – Mathematical models for communications channels [Molisch 6.2.2; Proakis 1-3]
- 3-2 Stochastic Modeling of Fading Multipath Channels
  - Multipath channel [Proakis 14-1]
  - Fading amplitude distribution (Rayleigh, Rice) [Molisch 5.4, 5.5]
  - Time-selective fading [Molisch 5.6]
  - Frequency-selective fading
  - WSSUS stochastic channel description [Molisch 6.3-6.5, Proakis 14]
- 3-3 Classification of Small-Scale Fading [Molisch 6.5]

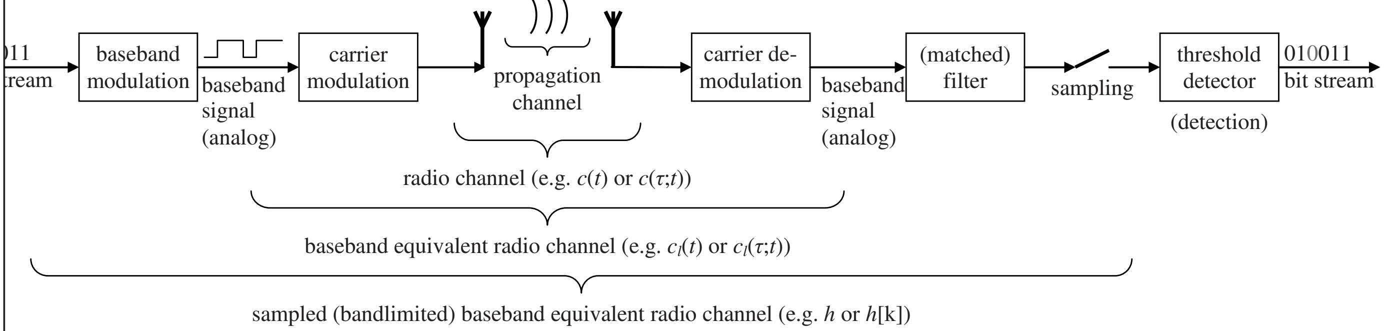
References

- A. F. Molisch: *Wireless Communications*, 2005, Wiley
- J. G. Proakis: *Digital Communications*, 3rd ed., 1995, McGraw Hill
- J. R. Barry, E. A. Lee, D. G. Messerschmitt: *Digital Communication*, 3rd ed., 2004, Kluwer
- A. Paulraj, R. Nabar, and D. Gore: *Introduction to Space-Time Wireless Communications*, 2003, Cambridge
- T. S. Rappaport: *Wireless Communications – Principles and Practice*, 2nd ed., 2002, Prentice Hall
- M. Pätzold: *Mobile Fading Channels*, 2002, Wiley

Figures (partly) extracted from these references

Signal Models

- “Signal processing” channel models can be described for different interfaces
- Application/**design objective** determines choice of appropriate model



Additive Noise Channel

Channel's frequency response is **flat** over signal bandwidth

- Simplest model – transmitted (TX) signal corrupted by additive noise

$$r(t) = \alpha s(t) + n'(t)$$

- $s(t)$ ... TX signal
  - is a **bandpass signal** $s(t) = \sqrt{2}\,\Re\{s_l(t)e^{j2\pi f_c t}\}$
- $r(t)$ ... received (RX) signal
- for (lowpass equivalent) baseband signals (i.e. complex envelopes of $s(t), r(t), n'(t)$)

$$r_l(t) = h s_l(t) + n'_l(t), \quad \text{with } h \in \mathbb{C}$$

Additive Noise Channel (cont'd)

- Noise is usually modeled as white, Gaussian (additive white Gaussian noise – AWGN)

$$\phi_{n'}(\tau) = \mathrm{E}\{n'(t)n'(t+\tau)\} = \frac{N_0}{2}\delta(\tau) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad S_{n'}(f) = \frac{N_0}{2}$$

Additive Noise Channel (cont'd)

- Sampled AWGN model (lowpass equivalent model)

$$r[k] = hs[k] + n[k] \quad (\text{all are} \in \mathbb{C})$$

- Noise characterization

$$\mathrm{E}\{n[k]n^*[l]\} = \sigma_n^2 \delta[k-l]$$

  - $n[k]$ is zero-mean circularly symmetric complex Gaussian (ZMCSCG)
    - Real and imaginary components are i.i.d. (independent, identically distributed)
    - σ_n^2 depends on (matched) filter at receiver front-end
    - Real and imaginary components have $\sigma_n^2/2$

Linear filter channel

Channel's frequency response is **frequency-selective** (i.e. non-flat), leading to (linear) signal distortions

- For time-**in**variant channels

$$\begin{aligned} r(t) &= s(t) * c(t) + n(t) \\ &= \int_{-\infty}^{\infty} c(\tau)s(t-\tau)d\tau + n(t) \end{aligned}$$

- $c(t)$ ... impulse response of linear filter

Linear filter channel (cont'd)

- Sampled case (lowpass equivalent model)

$$r[k] = \sum_{l=0}^{L-1} h[l]s[k-l] + n[k]$$

  - $h[k]$ incorporates
    - TX pulse shape
    - RX (matched) filter; ADC filter
    - (thus bandwidth corresponds to signal bandwidth)
    - physical channel
  - $n[k]$ ... AWGN (ZMCSCG)
- This is actually an equivalent, whitened matched filter (WMF) channel model [Barry/Lee/Messerschmitt]

Linear time-variant filter channel

- Characterized by **time-variant channel impulse response (CIR)** $c(\tau;t)$
  - response of channel at time t
  - to an impulse transmitted at time $t-\tau$
- τ ... "elapsed time", "age" variable

$$r(t) = s(t) * c(\tau;t) + n(t)$$

$$= \int_{-\infty}^{\infty} c(\tau;t)s(t-\tau)d\tau + n(t)$$

- model for multipath propagation

$$c(\tau;t) = \sum_{i=0}^{\infty} \alpha_i(t)\delta(\tau-\tau_i(t)) \quad (1)$$

Linear time-variant filter ch.

