Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2011/3/25

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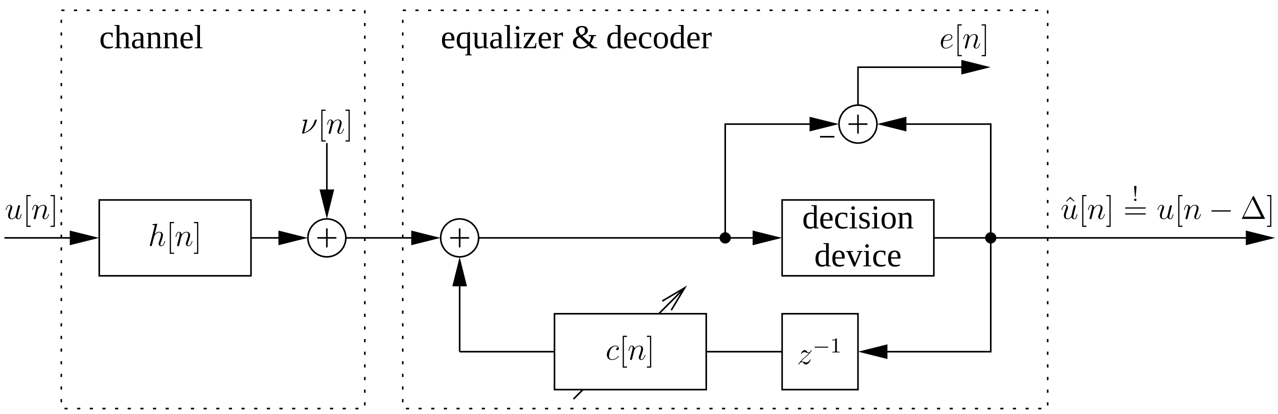
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (33 Points)

Consider the given data transmission scenario over a channel with impulse response $h[n]$ and additive noise $\nu[n]$. We use a decision-feedback equalizer (feedback only) in decision-directed operation mode.



The symbols to be transmitted are $u[n] \in \{-1, 1\}$, which occur with the same probability. Consecutive symbols can be assumed to be uncorrelated. Also the channel noise $\nu[n]$ is uncorrelated with the data $u[n-k]$. The decision device returns the sign of its input signal (zero is regarded as a positive number). The channel is given as

$$h[n] = -0.3\delta[n] + 1\delta[n-1] - 0.2\delta[n-2] + 0.6\delta[n-3].$$

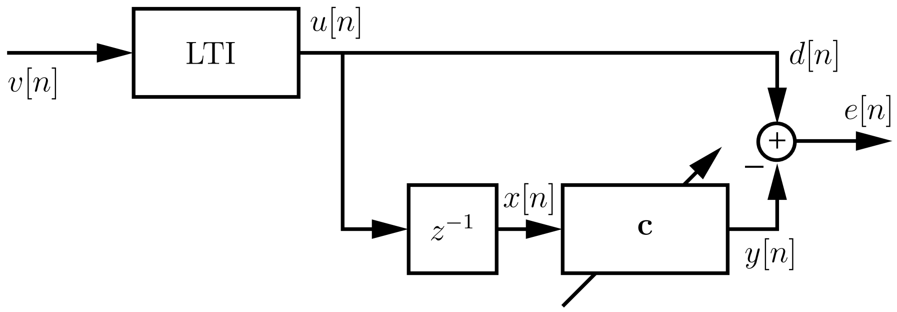
(a) Consider a transmission over the given channel *without* an equalizer (decision device only). What is the ideal delay Δ that minimizes ISI (Inter Symbol Interference)? For this delay Δ, calculate the worst-case ISI and answer whether the channel's eye is open or not.

(b) Find the optimum solution for an N-coefficient ($N > 0$) decision-feedback equalizer in the sense of a minimum mean-squared error (MinMSE) for a general delay Δ and a general channel impulse response.

(c) What is the best delay Δ for a decision-feedback equalizer with 2 coefficients $\mathbf{c} = [c_0, c_1]^T$ to minimize ISI? For this delay, give the optimum equalizer coefficients. Calculate the worst-case ISI and answer whether the equalizer can open the channel's eye or not.

