Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2012/7/2

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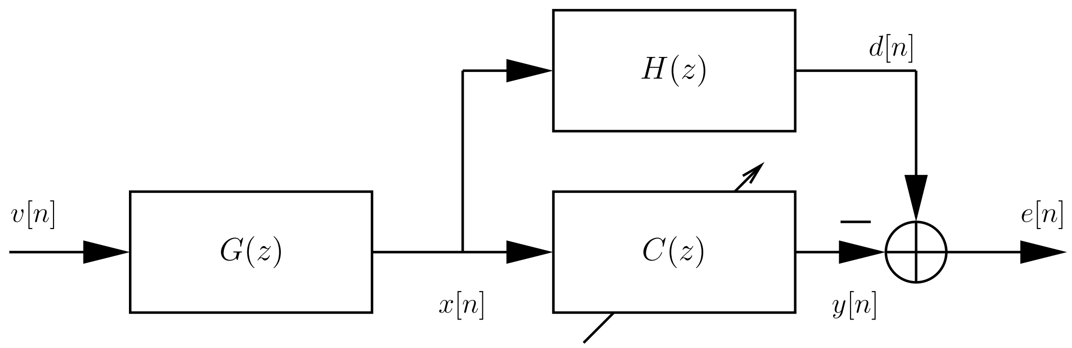
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Best wishes for a successful exam!

Problem 1 (32 Points)



The white-noise signal $v[n]$ with zero mean and variance $\sigma_v^2 = 1$ is filtered by the FIR filter $G(z)$ with impulse response

$$g[n] = \delta[n] - 0.4\delta[n-1].$$

The resulting signal $x[n]$ is used to identify the system $H(z)$ with impulse response

$$h[n] = 0.9\delta[n] + 0.3\delta[n-1]$$

by the adaptive filter $C(z)$, which has order 1, i.e., $N = 2$ coefficients.

(a) Determine the autocorrelation matrix $\mathbf{R}_{xx} = E\left\{\mathbf{x}[n]\mathbf{x}[n]^T\right\}$ of the signal $x[n]$ (Hint: Use the autocorrelation function $r_{xx}[k] = E\{x[n]x[n-k]\}$ to determine the entries of $\mathbf{R}_{xx}$).

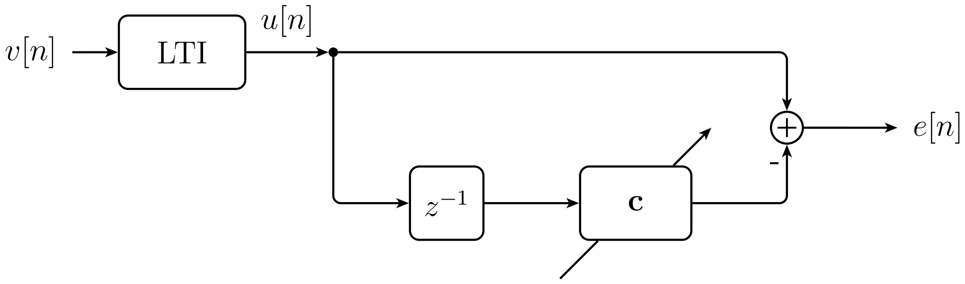
(b) Determine the crosscorrelation vector $\mathbf{p} = E\{d[n]\mathbf{x}[n]\}$.

(c) Determine the Wiener solution $\mathbf{c}_{\text{opt}}$ for the filter $C(z)$.

(d) Can you determine the Wiener solution $\mathbf{c}_{\text{opt}}$ without explicitly computing the entries of the autocorrelation matrix $\mathbf{R}_{xx}$ and the cross-correlation vector $\mathbf{p}$? If yes, show how. If no, explain why not.

Problem 2 (34 Points)

A prediction-error filter can be considered as a blind equalizer for AR processes.



The first 3 samples of the autocorrelation sequence of $u[n]$ are given as $r_{uu}[0] = 1.0$, $r_{uu}[1] = 0.\bar{5} = \frac{5}{9}$, and $r_{uu}[2] = \frac{17}{45} = 0.3\bar{7}$, where the bar denotes repeating decimals.

(a) For a general autocorrelation sequence and a general predictor order N, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(b) For a predictor with $N = 2$ coefficients, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(c) Assume that $u[n]$ is a second-order AR process. Write down the process-generator difference equation and calculate the variance σ_v^2 of the white-noise input $v[n]$.

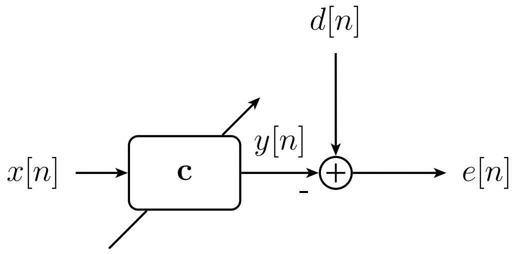
(d) Given the difference equation from **(c)**, compute the fourth sample of the autocorrelation sequence, $r_{uu}[3]$.

(e) Is the prediction error filter $P(z) = \frac{E(z)}{U(z)}$ a low-pass or a high-pass filter? Justify your answer! **Hint:** Plot the pole/zero diagram of $P(z)$, and, if necessary, sketch the magnitude response.

(f) Given the difference equation from **(c)** and a second-order predictor filter ($N = 2$), write down the autocorrelation sequence of the prediction error signal $e[n]$.

Problem 3 (34 Points)

Consider the following linear filtering problem:



The auto-correlation sequence of $x[n]$ and the cross-correlations between $x[n-k]$ and $d[n]$ are assumed to be known (we can build the auto-correlation matrix $\mathbf{R}_{xx}$ and the cross-correlation vector $\mathbf{p}$). The following adaptation rule (*coefficient-leakage gradient search*) is used to adapt the coefficient vector $\mathbf{c}[n]$:

$$\mathbf{c}[n] = (1 - \mu\alpha)\mathbf{c}[n-1] + \mu(\mathbf{p} - \mathbf{R}_{xx}\mathbf{c}[n-1])$$

where α is the leakage parameter ($0 < \alpha \ll 1$) and μ is the step-size parameter.

(a) Assume convergence. Where does this algorithm converge to?

(b) Transform the given adaptation rule in a way such that it adapts the misalignment vector (defined as $\mathbf{v}[n] = \mathbf{c}[n] - \mathbf{c}_\infty$). Hint: substitute for $\mathbf{p}$ using the result from **(a)**.

(c) Decouple the adaptation rule of **(b)** into a set of scalar adaptation expressions by using a unitary transform $\mathbf{q}[n] = \mathbf{Q}^H\mathbf{v}[n]$.

(d) Write the decoupled equation from **(c)** as individual exponential sequences and derive a condition on μ to ensure convergence towards $\mathbf{c}_\infty$, i.e., specify the range $\mu_{min} < \mu < \mu_{max}$ (assume α to be given).

(e) Assume we use a fixed step-size parameter μ. Compute the worst-case convergence time constant τ_{max}.