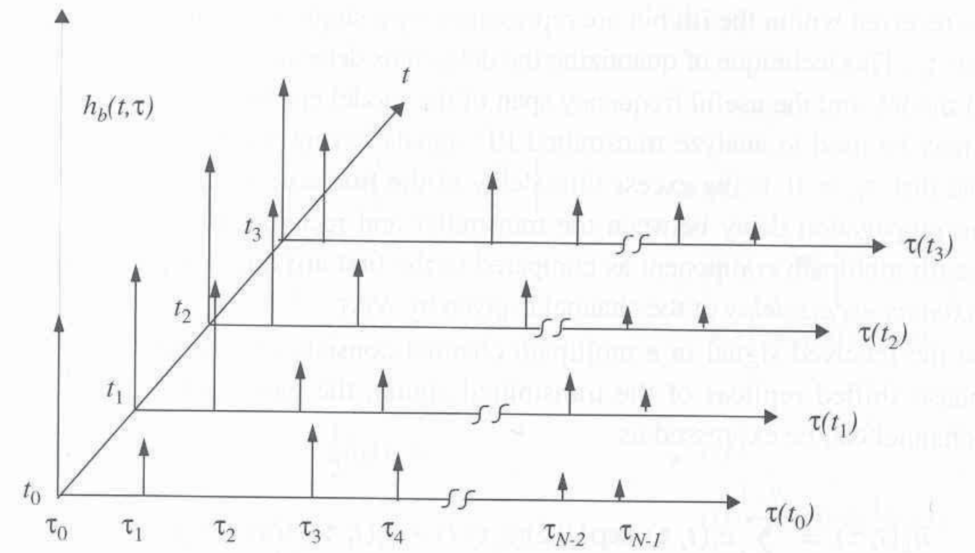


Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

[fig 5-4 Rap]

Stochastic modeling of fading multipath channels

- Motivated by their **randomly time-variant** nature and **large number** of multipath components
- Derivation of **lowpass equivalent** CIR from (1)

$$c_l(\tau;t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{-j2\pi f_c \tau_i(t)} \delta(\tau - \tau_i(t))$$
$$= \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)} \delta(\tau - \tau_i(t)) \quad (2)$$

considering discrete multipath components

- phase term $\varphi_i(t) = -2\pi f_c \tau_i(t)$ **varies** dramatically

Fading of an unmodulated carrier

- TX signal is unmodulated carrier (CW) $s_l(t) = 1$
- RX signal w/o noise: $y_l(t) = c_l(\tau; t) * 1 = c_l(t) \cdot 1$

$$c_l(t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)} = \sum_{i=0}^{\infty} \alpha_i(t) e^{-j2\pi f_c \tau_i(t)}$$

sum of vectors (phasors)
- amplitudes $\alpha_i(t)$ change slowly
- phases $\varphi_i(t)$ change by 2π if:
  - $\tau_i(t)$ changes by $1/f_c$
  - **i.e.:** path length changes by wavelength λ
- large number of multipath components

→ **model $c_l(t)$ as a random process!**

Fading of an unmodulated carrier (cont'd)

Modeling $c_l(t)$ as a random process:

- large number of multipath components are added
- by central limit theorem (CLT):
  - $c_l(t)$ is complex Gaussian
  - (CIR $c_l(\tau; t)$ is complex Gaussian)
- $c_l(t)$ has random phase and amplitude
- in **absence** of dominant component:
  $c_l(t)$ is zero-mean complex Gaussian

→ its envelope $|c_l(t)|$ is Rayleigh distributed

- **Rayleigh fading channel**

Fading of an unmodulated carrier (cont'd)

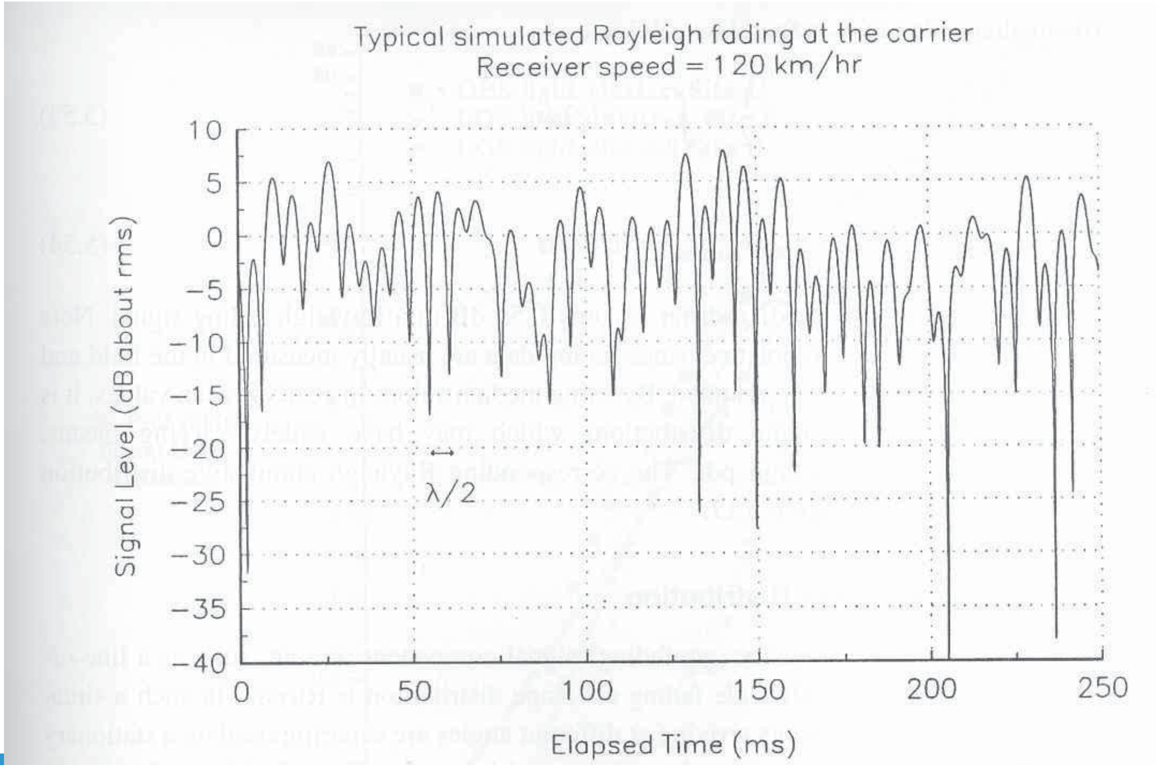


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

Fading of an unmodulated carrier (cont'd)

