Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2010/3/15

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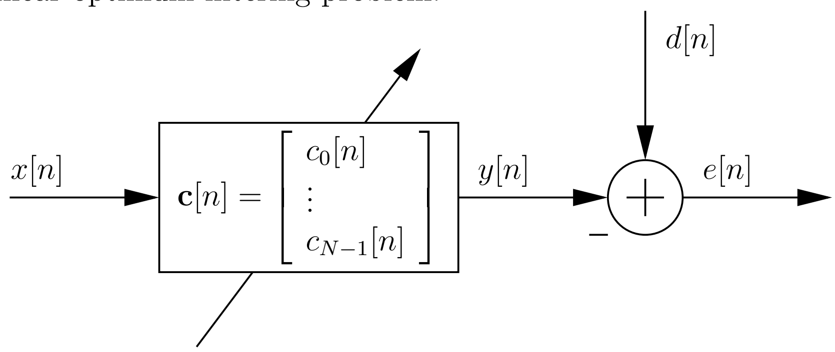
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (32 Points)

Consider a linear optimum filtering problem:



The *Leaky Gradient Search Method*

$$\mathbf{c}[n+1] = \beta\, \mathbf{c}[n] + \mu\, (\mathbf{p} - \mathbf{R}_x \mathbf{c}[n])$$

is used to adapt the coefficients of the adaptive filter. $\mathbf{R}_x$ is the auto-correlation matrix of the tap-input vector $\mathbf{x}[n] = \big[x[n], \ldots, x[n-N+1]\big]^\mathsf{T}$, and $\mathbf{p}$ is the cross-correlation vector between the desired signal $d[n]$ and the tap-input vector.

(a) Assume convergence of the algorithm. Where does the algorithm converge to?

(b) Derive a condition on the step size μ and on the leakage parameter β that ensures convergence, and specify the possible range for μ given β.

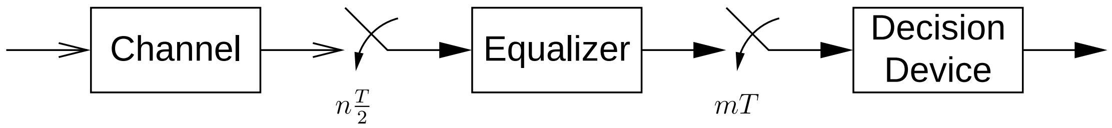
(c) The *Principle of Orthogonality* says:

> "The estimate $y[n]$ of the desired signal $d[n]$ is optimal in the sense of a minimum mean squared error if, and only if, the error $e[n]$ is orthogonal to the input $x[n-m]$ for $m = 0 \ldots N-1$".

(i) Prove this principle. (ii) Show whether this principle applies for the solution found by the leaky gradient search method, or not.

Problem 2 (33 Points)

Consider the $\frac{T}{2}$-*fractionally-spaced equalizer* illustrated below, where the incoming signal is sampled at a rate twice the symbol rate. The decision device is synchronized with the *even-indexed* samples.



The discrete-time FIR model of the communication channel is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} = 1 + 2\,z^{-1} + 3/4\,z^{-2} + 1/4\,z^{-3},$$

where the unit delay z^{-1} corresponds to $\frac{T}{2}$.

(a) Design a *least-squares* equalizer of order 1 (2 coefficients)