Problem 2 (32 Points)

A prediction-error filter can be considered as a blind equalizer for AR processes.



The first 3 samples of the auto-correlation sequence of $u[n]$ are given as $r_{uu}[0] = 1.0$, $r_{uu}[1] = 0.\bar{5} = \frac{5}{9}$, and $r_{uu}[2] = 0.3\bar{7} = \frac{17}{45}$ where the bars denote repeating decimals.

(a) For a predictor with $N = 1$ coefficient, find the MSE-optimal coefficient of the adaptive filter.

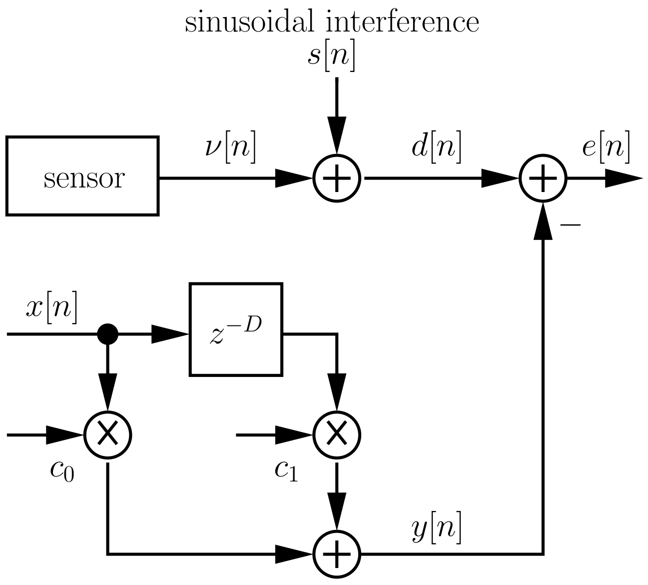
(b) For a predictor with $N = 2$ coefficients, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(c) Assume, $u[n]$ is an AR process of 2nd order. Write down the process-generator difference equation and calculate the variance σ_v^2 of the white-noise input $v[n]$.

(d) Next, assume that the coefficient vector $\mathbf{c}$ ($N = 2$) is updated constantly, such that it is MSE-optimal. Would the coefficients change if $v[n]$ is not white? How would they change? What would happen to the coefficients if $v[n]$ is white, but the LTI system is an FIR filter?

Problem 3 (34 Points)

Consider the following adaptive *sinusoidal-interference canceler*:



The input signal is given by

$$x[n] = \cos\left(2\pi \frac{f_{ac}}{f_s} n + \varphi\right).$$

The frequency $f_{ac} = 50$ Hz, and the sampling frequency $f_s = 8000$ Hz. The phase offset φ is considered as a random variable uniformly distributed over $(-\pi, \pi]$. The delay in the adaptive transversal filter is $D \in \mathbb{N}$. The sensor signal $\nu[n]$ is unknown, but it is uncorrelated with $x[n]$. The sinusoidal interference is

$$s[n] = \frac{1}{2} \cos\left(2\pi \frac{f_{ac}}{f_s} n + \varphi - \frac{\pi}{4}\right).$$

(a) Determine the auto-correlation sequence $r_{xx}[k]$ of the above given $x[n]$ as well as the auto-correlation matrix $\mathbf{R_{xx}}$ of the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$. (Do not just give the final result for $r_{xx}[k]$, but carefully show how to find the result by means of integration.)

(b) Determine the eigenvalues λ_i of $\mathbf{R_{xx}}$, and give the condition number $\kappa = \frac{\lambda_{max}}{\lambda_{min}}$. Find the (smallest) delay $D \in \mathbb{N}$ that minimizes κ. Explain, what theoretical and practical advantages such a selection of D can bring.

(c) Determine the cross-correlation vector $\mathbf{p}$ between the signal $d[n]$ and the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$.

(d) Determine the Wiener-Hopf solution for the two coefficients c_0 and c_1. (Take the D found before or use a general D if you have not found one before.)