- Rayleigh distribution:

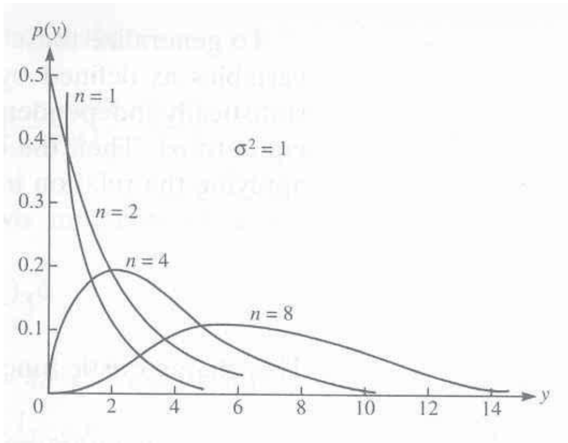
$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)} \quad \text{for } r \geq 0$$

  - characterized by σ^2: variance of **underlying Gaussian processes** $X_1, X_2 \sim \mathcal{N}(0, \sigma^2)$, where $X_1 = \Re\{c_l(t)\}$ and $X_2 = \Im\{c_l(t)\}$

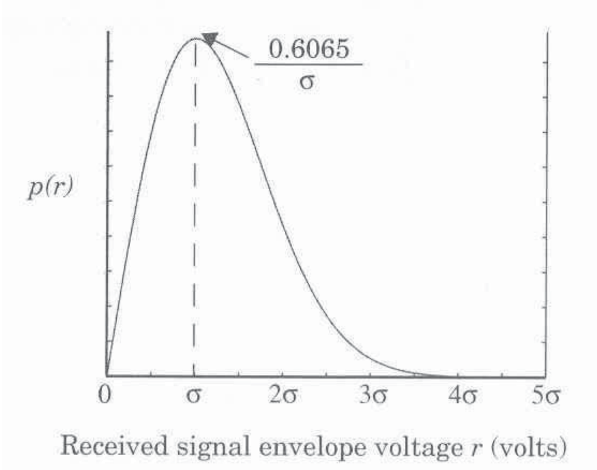
derivation of Rayleigh distribution ...

- $Y = X_1^2 + X_2^2$ ... has χ^2-PDF of 2 degrees of freedom
- $R = \sqrt{X_1^2 + X_2^2}$ ... amplitude $|c_l(t)|$ has Rayleigh PDF

Fading of an unmodulated carrier (cont'd)



central chi-square PDF
of n degrees of freedom



Rayleigh PDF
characterized by σ

Fading of an unmodulated carrier (cont'd)

- in **presence** of a dominant component:
  $c_l(t)$ is non-zero-mean complex Gaussian

$\rightarrow$ its envelope $|c_l(t)|$ is Ricean distributed

- **Ricean fading channel**

- Ricean distribution:

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+s^2}{2\sigma^2}} I_0\left(\frac{rs}{\sigma^2}\right) \qquad \text{for } r \geq 0$$

  $I_0(x)$ ... zero-order modified Bessel function of first kind

- characterized by
  σ^2 ... variance of underlying Gaussian processes and
  $s^2 = m_1^2 + m_2^2$ ... power of mean (i.e. $s^2 = |\mathrm{E}\{c_l(t)\}|^2$)

Fading of an unmodulated carrier (cont'd)

- Shape of Ricean distribution defined by

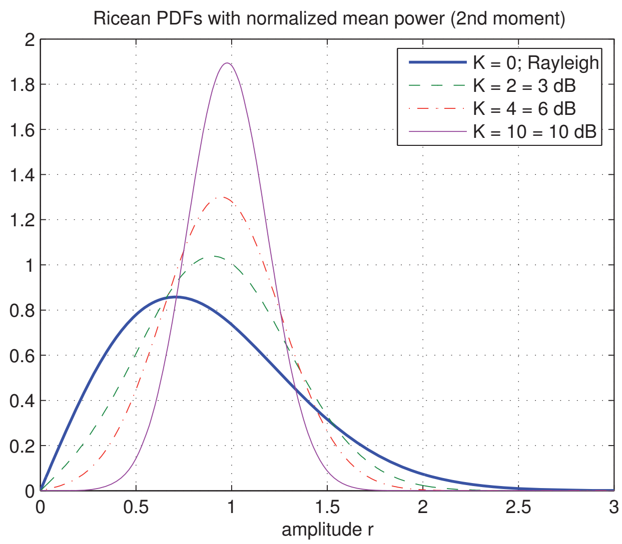
$$K = \frac{s^2}{2\sigma^2}$$

$$K\ [dB] = 10 \log \frac{s^2}{2\sigma^2}$$

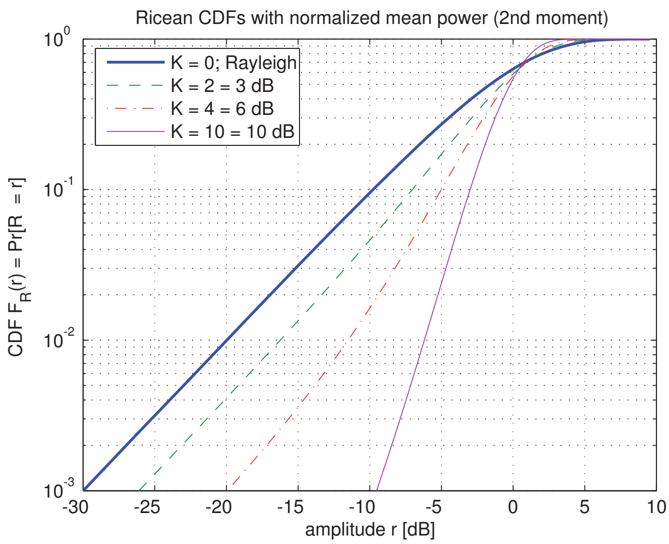
Ricean K-factor

- Ratio of deterministic signal power (mean) and variance of multipath (scattered components)
- For $K = 0 = -\infty$ dB:
  Ricean distribution equivalent to Rayleigh