$$C(z) = c_0 + c_1 z^{-1}$$

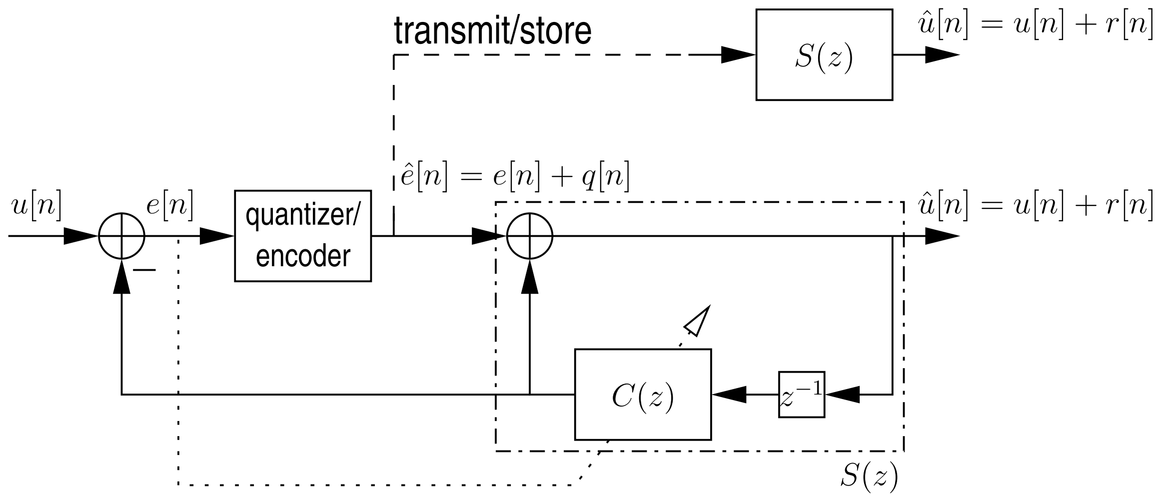
so that the cascade of channel and equalizer approximates $H(z)C(z) \approx 1$ (delay-free, perfect equalization).

(b) Design a *least-squares* equalizer with 2 coefficients that approximates $H(z)C(z) \approx z^{-2}$ (perfect equalization with delay of one symbol).

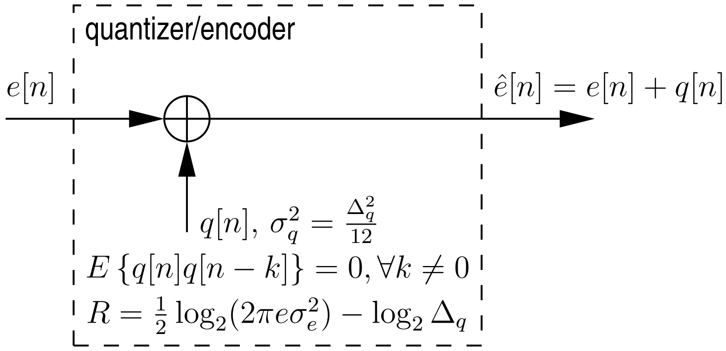
(c) Calculate the noise gains of both solutions. Which solution would you prefer and why?

Problem 3 (35 Points)

Consider the closed-loop predictor in the figure below. Such a predictor predicts from the reconstructed samples instead of the original samples by incorporating the synthesis filter $S(z)$ into the encoder. From the given signal flow graph it can be seen that the reconstruction error at the decoder output is the same as the error introduced by the quantizer $r[n] = q[n]$.



For our calculations, we use the following quantizer/encoder model, and we **assume** $u[n]$ **and** $q[n]$ **to be uncorrelated** with each other. Δ_q is the quantization cell size, and R is the average bit rate in bits/sample of the lossless encoder-decoder pair (not shown in the figures).



(a) For a process $u[n]$ with a general auto-correlation sequence $r_{uu}[k]$ and a given quantization noise variance σ_q^2, derive the design equation for an Nth-order (i.e., N coefficients in $C(z)$) MSE-optimal closed-loop predictor in matrix/vector notation. Hint: to start with, reduce the signal flow graph to the essentials (inputs $u[n]$ and $q[n]$ $\Rightarrow$ output $e[n]$).

(b) Let $u[n]$ be samples of an AR process with the following generator difference equation

$$u[n] = v[n] + u[n-1] - 0.25\, u[n-2].$$

$v[n]$ are samples of white, zero-mean, Gaussian noise. The variance of the AR process $u[n]$ is known as $\sigma_u^2 = 1$. Derive the first three samples of the autocorrelation sequence $r_{uu}[k]$, $k = 0, 1, 2$. Also, compute the variance of the white-noise input, σ_v^2

(c) For the AR process $u[n]$ given in **(b)** and a quantization cell size $\Delta_q = 0.5$, calculate the coefficients of a 2nd-order MSE-optimal closed-loop predictor (2 coefficients: c_0 and c_1). Also, calculate the bit rate R of the predictive encoder.