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Exam *Adaptive Systems* on 2007/7/3

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Exam duration: 120 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam! Good luck!

Problem 1 (33 Points)

Consider the following identification problem:

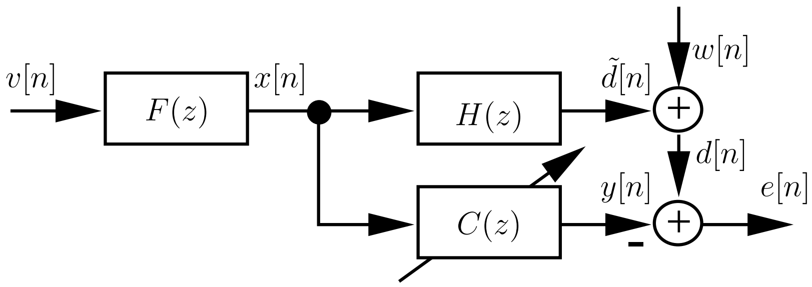


Figure 1: Adaptive system with correlated and uncorrelated white noise.

The white Gaussian noise process $v[n]$ with zero mean and variance $\sigma_v^2 = 1$ is filtered by the FIR filter $F(z)$, which has an impulse response of $f[n] = -0.25\delta[n] + 1\delta[n-1]$ (moving average (MA) process). The resulting output $x[n]$ is used to identify the system $H(z)$, which has an impulse response of $h[n] = 0.25\delta[n-1]$, by the adaptive filter $C(z)$ with 3 coefficients, i.e., $\mathbf{c} = [c_0, c_1, c_2]^T$. The desired output $d[n]$ is corrupted by the additive white Gaussian noise $w[n]$ with zero mean and variance $\sigma_w^2 = 0.05$.

(a) Determine **the values** of the auto-correlation matrix $\mathbf{R} = E\left\{\mathbf{x}[n]\mathbf{x}[n]^T\right\}$.

(b) Determine **the values** of the cross-correlation vector $\mathbf{p} = E\{d[n]\mathbf{x}[n]\}$.

(c) Determine the optimal coefficient vector $\mathbf{c}_{opt}$ for the filter $C(z)$ in the sense of the Wiener solution.

(d) Determine the minimum mean-squared error (MMSE) J_{min} of the system.

Problem 2 (33 Points)

The *Gradient Method*

$$\mathbf{c}[n] = \mathbf{c}[n-1] + \mu \left(\mathbf{p} - \mathbf{R}\mathbf{c}[n-1]\right)$$

should be used to adapt the coefficients of an adaptive filter.

(a) Simplify the adaptation algorithm by substituting $\mathbf{p}$. Additionally, introduce the misalignment vector $\mathbf{v}[n]$.

(b) Assume that the autocorrelation function is given by $r_{xx}[k] = -0.5\delta[k+1]+\delta[k]-0.5\delta[k-1]$. Determine the autocorrelation matrix $\mathbf{R}_{xx}$ of the system.

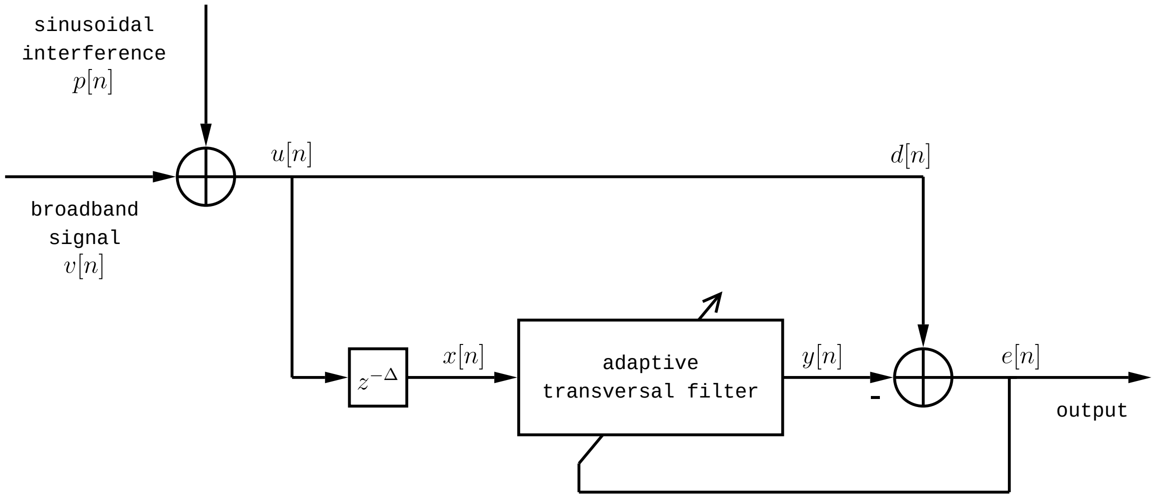
(c) The eigenvectors of the autocorrelation matrix $\mathbf{R}_{xx}$ are given by $\mathbf{q}_1 = \left[-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]^T$ and $\mathbf{q}_2 = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]^T$. Determine the eigenvalues λ_1 and λ_2 of the autocorrelation matrix $\mathbf{R}_{xx}$.

(d) Determine the stability bounds (the concrete values) of the stepsize parameter μ.

(e) Assume that the stepsize parameter is $\mu = 0.1$. Determine the convergence time constants (the concrete values) τ_1 and τ_2.

Problem 3 (34 Points)

Consider the following adaptive system to attenuate a sinusoidal interference:



The broadband signal $v[n]$ has the following autocorrelation sequence

$$r_{vv}[k] = \frac{1}{2}\delta[k+1] + \delta[k] + \frac{1}{2}\delta[k-1]$$

and the statistically independent interference is known as

$$p[n] = \sin\left(-\frac{\pi}{3}n + \varphi\right)$$

with φ a random variable uniformly distributed between $-\pi$ and π. The adaptive transversal filter has 2 coefficients (c_0 and c_1) and the predictor delay is $\Delta = 3$, i.e., $x[n] = u[n-3]$.

(a) Determine $r_{uu}[k]$, i.e., the autocorrelation sequence of $u[n]$ (Hint: You may find the relation $\sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$ useful).

(b) Calculate the optimum coefficients $\mathbf{c}_{opt}$ of the adaptive filter in the sense of a minimum mean-square error (Hint: $\mathbf{R} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\mathbf{R}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$).

(c) Determine and sketch the pole/zero diagram of the prediction error filter $H_e(z)$ (i.e., the system with input $u[n]$ and output $e[n]$) when the filter operates in its optimum.