Fading of an unmodulated carrier (cont'd)



Ricean (and Rayleigh) PDFs



Ricean (and Rayleigh) CDFs

Time-selective fading

- Characterization of the time variability

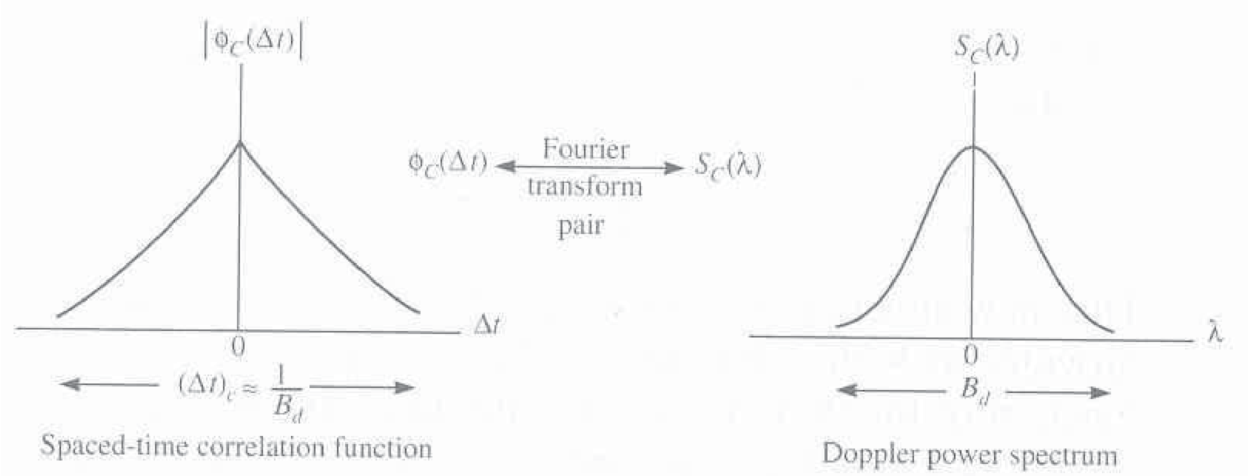
$$s_l(t) = 1 \quad \rightarrow \quad y_l(t) = c_l(\tau; t) * 1 = c_l(t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)}$$

- Characterize autocorrelation function of $c_l(t)$
  - assume $c_l(t)$ is complex Gaussian
  - assume $c_l(t)$ is wide-sense stationary (WSS)
- Define: **spaced-time correlation function**

$$\phi_c(\Delta t) = \mathrm{E}\{c_l^*(t) c_l(t + \Delta t)\} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad S_c(\nu)$$

- **Doppler power spectrum** $S_c(\nu) = \int_{-\infty}^{\infty} \phi_c(\Delta t) e^{-j2\pi\nu\Delta t} d\Delta t$

Time-selective fading (cont'd)



Relationship between $\phi_C(\Delta t)$ and $S_C(\lambda)$.

Doppler power spectrum: **average power output** of channel **as a function of Doppler frequency**

Time-selective fading (cont'd)

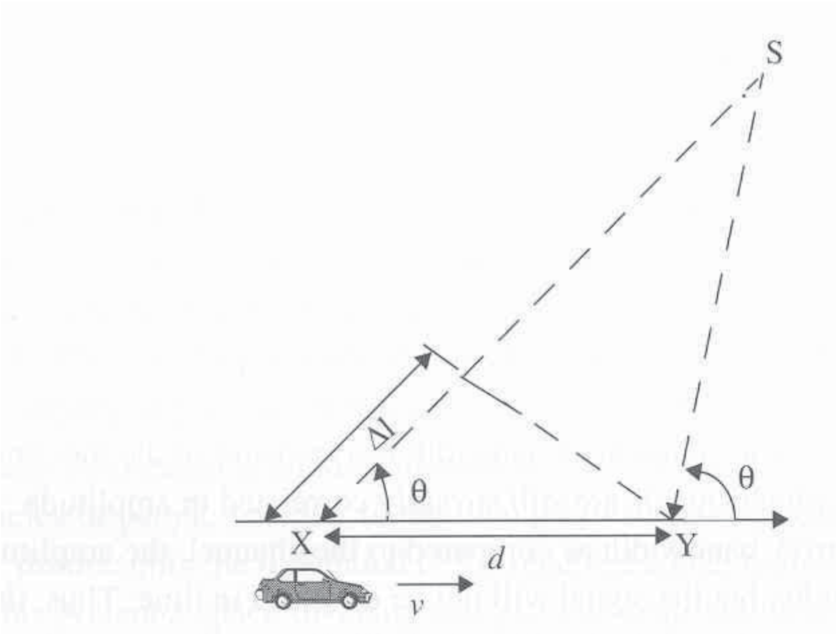


Illustration of Doppler effect.

