Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2008/12/15

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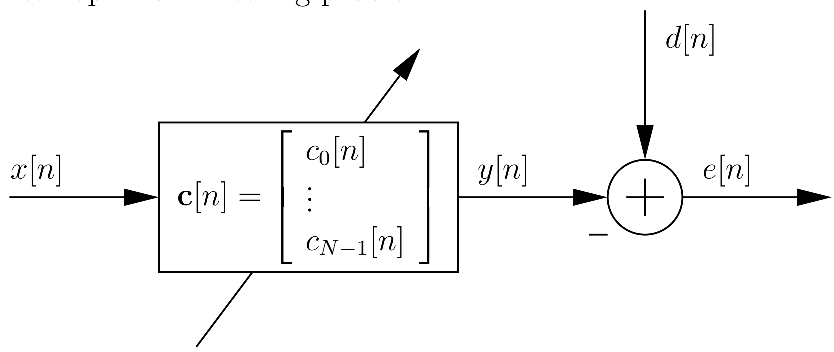
Exam duration: 180 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (32 Points)

Consider a linear optimum filtering problem:



The *Leaky Gradient Search Method*

$$\mathbf{c}[n+1] = \beta\,\mathbf{c}[n] + \mu\,(\mathbf{p} - \mathbf{R}_x\mathbf{c}[n])$$

is used to adapt the coefficients of the adaptive filter. $\mathbf{R}_x$ is the auto-correlation matrix of the tap-input vector $\mathbf{x}[n] = \big[x[n], \ldots, x[n-N+1]\big]^{\mathsf{T}}$, and $\mathbf{p}$ is the cross-correlation vector between the desired signal $d[n]$ and the tap-input vector.

(a) Assume convergence of the algorithm. Where does the algorithm converge to?

(b) Derive a condition on the step size μ and on the leakage parameter β that ensures convergence, and specify the possible range for μ given β.

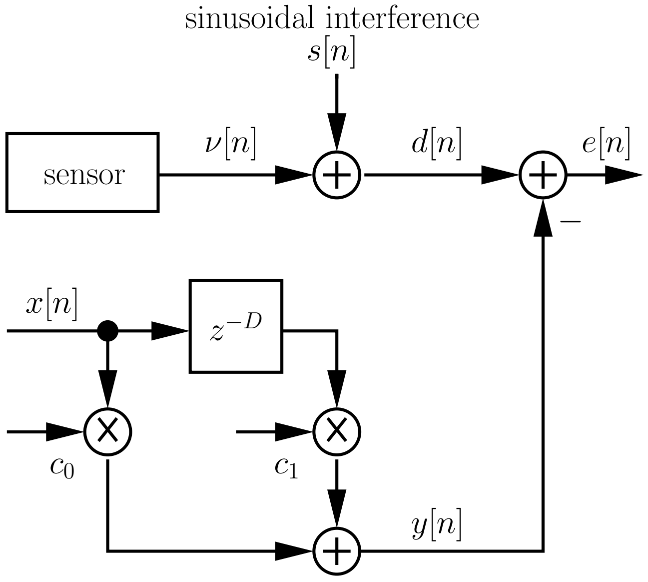
(c) The *Principle of Orthogonality* says:

> "The estimate $y[n]$ of the desired signal $d[n]$ is optimal in the sense of a minimum mean squared error if, and only if, the error $e[n]$ is orthogonal to the input $x[n-m]$ for $m = 0 \ldots N-1$".

(i) Prove this principle. (ii) Show whether this principle applies for the solution found by the leaky gradient search method, or not.

Problem 2 (35 Points)

Consider the following adaptive *sinusoidal-interference canceler*:



The input signal is given by $x[n] = \cos(2\pi \frac{f_{ac}}{f_s} n + \varphi)$. The frequency $f_{ac} = 50$ Hz, and the sampling frequency $f_s = 12000$ Hz. The phase offset φ is considered as a random variable uniformly distributed over $(-\pi, \pi]$. The sensor signal $\nu[n]$ is unknown, but it is uncorrelated with $x[n]$. The sinusoidal interference is $s[n] = \frac{1}{2}\cos(2\pi \frac{f_{ac}}{f_s} n + \varphi - \frac{\pi}{4})$. The delay in the adaptive filter is $D \in \mathbb{N}$.