[Rappaport fig. 5-1]

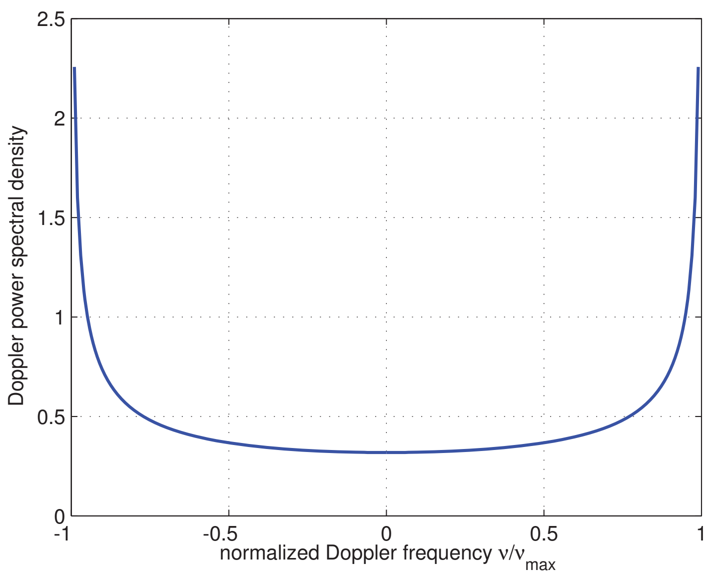
Time-selective fading (cont'd)

Jakes model for Doppler power spectrum

- assumes mobile moving at const. velocity v
- uniformly distributed scattering **around** mobile
- Jakes Doppler spectrum:

$$S_c(\nu) = \frac{1}{\pi} \frac{1}{\sqrt{\nu_{max}^2 - \nu^2}}$$

(for normalized power)



Time-selective fading (cont'd)

Characterization of **time-selective fading** by parameters

- RMS Doppler spread

$$\nu_{rms} = \sqrt{\overline{\nu^2} - \overline{\nu}^2}$$

  second centralized moment of normalized Doppler PSD

- mean and mean squared Doppler spread

$$\overline{\nu} = \frac{\int \nu S_c(\nu) d\nu}{\int S_c(\nu) d\nu} \qquad \overline{\nu^2} = \frac{\int \nu^2 S_c(\nu) d\nu}{\int S_c(\nu) d\nu}$$

- Coherence time

$$T_c \approx \frac{1}{\nu_{rms}}$$

Frequency-selective fading

f a **(time-invariant)** multipath channel

- Characterization of the time dispersion: CIR $c_l(\tau)$

$$s_l(t) = \delta(t) \quad \rightarrow \quad y_l(t) = c_l(\tau; t) * \delta(t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)} \delta(t - \tau_i(t))$$

$$c_l(\tau) = \sum_{i=0}^{\infty} \alpha_i e^{j\varphi_i} \delta(\tau - \tau_i)$$

- ACF: **Uncorrelated scattering** assumption:

$$\mathrm{E}\{c_l^*(\tau_1) c_l(\tau_2)\} = S_c(\tau_1)\delta(\tau_1 - \tau_2)$$

$S_c(\tau)$ ... **multipath intensity profile** (= delay power spectrum; = average power delay profile)

Frequency-selective fading (cont'd)

- Time-dispersion implies frequency-selectivity
  - Equivalent channel characterization by **channel transfer function** (TF) $C_l(f)$

$$c_l(\tau) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad C_l(f) = \int_{-\infty}^{\infty} c_l(\tau) e^{-j2\pi f \tau} d\tau$$

- ACF of channel TF

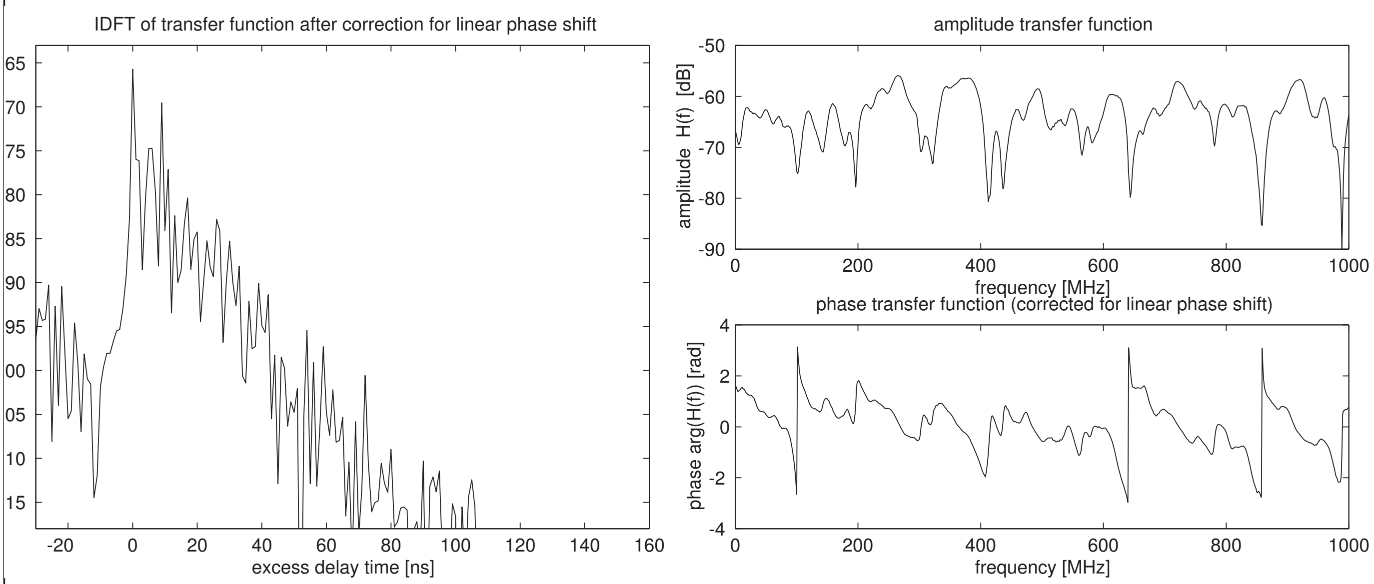
$$S_c(\tau) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \phi_C(\Delta f) = \mathrm{E}\{C_l^*(f) C_l(f + \Delta f)\}$$

$\phi_C(\Delta f)$ ... spaced-frequency correlation function

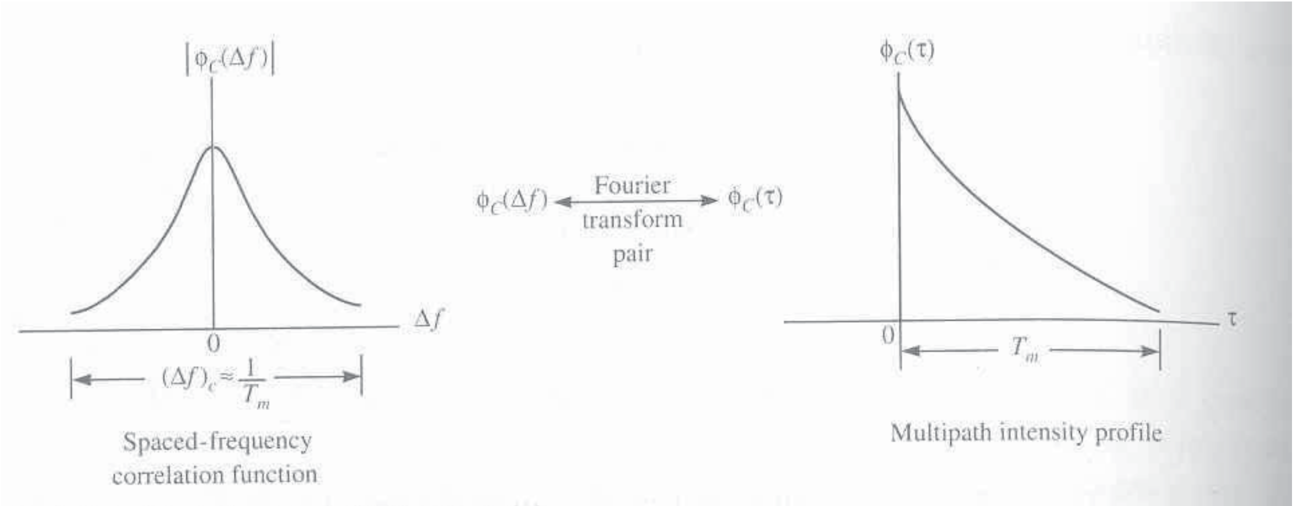
  - TF $C_l(f)$ is **wide-sense stationary** (WSS in f) if CIR $c_l(\tau)$ fulfills "uncorrelated scattering" (US in τ)

Frequency-selective fading (cont'd)

- Channel IR vs. channel frequency response



Frequency-selective fading (cont'd)



Relationship between $\phi_C(\Delta f)$ and $\phi_c(\tau)$.

Multipath intensity profile: average power output of channel as a function of delay

Frequency-selective fading (cont'd)

Characterization by parameters

- RMS delay spread

$$\tau_{rms} = \sqrt{\overline{\tau^2} - \overline{\tau}^2}$$

second centralized moment of normalized multipath intensity profile

- mean and mean squared delay spread

$$\overline{\tau} = \frac{\int \tau S_c(\tau) d\tau}{\int S_c(\tau) d\tau} \qquad \overline{\tau^2} = \frac{\int \tau^2 S_c(\tau) d\tau}{\int S_c(\tau) d\tau}$$

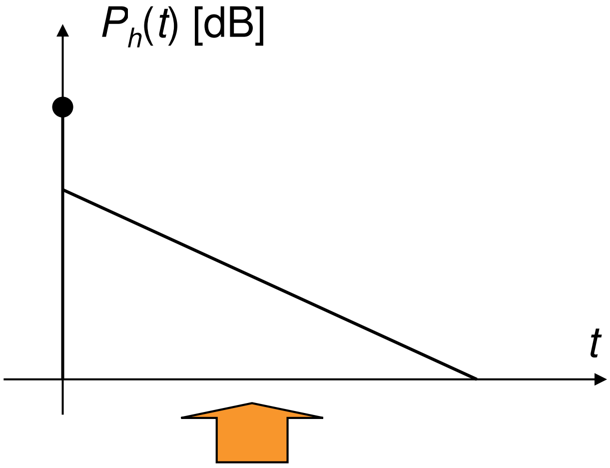
- Coherence bandwidth

$$B_c \approx \frac{1}{\tau_{rms}}$$

Frequency-selective fading (cont'd)

Characterization of **multipath intensity profile** (simplified; suitable for indoor channels)

- Exponentially decaying part
- Line-of-sight (LOS) component
- Defined by **channel parameters**



Channel parameters:
total power P_0
K-factor (rel. strength of LOS)
RMS delay spread (duration)

The WSSUS channel

- **joint modeling** of
  - time dispersion (= frequency selectivity)
  - and time variability (= Doppler spread)
- Define: ACF of time-variant CIR $c_l(\tau; t)$

$$\mathrm{E}\{c_l^*(\tau_1; t) c_l(\tau_2; t + \Delta t)\} = \phi_c(\tau_1; \Delta t)\delta(\tau_1 - \tau_2)$$

  assumes:
  - time-variations are **wide-sense stationary** (WSS)
  - attenuation and phase shifts are independent at τ_1 and τ_2: **uncorrelated scattering** (US)
- for $\Delta t = 0$: $\phi_c(\tau; \Delta t) = S_c(\tau)$ multpath intensity profile
- $\phi_c(\tau; \Delta t)$ ... lagged-time correlation function

The WSSUS channel (cont'd)

An equivalent representation of the t-var. CIR $c_l(\tau;t)$:

- **Time-variant channel transfer function** (TF) $C_l(f;t)$

$$c_l(\tau;t) \quad \overset{\mathcal{F}_\tau}{\longleftrightarrow} \quad C_l(f;t) = \int_{-\infty}^{\infty} c_l(\tau;t)e^{-j2\pi f\tau}d\tau$$