(a) Determine the auto-correlation matrix $\mathbf{R}_x$ of the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$. (Do not just give the result, but show how to find the result by integration.)

(b) Determine the eigenvalues λ_i of $\mathbf{R}_x$, and give the condition number $\kappa = \frac{\lambda_{max}}{\lambda_{min}}$. Find the (smallest) delay $D \in \mathbb{N}$ that minimizes κ. Explain, what theoretical/practical advantage such a selection of D may bring.

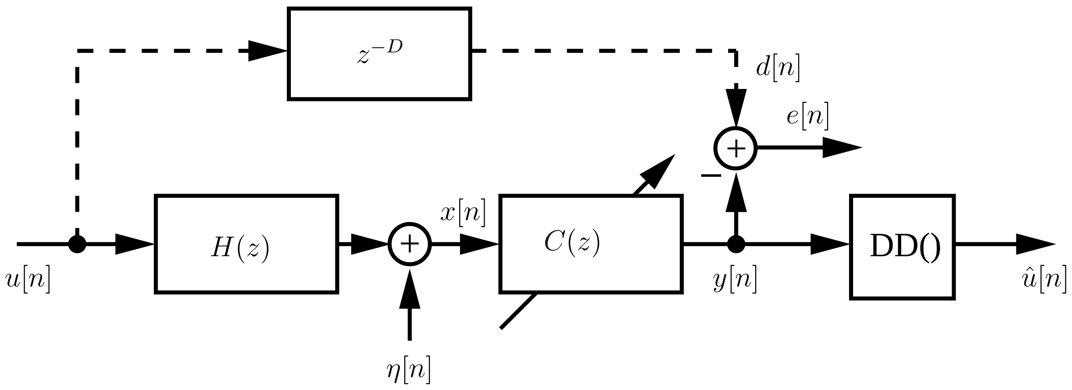
(c) Determine the cross-correlation vector $\mathbf{p}$ between the signal $d[n]$ and the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$.

(d) Determine the Wiener-Hopf solution for the two coefficients c_0 and c_1. (Take the D found before or use a general D if you have not found one before.)

(e) Show whether your found solution for c_0 and c_1 achieves perfect cancelation $e[n] \stackrel{?}{=} \nu[n]$, or not.

Problem 3 (33 Points)

Consider the following channel-equalization problem:



The symbols to be transmitted are $u[n] \in \{-1, +1\}$, originate from a stationary process, have zero mean, and are uncorrelated with each other $E\{u[n]u[n-m]\} = 0,\ \forall m \neq 0$. The channel noise $\eta[n]$ is white, stationary, and uncorrelated with the data $E\{\eta[n]u[n-m]\} = 0,\ \forall m$. $H(z)$ is the discrete-time FIR model of the communication channel, $C(z)$ is an FIR equalizer, and the decision device (DD) is

$$\mathrm{DD}(y[n]) = \begin{cases} 1, & y[n] \geq 0 \\ -1, & y[n] < 0 \end{cases} .$$

(a) Using vector/matrix notation, derive the equation to obtain a *minimum mean squared error (MinMSE) equalizer* with N coefficients for a general FIR channel model and a given data variance σ_u^2 and a given channel noise variance σ_η^2. Define all used vectors and matrices.

(b) Let the discrete-time FIR model of the communication channel be

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} = 1 + 1/2\, z^{-1} + 1/3\, z^{-2} + 1/4\, z^{-3}.$$

Without an equalizer, what delay D should be used to minimize the intersymbol interference (ISI)? Derive whether the channel's eye is open or closed.

(c) For the above given channel, a delay of $D = 0$, and a noise variance of $\sigma_\eta^2 = 1$, compute the coefficients of a MinMSE equalizer with $N = 2$ coefficients.

(d) Derive wheteher the eye of the cascade of the given channel and the obtained 2-coefficient MinMSE equalizer is open or closed.