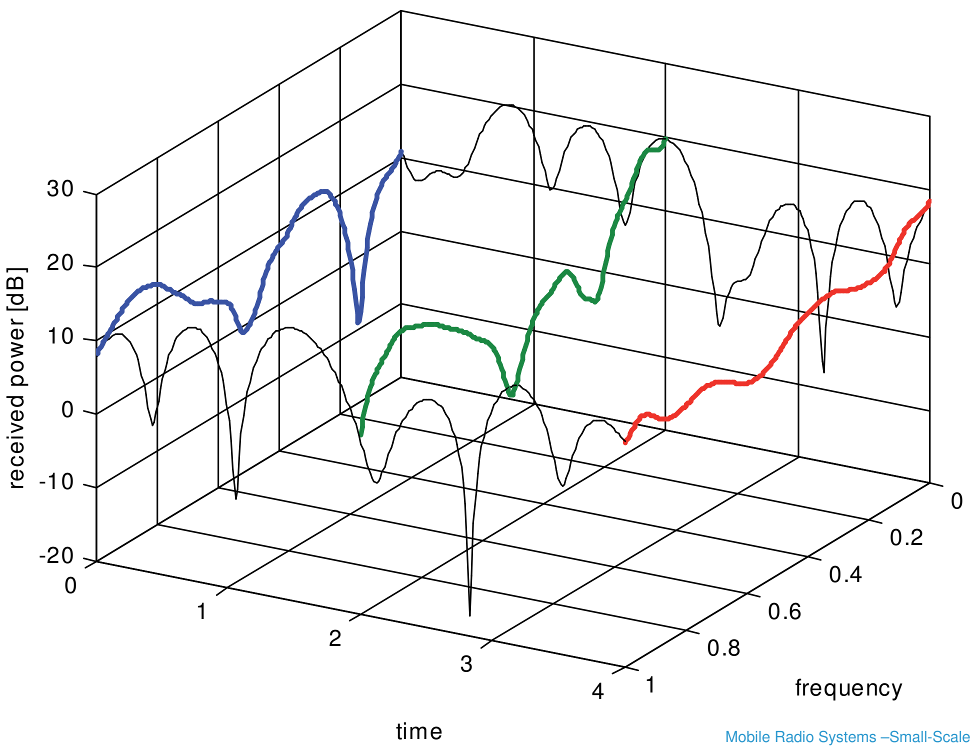
  - from US property follows WSS in f-domain
- equivalent characterization (ACF)

$$\phi_C(\Delta f;\Delta t) = \mathrm{E}\{C_l^*(f;t)C_l(f+\Delta f;t+\Delta t)\}$$

  spaced-frequency spaced-time correlation function (WSSWSS!)

The WSSUS channel (cont'd)

- time- and frequency-selective transfer function

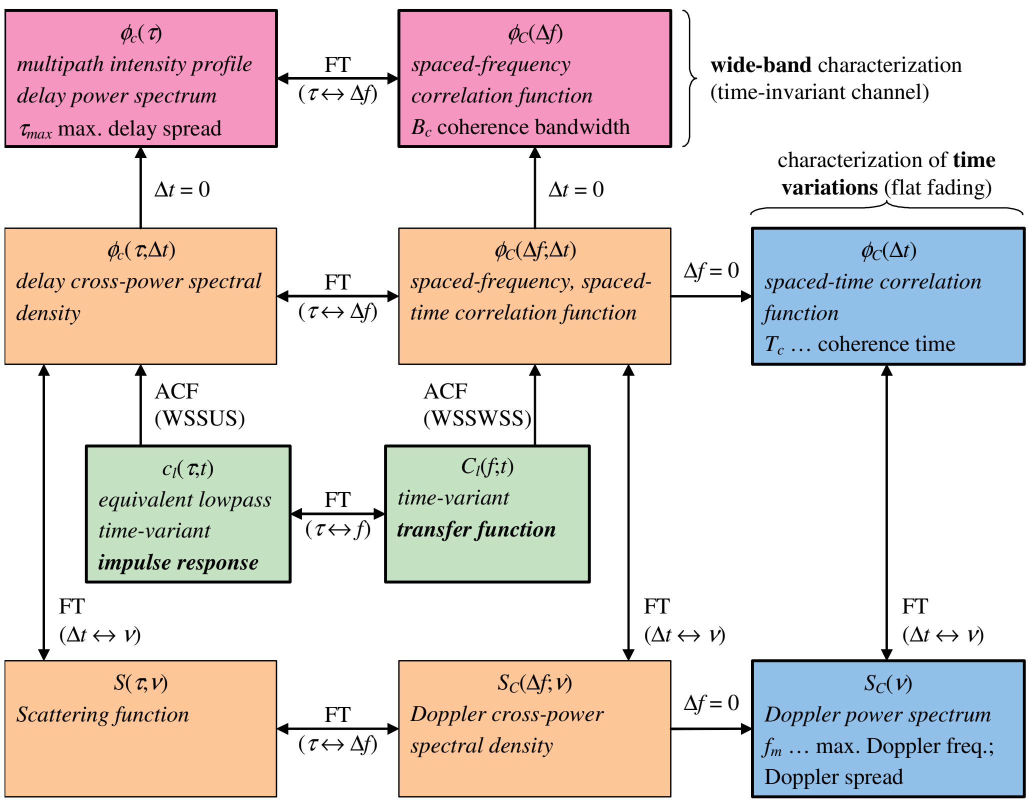


The WSSUS channel (cont'd)

- Equivalent **representations**: time-variant system functions – Bello functions [Bello63]
  - $c_l(\tau;t)$ and $C_l(f;t)$ **and two more** Fourier transformed functions w.r.t. $t \leftrightarrow \nu$ and $f \leftrightarrow \tau$
- Equivalent (2-nd order) **characterizations**: correlation functions of Bello functions
  - $\phi_c(\tau;\Delta t)$ and $\phi_C(\Delta f;\Delta t)$ **and two more** Fourier transformed functions w.r.t. $\Delta t \leftrightarrow \nu$ and $\Delta f \leftrightarrow \tau$

overview shown on next slide

The WSSUS channel (cont'd)



The WSSUS channel (cont'd)

- Doppler-delay scattering function

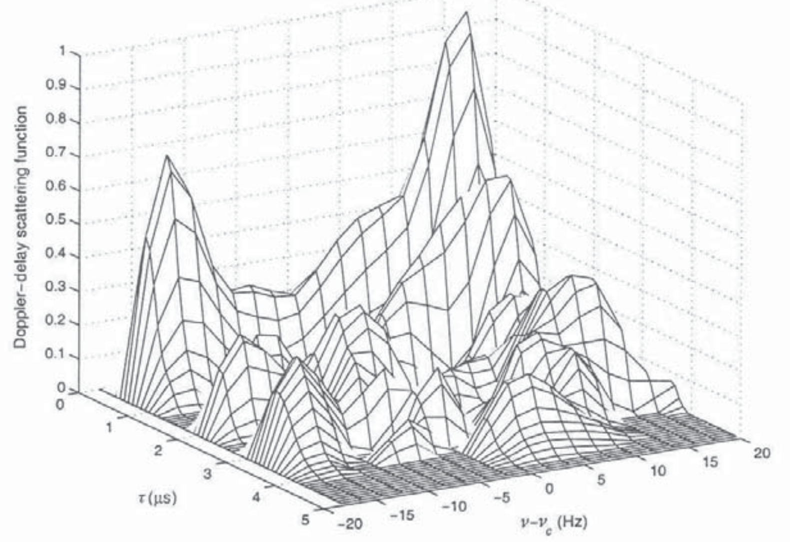


Figure 2.9: The Doppler-delay scattering function represents the average power in the Doppler-delay dimensions.

[Paulraj; fig 2-9]

Channel as a space-time random field

- **Homogenous (HO) channel** is (locally) stationary in space
  - characterization:

$$\mathrm{E}\{c_l^*(\tau;t;\mathbf{d})c_l(\tau;t;\mathbf{d}+\Delta\mathbf{d})\} = \phi_{\mathbf{d}}(\tau;t;\Delta\mathbf{d})$$

  - agrees with discrete scattering model: each scatterer has discrete ToA τ_i and AoA θ_i
- space-angle transform: assume $\mathbf{d}$ lies on x-axis; parameterized by x (and dropping t)

$$c_l(\tau;x) = \int_{-\infty}^{\infty} c_l(\tau;\theta)e^{-j2\pi\sin(\theta)\frac{x}{\lambda}}d\theta$$

- we may define the **angle-delay scattering function** $S_c(\tau;\theta)$

Channel as a space-time random field (cont'd)

- Angle-delay scattering function

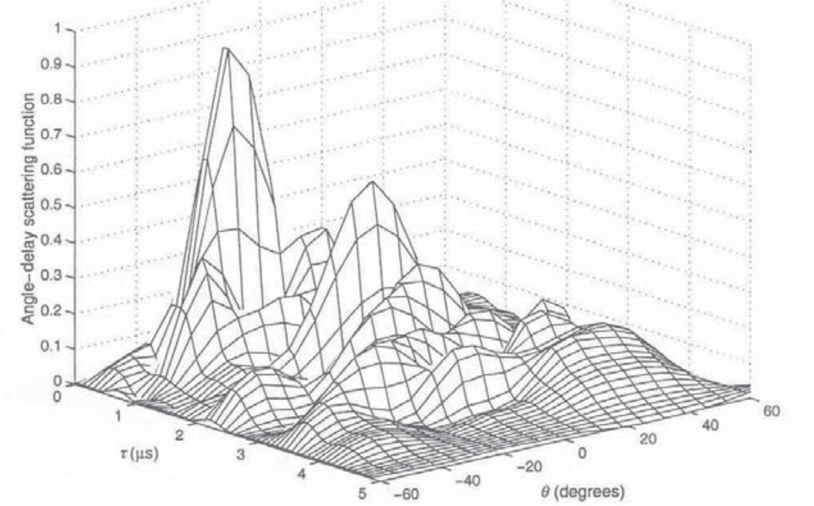


Figure 2.10: The angle-delay scattering function represents the average power in the angle-delay dimensions.

[Paulraj; fig 2-10]

Channel as a space-time random field (cont'd)

$S_c(\theta)$ ... **angle power spectrum**: average power vs. angle of arrival

- Characterization by parameters:
- RMS angle spread: $\theta_{rms} = \sqrt{\overline{\theta^2} - \overline{\theta}^2}$
  second centralized moment of normalized angle power spectrum
- mean and mean squared angle spread

$$\overline{\theta} = \frac{\int \theta S_c(\theta) d\theta}{\int S_c(\theta) d\theta} \qquad \overline{\theta^2} = \frac{\int \theta^2 S_c(\theta) d\theta}{\int S_c(\theta) d\theta}$$

- Coherence **distance** $D_c \propto \frac{1}{\theta_{rms}}$

3-3 Classification of Small-Scale Fading

- Compares system and channel parameters

Classification	w.r.t. **symbol period** T_s	w.r.t. **bandwidth** $B_s \propto 1/T_s$
dispersiveness		
flat fading	$T_s \gg \tau_{rms}$	$B_s \ll B_c$
frequency selective	$T_s < \tau_{rms}$	$B_s > B_c$
time variations		
slow fading	$T_s \ll T_c$	$B_s \gg \nu_{rms}$
fast fading	$T_s > T_c$	$B_s < \nu_{rms}$

Classification example – GSM

- **Key air-interface parameters:**
  - Carrier frequency ... 900 MHz, 1.8 GHz
  - Bandwidth ... 200 kHz
  - Frame; slot length ... ∼4.6 ms; ∼0.6 ms
- Time dispersiveness
  - τ_{rms} (typical urban and suburban) ... 100–800 ns
  - corresponds to $B_c \approx$ 1.2–10 MHz
  - **flat fading**
- Time variability
  - assume $v = 50$ m/s at $f_c = 1$ GHz $\rightarrow \nu_{\max} = 167$ Hz
  - corresponds to $T_c \approx$ 6 ms
  - **Time-invariant during slot**

Classification example – WLAN

- **Key air-interface parameters:**
  - Carrier frequency ... 2.4; 5 GHz
  - Bandwidth ... 17 MHz (sampling f: $f_s = 20$ MHz)
  - OFDM symbol length ... 4 μs
- Time dispersiveness
  - τ_{rms} (indoor) ... 10–300 ns
  - corresponds to $B_c \approx$ 3–100 MHz
  - **frequency selective**
- Time variability
  - assume $v = 2$ m/s at $f_c = 5$ GHz $\rightarrow \nu_{\max} = 33$ Hz
  - corresponds to $T_c \approx$ 30 ms; several 1000 symbols
  - **Time-invariant during packet**